Chapter 656

Confidence Intervals for the Ratio of Two Variances using Variances

Introduction

This routine calculates the group sample sizes necessary to achieve a specified interval width or distance from the variance ratio to the confidence limit at a stated confidence level for a confidence interval about the variance ratio when the underlying data distribution is normal.

Caution: This procedure assumes that the variances of the future samples will be the same as the variances that are specified. The Confidence Intervals for the Ratio of Two Variances using Relative Error controls the width or distance from the variance ratio to the limits by controlling the width or distance as a percent of the true variance ratio.

Technical Details

For a ratio of two variances from normal distributions, a two-sided, 100(1 – α)% confidence interval is calculated by

$$\left[\frac{s_1^2}{s_2^2}\frac{1}{F_{\alpha/2,n_1-1,n_2-1}}, \frac{s_1^2}{s_2^2}F_{\alpha/2,n_2-1,n_1-1}\right]$$

A one-sided 100(1 – α)% upper confidence limit is calculated by

$$\frac{s_1^2}{s_2^2}F_{\alpha,n_2-1,n_1-1}$$

Similarly, the one-sided 100(1 – α)% lower confidence limit is

$$\frac{s_1^2}{s_2^2}\frac{1}{F_{\alpha,n_1-1,n_2-1}}$$

For two-sided intervals, the distance from the variance ratio to each of the limits is different. Thus, instead of specifying the distance to the limits we specify the width of the interval, *W*.

The basic equation for determining sample size for a two-sided interval when *W* has been specified is

$$W = \frac{s_1^2}{s_2^2}F_{\alpha/2,n_2-1,n_1-1} - \frac{s_1^2}{s_2^2}\frac{1}{F_{\alpha/2,n_1-1,n_2-1}}$$

For one-sided intervals, the distance from the variance ratio to limit, *D*, is specified.

The basic equation for determining sample size for a one-sided upper limit when *D* has been specified is

$$D = \frac{s_1^2}{s_2^2} F_{\alpha, n_2 - 1, n_1 - 1} - \frac{s_1^2}{s_2^2}$$

The basic equation for determining sample size for a one-sided lower limit when *D* has been specified is

$$D = \frac{s_1^2}{s_2^2} - \frac{s_1^2}{s_2^2} \frac{1}{F_{\alpha, n_1 - 1, n_2 - 1}}$$

These equations can be solved for any of the unknown quantities in terms of the others.

Confidence Level

The confidence level, 1 – α, has the following interpretation. If thousands of random samples of size n_1 and n_2 are drawn from populations 1 and 2, respectively, and a confidence interval for the variance ratio is calculated for each pair of samples, the proportion of those intervals that will include the true variance ratio is 1 – α.

Example 1 – Calculating Sample Size

Suppose a study is planned in which the researcher wishes to construct a two-sided 95% confidence interval for the variance ratio such that the width of the interval is no wider than 0.5. The confidence level is set at 0.95, but 0.99 is included for comparative purposes. The variance estimates to be used are 5 for Group 1, and 10 for Group 2. Instead of examining only the interval width of 0.5, a series of widths from 0.2 to 0.8 will also be considered.

The goal is to determine the necessary sample size.

Setup

If the procedure window is not already open, use the PASS Home window to open it. The parameters for this example are listed below and are stored in the **Example 1** settings file. To load these settings to the procedure window, click **Open Example Settings File** in the Help Center or File menu.

Design Tab

Solve For .... **Sample Size**
Interval Type .... **Two-Sided**
Confidence Level (1 - Alpha) **0.95 0.99**
Group Allocation .... **Equal (N1 = N2)**
Confidence Interval Width (Two-Sided) .... **0.2 to 0.8 by 0.1**
V1 (Variance Group 1).................................. **5**
V2 (Variance Group 2).................................. **10**

Output

Click the Calculate button to perform the calculations and generate the following output.

Numeric Reports

Numeric Results

Solve For: Sample Size
Interval Type: Two-Sided

	Sample Size			Confidence Interval Width		Variances		Confidence Interval Limits	
Confidence Level	N1	N2	N	Target	Actual	V1	V2	Lower	Upper
0.95	392	392	784	0.2	0.200	5	10	0.41	0.61
0.95	178	178	356	0.3	0.300	5	10	0.37	0.67
0.95	104	104	208	0.4	0.398	5	10	0.34	0.74
0.95	69	69	138	0.5	0.498	5	10	0.31	0.81
0.95	50	50	100	0.6	0.597	5	10	0.28	0.88
0.95	39	39	78	0.7	0.691	5	10	0.26	0.95
0.95	31	31	62	0.8	0.796	5	10	0.24	1.04
0.99	675	675	1350	0.2	0.200	5	10	0.41	0.61
0.99	307	307	614	0.3	0.300	5	10	0.37	0.67
0.99	178	178	356	0.4	0.399	5	10	0.34	0.74
0.99	118	118	236	0.5	0.498	5	10	0.31	0.81
0.99	85	85	170	0.6	0.598	5	10	0.28	0.88
0.99	65	65	130	0.7	0.699	5	10	0.26	0.96
0.99	53	53	106	0.8	0.791	5	10	0.24	1.03

