Chapter 640

Confidence Intervals for One Standard Deviation Using Standard Deviation

Introduction

This routine calculates the sample size necessary to achieve a specified interval width or distance from the standard deviation to the confidence limit at a stated confidence level for a confidence interval about the standard deviation when the underlying data distribution is normal.

Caution: This procedure assumes that the standard deviation of the future sample will be the same as the standard deviation that is specified. If the standard deviation to be used in the procedure is estimated from a previous sample or represents the population standard deviation, the Confidence Intervals for One Standard Deviation with Tolerance Probability procedure should be considered. That procedure controls the probability that the width or distance from the standard deviation to the confidence limit will be less than or equal to the value specified. The Confidence Intervals for One Standard Deviation using Relative Error controls the width or distance from the standard deviation to the limit by controlling the distance as a percent of the true standard deviation.

Technical Details

For a single standard deviation from a normal distribution with unknown mean, a two-sided, 100(1 – α)% confidence interval is calculated by

$$\left[s\left\{\frac{n-1}{\chi^2_{1-\alpha/2,n-1}}\right\}^{1/2}, s\left\{\frac{n-1}{\chi^2_{\alpha/2,n-1}}\right\}^{1/2} \right]$$

A one-sided 100(1 – α)% upper confidence limit is calculated by

$$s\left\{\frac{n-1}{\chi^2_{\alpha,n-1}}\right\}^{1/2}$$

Similarly, the one-sided 100(1 – α)% lower confidence limit is

$$s\left\{\frac{n-1}{\chi^2_{1-\alpha,n-1}}\right\}^{1/2}$$

For two-sided intervals, the distance from the standard deviation to each of the limits is different. Thus, instead of specifying the distance to the limits we specify the width of the interval, W.

The basic equation for determining sample size for a two-sided interval when *W* has been specified is

$$W = s\left\{\frac{n-1}{\chi^2_{\alpha/2,n-1}}\right\}^{1/2} - s\left\{\frac{n-1}{\chi^2_{1-\alpha/2,n-1}}\right\}^{1/2}$$

For one-sided intervals, the distance from the standard deviation to limits, *D*, is specified.

The basic equation for determining sample size for a one-sided upper limit when *D* has been specified is

$$D = s\left\{\frac{n-1}{\chi^2_{\alpha/2,n-1}}\right\}^{1/2} - s$$

The basic equation for determining sample size for a one-sided lower limit when *D* has been specified is

$$D = s - s\left\{\frac{n-1}{\chi^2_{1-\alpha/2,n-1}}\right\}^{1/2}$$

These equations can be solved for any of the unknown quantities in terms of the others.

Confidence Level

The confidence level, 1 – α, has the following interpretation. If thousands of samples of *n* items are drawn from a population using simple random sampling and a confidence interval is calculated for each sample, the proportion of those intervals that will include the true population standard deviation is 1 – α.

Example 1 – Calculating Sample Size

Suppose a study is planned in which the researcher wishes to construct a two-sided 95% confidence interval for the standard deviation such that the width of the interval is no wider than 20 units. The confidence level is set at 0.95, but 0.99 is included for comparative purposes. The standard deviation estimate, based on the range of data values, is 34. Instead of examining only the interval width of 20, a series of widths from 16 to 24 will also be considered.

The goal is to determine the necessary sample size.

Setup

If the procedure window is not already open, use the PASS Home window to open it. The parameters for this example are listed below and are stored in the **Example 1** settings file. To load these settings to the procedure window, click **Open Example Settings File** in the Help Center or File menu.

Design Tab	
Solve For	**Sample Size**
Interval Type	**Two-Sided**
Confidence Level	**0.95 0.99**
Confidence Interval Width (Two-Sided)	**16 to 24 by 1**
S (Standard Deviation)	**34**

Output

Click the Calculate button to perform the calculations and generate the following output.

