Chapter 295

Control Charts for Variability (Simulation)

Introduction

This procedure allows you to study the run length distribution of R, S, and S with probability limits process control charts using simulation. This procedure can also be used to study charts with two or more observations at each sample. The in-control standard deviation can be input directly, or a specified number of in-control preliminary samples can be simulated based on a user-determined in-control distribution. This procedure can also be used to determine the necessary sample size to obtain a given run length.

Simulation Details

If the in-control standard deviation is assumed to be known (standard), the steps to the simulation process are as follows (assume a sample consists of *n* observations).

1. An out-of-control sample of size *n* is generated from a normal distribution with the specified out-of-control standard deviation.
2. The standard deviation of the sample is calculated and compared to the control limits.
3. If the sample results in an out-of-control signal, the sample number is recorded as the run length for that simulation. If the sample does not result in an out-of-control signal, return to Step 1.
4. Steps 1 through 3 are repeated until the number of simulations (*Nsim*) is reached. The result is *Nsim* run lengths.
5. The average or median or specified percentile of the run length distribution is reported.

If the in-control standard deviation is to be simulated based on in-control preliminary samples (*NPrelim*), the steps to the simulation process are as follows (assume a sample consists of *n* observations).

1. *NPrelim* in-control samples of size *n* are generated from a normal distribution with specified standard deviation of the in-control distribution.
2. The in-control standard deviation is calculated based on the *NPrelim* simulated in-control samples.
3. An out-of-control sample of size *n* is generated from a normal distribution with the specified out-of-control standard deviation.
4. The standard deviation of the sample is calculated and compared to the control limits.
5. If the sample results in an out-of-control signal, the sample number is recorded as the run length for that simulation. If the sample does not result in an out-of-control signal, return to Step 3.
6. Steps 1 through 5 are repeated until the number of simulations (*Nsim*) is reached. The result is *Nsim* run lengths.
7. The average or median or specified percentile of the run length distribution is reported.

© NCSS, LLC. All Rights Reserved.

Formulas for Constructing Control Charts

The formula used for each set of control chart limits depends on whether the standard deviation is assumed to be a known, standard value, or estimated from in-control preliminary samples.

R Chart Limits

If the standard deviation is a known, standard value, the lower and upper control limits for the R chart are calculated (see Montgomery, 1991) using the formulae

$$LCL = d_2\sigma - z_{1-\alpha/2}d_3\sigma$$

$$UCL = d_2\sigma + z_{1-\alpha/2}d_3\sigma$$

where $d_2 = R / \sigma$ and $d_3 = \sigma_R / \sigma$.

If the limits are to be created from in-control preliminary samples, the lower and upper control limits for the R chart are calculated using the formulae

$$LCL = \bar{R} - z_{1-\alpha/2}d_3\frac{\bar{R}}{d_2}$$

$$UCL = \bar{R} + z_{1-\alpha/2}d_3\frac{\bar{R}}{d_2}$$

The statistic that is compared to the limits at each subgroup is the subgroup range

$$R_i = x_{(n)} - x_{(1)}$$

S Chart Limits

If the standard deviation is a known, standard value, the lower and upper control limits for the S chart are calculated (see Montgomery, 1991) using the formulae

$$LCL = c_4\sigma - z_{1-\alpha/2}\sigma\sqrt{1-c_4^2}$$

$$UCL = c_4\sigma + z_{1-\alpha/2}\sigma\sqrt{1-c_4^2}$$

where

$$c_4 = \sqrt{\frac{2}{n-1}}\frac{\Gamma\left(\frac{n}{2}\right)}{\Gamma\left(\frac{n-1}{2}\right)}$$

© NCSS, LLC. All Rights Reserved.