Confidence Level	The proportion of confidence intervals (constructed with this same confidence level, sample size, etc.) that would contain the true ratio in population proportions.
N1 and N2	The number of items sampled from each population.
N	The total sample size. N = N1 + N2.
Confidence Interval Width	The distance from the lower limit to the upper limit.
Target Width	The value of the width that is entered into the procedure.
Actual Width	The value of the width that is obtained from the procedure.
V1	The assumed sample variance for Group 1, where the ratio is V1/V2.
V2	The assumed sample variance for Group 2.
Confidence Interval Limits	The lower and upper limits of the confidence interval for the true variance ratio (Population Variance 1 / Population Variance 2).

Summary Statements

A parallel two-group design will be used to obtain a two-sided 95% confidence interval for the ratio of two variances (Variance 1 / Variance 2). The standard F-distribution-based formula will be used to calculate the confidence interval. The sample variance for Group 1 (numerator) is assumed to be 5 and the sample variance for Group 2 (denominator) is assumed to be 10. To produce a confidence interval with a width of no more than 0.2, the number of subjects needed will be 392 in Group 1 and 392 in Group 2.

Dropout-Inflated Sample Size

	Sample Size			Dropout-Inflated Enrollment Sample Size			Expected Number of Dropouts		
Dropout Rate	**N1**	**N2**	**N**	**N1'**	**N2'**	**N'**	**D1**	**D2**	**D**
20%	392	392	784	490	490	980	98	98	196
20%	178	178	356	223	223	446	45	45	90
20%	104	104	208	130	130	260	26	26	52
20%	69	69	138	87	87	174	18	18	36
20%	50	50	100	63	63	126	13	13	26
20%	39	39	78	49	49	98	10	10	20
20%	31	31	62	39	39	78	8	8	16
20%	675	675	1350	844	844	1688	169	169	338
20%	307	307	614	384	384	768	77	77	154
20%	178	178	356	223	223	446	45	45	90
20%	118	118	236	148	148	296	30	30	60
20%	85	85	170	107	107	214	22	22	44
20%	65	65	130	82	82	164	17	17	34
20%	53	53	106	67	67	134	14	14	28

Dropout Rate The percentage of subjects (or items) that are expected to be lost at random during the course of the study and for whom no response data will be collected (i.e., will be treated as "missing"). Abbreviated as DR.

N1, N2, and N The evaluable sample sizes at which the confidence interval is computed. If N1 and N2 subjects are evaluated out of the N1' and N2' subjects that are enrolled in the study, the design will achieve the stated confidence interval.

N1', N2', and N' The number of subjects that should be enrolled in the study in order to obtain N1, N2, and N evaluable subjects, based on the assumed dropout rate. After solving for N1 and N2, N1' and N2' are calculated by inflating N1 and N2 using the formulas N1' = N1 / (1 - DR) and N2' = N2 / (1 - DR), with N1' and N2' always rounded up. (See Julious, S.A. (2010) pages 52-53, or Chow, S.C., Shao, J., Wang, H., and Lokhnygina, Y. (2018) pages 32-33.)

D1, D2, and D The expected number of dropouts. D1 = N1' - N1, D2 = N2' - N2, and D = D1 + D2.

Dropout Summary Statements

Anticipating a 20% dropout rate, 490 subjects should be enrolled in Group 1, and 490 in Group 2, to obtain final group sample sizes of 392 and 392, respectively.