Numeric Reports

Numeric Results

Solve For: Sample Size
Interval Type: Two-Sided

Confidence Level	Sample Size N	Confidence Interval Width		Standard Deviation S	Confidence Interval Limits	
		Target	Actual		Lower	Upper
0.95	40	16	15.806	34	27.851	43.657
0.99	67	16	15.873	34	27.715	43.588
0.95	36	17	16.774	34	27.577	44.351
0.99	60	17	16.870	34	27.420	44.289
0.95	32	18	17.944	34	27.258	45.202
0.99	54	18	17.891	34	27.128	45.019
0.95	30	19	18.629	34	27.078	45.707
0.99	49	19	18.900	34	26.850	45.750
0.95	27	20	19.819	34	26.776	46.595
0.99	45	20	19.842	34	26.599	46.441
0.95	25	21	20.751	34	26.548	47.299
0.99	41	21	20.939	34	26.317	47.256
0.95	23	22	21.827	34	26.295	48.122
0.99	38	22	21.892	34	26.080	47.972
0.95	22	23	22.430	34	26.158	48.588
0.99	35	23	22.986	34	25.818	48.804
0.95	20	24	23.803	34	25.857	49.659
0.99	33	24	23.813	34	25.627	49.440

Confidence Level	The proportion of confidence intervals (constructed with this same confidence level, sample size, etc.) that would contain the population standard deviation.
N	The size of the sample drawn from the population.
Confidence Interval Width	The distance from the lower limit to the upper limit.
Target Width	The value of the width that is entered into the procedure.
Actual Width	The value of the width that is obtained from the procedure.
S	The sample standard deviation.
Lower Confidence Interval Limit	The lower limit of the confidence interval.
Upper Confidence Interval Limit	The upper limit of the confidence interval.

Summary Statements

A single-group design will be used to obtain a two-sided 95% confidence interval for a single standard deviation. The standard Chi-square-based formula will be used to calculate the confidence interval. The sample standard deviation is assumed to be 34. To produce a confidence interval with a width of no more than 16, 40 subjects will be needed.

Dropout-Inflated Sample Size

Dropout Rate	Sample Size N	Dropout-Inflated Enrollment Sample Size N'	Expected Number of Dropouts D
20%	40	50	10
20%	67	84	17
20%	36	45	9
20%	60	75	15
20%	32	40	8
20%	54	68	14
20%	30	38	8
20%	49	62	13
20%	27	34	7
20%	45	57	12
20%	25	32	7
20%	41	52	11
20%	23	29	6
20%	38	48	10
20%	22	28	6
20%	35	44	9
20%	20	25	5
20%	33	42	9

Dropout Rate	The percentage of subjects (or items) that are expected to be lost at random during the course of the study and for whom no response data will be collected (i.e., will be treated as "missing"). Abbreviated as DR.
N	The evaluable sample size at which the confidence interval is computed. If N subjects are evaluated out of the N' subjects that are enrolled in the study, the design will achieve the stated confidence interval.
N'	The total number of subjects that should be enrolled in the study in order to obtain N evaluable subjects, based on the assumed dropout rate. After solving for N, N' is calculated by inflating N using the formula N' = N / (1 - DR), with N' always rounded up. (See Julious, S.A. (2010) pages 52-53, or Chow, S.C., Shao, J., Wang, H., and Lokhnygina, Y. (2018) pages 32-33.)
D	The expected number of dropouts. D = N' - N.

Dropout Summary Statements

Anticipating a 20% dropout rate, 50 subjects should be enrolled to obtain a final sample size of 40 subjects.