If the limits are to be created from in-control preliminary samples, the lower and upper control limits for the S chart are calculated using the formulae

$$LCL = \bar{S} - z_{1-\alpha/2}\frac{\bar{S}}{c_4}\sqrt{1-c_4^2}$$

$$UCL = \bar{S} + z_{1-\alpha/2}\frac{\bar{S}}{c_4}\sqrt{1-c_4^2}$$

The statistic that is compared to the limits at each subgroup is the subgroup standard deviation

$$s_i = \sqrt{\frac{\sum_{j=1}^{n}(x_{ij}-\bar{x}_i)^2}{n-1}}$$

S Chart with Probability Limits

If the standard deviation is a known, standard value, the lower and upper control limits for the S chart with probability limits are calculated (see Ryan, 1989) using the formulae

$$LCL = \sigma\sqrt{\frac{\chi^2_{\alpha/2}}{n-1}}$$

$$UCL = \sigma\sqrt{\frac{\chi^2_{1-\alpha/2}}{n-1}}$$

If the limits are to be created from in-control preliminary samples, the lower and upper control limits for the S chart with probability limits are calculated using the formulae

$$LCL = \frac{\bar{S}}{c_4}\sqrt{\frac{\chi^2_{\alpha/2}}{n-1}}$$

$$UCL = \frac{\bar{S}}{c_4}\sqrt{\frac{\chi^2_{1-\alpha/2}}{n-1}}$$

The statistic that is compared to the limits at each subgroup is the same as that of the previous S Chart, namely, the subgroup standard deviation

$$s_i = \sqrt{\frac{\sum_{j=1}^{n}(x_{ij}-\bar{x}_i)^2}{n-1}}$$

© NCSS, LLC. All Rights Reserved.

Example 1 – Run Length Distribution

A researcher wishes to examine the run length distribution for a process monitored by an R chart. Ten observations are to make up the sample examined at each hour. The in-control standard deviation is known to be 3.8. The researcher would like to see the run length distribution if the out-of-control standard deviation is 4.6. A probability of 0.00135 is to be used in the control chart for the boundaries.

Setup

If the procedure window is not already open, use the PASS Home window to open it. The parameters for this example are listed below and are stored in the **Example 1** settings file. To load these settings to the procedure window, click **Open Example Settings File** in the Help Center or File menu.

Design Tab	
Solve For	**Run Length Distribution**
Simulations	**5000**
Random Seed	**4822726** (for Reproducibility)
Maximum Run Length	**5000**
Run Length Summary 1	**ARL (Average Run Length)**
Run Length Summary 2	**MRL (Median Run Length)**
n (Sample Size)	**10**

Distributions, Tests Tab	
In-Control Distributions Specified By	**Standard Deviation Directly**
In-Control Standard Deviation	**3.8**
Out-of-Control Standard Deviation	**4.6**
R	**Checked**
S	**Unchecked**
S with Probability Limits	**Unchecked**
Two-Sided or Upper Limit Only	**Upper Limit Only**
Specify Probability Using	**Probability**
Probability	**0.00135**

© NCSS, LLC. All Rights Reserved.

Output

Click the Calculate button to perform the calculations and generate the following output.

Numeric Results for Control Charts for Process Variability

Solve For:	Run Length Distribution
Control Limits:	Determined by specifying a standard in-control standard deviation directly
Out-of-Control Distribution:	Normal with Out-of-Control SD

Chart Type	Average Run Length	Median Run Length	Sample Size n	In-Control SD	Outside Limits One-Sided Probability	Z-Multiplier	Control Limits Lower LCL	Control Limits Upper UCL	Out-of-Control SD
R	21.9	15	10	3.8	0.00135	3.205	0	20.7821	4.6

Simulations: 5000. Run Time: 1.67 seconds.
User-Entered Random Seed: 4822726

Average Run Length	The mean of the run lengths across all simulations.
Median Run Length	The median of the run lengths across all simulations.
n	The number of units measured in each sample.
In-Control SD	The assumed known standard deviation that is used in the calculation of limits.
One-Sided Probability	The probability of a sample range or standard deviation outside the control limits.
Z-Multiplier	The Z-multiplier which corresponds to the One-sided probability of a single sample range or standard deviation outside the control limits.
LCL and UCL	The lower and upper control chart limits, respectively.
Out-of-Control SD	The standard deviation of the normal distribution from which out-of-control samples are drawn.

Summary Statements

An R control chart will be used to monitor the process variability for an out-of-control signal. The chart is assumed to have lower and upper control limits of 0 and 20.7821, respectively. With samples of size 10 from an (out-of-control) Normal distribution with standard deviation 4.6, the average run length is 21.9 and the median run length is 15. These results are based on 5000 simulations (Monte Carlo samples).