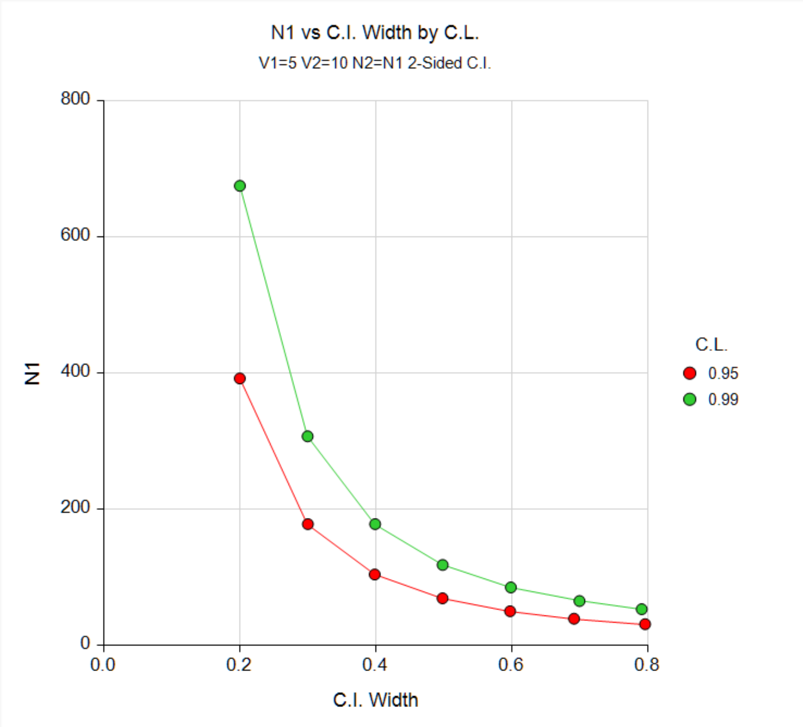
References

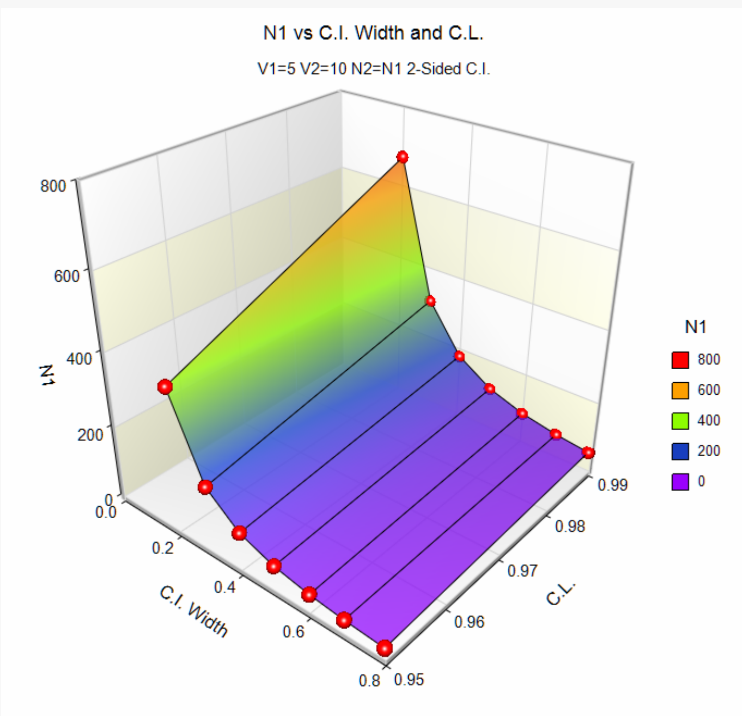
Ostle, B. and Malone, L.C. 1988. Statistics in Research. Iowa State University Press. Ames, Iowa.
Zar, Jerrold H. 1984. Biostatistical Analysis (Second Edition). Prentice-Hall. Englewood Cliffs, New Jersey.

This report shows the calculated sample size for each of the scenarios.

Plots Section

Plots





These plots show the group sample size versus the confidence interval width for the two confidence levels.

Example 2 – Validation using Sachs (1984)

Sachs (1984) page 261 gives an example of a calculation for a confidence interval for the variance ratio when the confidence level is 90%, the variances are 8 and 3, and the interval width is 4.56. The necessary sample size is 20 per group.

Setup

If the procedure window is not already open, use the PASS Home window to open it. The parameters for this example are listed below and are stored in the **Example 2** settings file. To load these settings to the procedure window, click **Open Example Settings File** in the Help Center or File menu.

Design Tab

Solve For ....**Sample Size**
Interval Type ....**Two-Sided**
Confidence Level (1 - Alpha) **0.90**
Group Allocation ....**Equal (N1 = N2)**
Confidence Interval Width (Two-Sided) **4.56**
V1 (Variance Group 1) **8**
V2 (Variance Group 2) **3**

Output

Click the Calculate button to perform the calculations and generate the following output.

Numeric Results

Solve For: Sample Size
Interval Type: Two-Sided

	Sample Size			Confidence Interval Width		Variances		Confidence Interval Limits	
Confidence Level	N1	N2	N	Target	Actual	V1	V2	Lower	Upper
0.9	20	20	40	4.56	4.552	8	3	1.23	5.78

PASS also calculated the necessary sample size to be 20 per group.