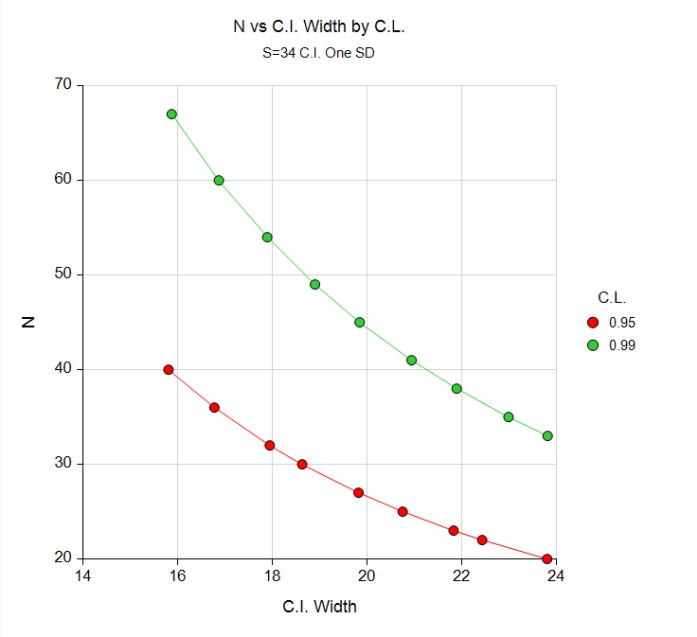
References

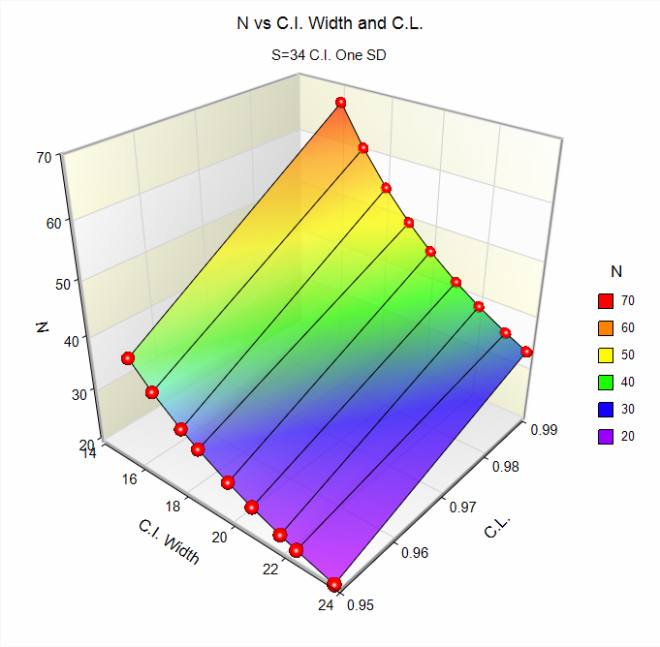
Hahn, G. J. and Meeker, W.Q. 1991. Statistical Intervals. John Wiley & Sons. New York.

This report shows the calculated sample size for each of the scenarios.

Plots Section

Plots





These plots show the sample size versus the confidence interval width for the two confidence levels.

Example 2 – Validation using Hahn and Meeker

Hahn and Meeker (1991) page 56 give an example of a calculation for a confidence interval on the standard deviation when the confidence level is 95%, the standard deviation is 1.31, and the interval width is 2.9795. The necessary sample size is 5.

Setup

If the procedure window is not already open, use the PASS Home window to open it. The parameters for this example are listed below and are stored in the **Example 2** settings file. To load these settings to the procedure window, click **Open Example Settings File** in the Help Center or File menu.

Design Tab	
Solve For	**Sample Size**
Interval Type	**Two-Sided**
Confidence Level	**0.95**
Confidence Interval Width (Two-Sided)	**2.9795**
S (Standard Deviation)	**1.31**

Output

Click the Calculate button to perform the calculations and generate the following output.

Numeric Results

Solve For: Sample Size
Interval Type: Two-Sided

Confidence Level	Sample Size N	Confidence Interval Width		Standard Deviation S	Confidence Interval Limits	
		Target	Actual		Lower	Upper
0.95	5	2.98	2.979	1.31	0.785	3.764

PASS also calculated the necessary sample size to be 5.