Individual Summaries

Control Limits:	Determined by specifying a standard in-control standard deviation directly
Out-of-Control Distribution:	Normal with Out-of-Control SD
Simulations:	5000
Random Seed:	4822726 (User-Entered)

R Details

Chart Type	Average Run Length	Median Run Length	Sample Size n	In-Control SD	Outside Limits One-Sided Probability	Z-Multiplier	Control Limits Lower LCL	Control Limits Upper UCL	Out-of-Control SD
R	21.9	15	10	3.8	0.00135	3.205	0	20.7821	4.6

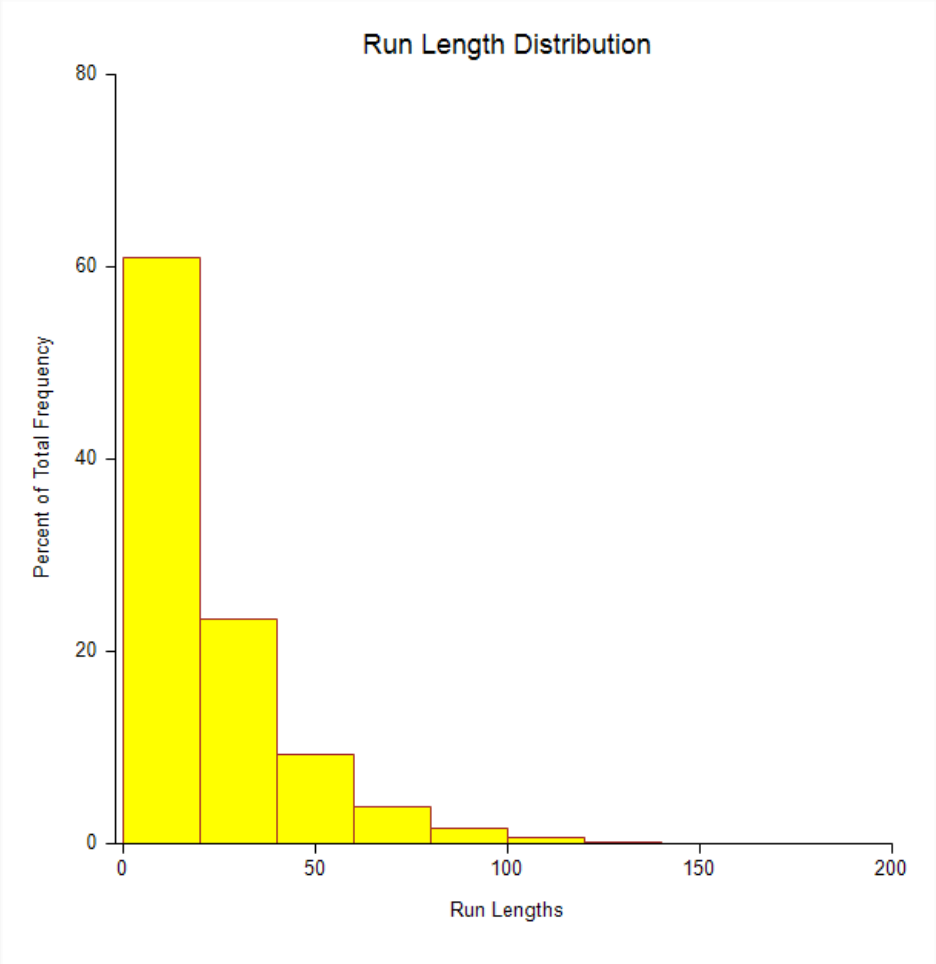
Average Run Length 95% CI: (21.3, 22.5)
Median Run Length 95% CI: (15, 16)

Average Run Length and Percentiles

Avg	1%	5%	10%	25%	50%	75%	90%	95%	99%
21.9	1	2	3	7	15	30.8	51	65	98

© NCSS, LLC. All Rights Reserved.

Histogram



References

Ryan, T.P. 1989. Statistical Methods for Quality Improvement. Wiley. New York.
Montgomery, D.C. 1991. Introduction to Statistical Quality Control. Wiley. New York.

The results will vary slightly because they are based on simulation. The plot shows the distribution of run lengths of 5000 simulated runs.

© NCSS, LLC. All Rights Reserved.

Example 2 – Comparing Tests

Continuing with the Example 1 parameters, the researchers would like to compare the various control chart tests available.

Setup

If the procedure window is not already open, use the PASS Home window to open it. The parameters for this example are listed below and are stored in the **Example 2** settings file. To load these settings to the procedure window, click **Open Example Settings File** in the Help Center or File menu.

Design Tab	
Solve For	**Run Length Distribution**
Simulations	**5000**
Random Seed	**4349025** (for Reproducibility)
Maximum Run Length	**5000**
Run Length Summary 1	**ARL (Average Run Length)**
Run Length Summary 2	**MRL (Median Run Length)**
n (Sample Size)	**10**

Distributions, Tests Tab	
In-Control Distributions Specified By	**Standard Deviation Directly**
In-Control Standard Deviation	**3.8**
Out-of-Control Standard Deviation	**4.6**
R	**Checked**
S	**Checked**
S with Probability Limits	**Checked**
Two-Sided or Upper Limit Only	**Upper Limit Only**
Specify Probability Using	**Probability**
Probability	**0.00135**

© NCSS, LLC. All Rights Reserved.

Output

Click the Calculate button to perform the calculations and generate the following output.

Numeric Results for Control Charts for Process Variability

Solve For:	Run Length Distribution
Control Limits:	Determined by specifying a standard in-control standard deviation directly
Out-of-Control Distribution:	Normal with Out-of-Control SD

Chart Type	Average Run Length	Median Run Length	Sample Size n	In-Control SD	Outside Limits One-Sided Probability	Z-Multiplier	Control Limits Lower LCL	Control Limits Upper UCL	Out-of-Control SD
R	21.5	15	10	3.8	0.00135	3.205	0	20.7821	4.6
S	21.5	15	10	3.8	0.00135	3.205	0	6.3436	4.6
S Prob	34.2	24	10	3.8	0.00135		0	6.5931	4.6

Simulations: 5000. Run Time: 5.33 seconds.
User-Entered Random Seed: 4349025

The R and S control chart tests show the process is out-of-control a bit sooner than the S test with probability limits.

© NCSS, LLC. All Rights Reserved.

Example 3 – Validation Using Ryan (1989)

Ryan (1989), page 93, indicates that in-control processes with a sample size of 4 at each look will have a false signal on average after 222 samples for the S test, while the S with Probability Limits test will only give a false positive after 741 (1/0.00135) samples.

For reproducibility, we'll use a random seed of 4945379.

Setup

If the procedure window is not already open, use the PASS Home window to open it. The parameters for this example are listed below and are stored in the **Example 3** settings file. To load these settings to the procedure window, click **Open Example Settings File** in the Help Center or File menu.

Design Tab	
Solve For	**Run Length Distribution**
Simulations	**10000**
Random Seed	**4945379** (for Reproducibility)
Maximum Run Length	**5000**
Run Length Summary 1	**ARL (Average Run Length)**
Run Length Summary 2	**MRL (Median Run Length)**
n (Sample Size)	**4**

Distributions, Tests Tab	
In-Control Distributions Specified By	**Standard Deviation Directly**
In-Control Standard Deviation	**3.8**
Out-of-Control Standard Deviation	**3.8**
R	**Unchecked**
S	**Checked**
S with Probability Limits	**Checked**
Two-Sided or Upper Limit Only	**Upper Limit Only**
Specify Probability Using	**Probability**
Probability	**0.00135**

© NCSS, LLC. All Rights Reserved.

Output

Click the Calculate button to perform the calculations and generate the following output.

Numeric Results for Control Charts for Process Variability

Solve For:	Run Length Distribution
Control Limits:	Determined by specifying a standard in-control standard deviation directly
Out-of-Control Distribution:	Normal with Out-of-Control SD

Chart Type	Average Run Length	Median Run Length	Sample Size n	In-Control SD	Outside Limits One-Sided Probability	Z-Multiplier	Control Limits Lower LCL	Control Limits Upper UCL	Out-of-Control SD
S	221.2	156	4	3.8	0.00135	3.205	0	7.9334	3.8
S Prob	736.8	503	4	3.8	0.00135		0	8.6738	3.8

Simulations: 10000. Run Time: 49.54 seconds.
User-Entered Random Seed: 4945379

The average run lengths are very close to those presented in Ryan (1989).

© NCSS, LLC. All Rights Reserved.