Chapter 560

Factorial Analysis of Variance

Introduction

A common task in research is to compare the average response across levels of one or more factor variables. Examples of factor variables are income level of two regions, nitrogen content of three lakes, or drug dosage. The factorial analysis of variance compares the means of two or more factors. *F* tests are used to determine statistical significance of the factors and their interactions. The tests are non-directional in that the null hypothesis specifies that all means are equal and the alternative hypothesis simply states that at least one mean is different. This ***PASS*** module performs power analysis and sample size estimation for an analysis of variance design with up to three fixed factors.

In the following example, the responses of a weight loss experiment are arranged in a two-factor, fixed-effect, design. The first factor is diet (D1 and D2) and the second factor is dose level of a dietary drug (low, medium, and high). The twelve individuals available for this study were assigned at random to one of the six treatment groups (cells) so that there were two per group. The response variable was an individual's weight loss after four months.

Table of Individual Weight Losses

	Dietary Drug Dose Level		
Diet	Low	Medium	High
D1	14, 16	15, 18	23, 28
D2	18, 21	18, 22	38, 39

Important features to note are that each table entry represents a different individual and that the response variable (weight loss) is continuous, while the factors (Diet and Dose) are discrete.

Means can be calculated for each cell of the table. These means are shown in the table below. Note that we have added an additional row and column for the row, column, and overall means. The six means in the interior of this table are called the *cell means*.

Table of Means

	Dietary Drug Dose Level			
Diet	**Low**	**Medium**	**High**	**Total**
D1	15.00	16.50	25.50	**19.00**
D2	19.50	20.00	38.50	**26.00**
Total	**17.25**	**18.25**	**32.00**	**22.50**

The Linear Model

A mathematical model may be formulated that underlies this experimental design. This model expresses each cell mean, μ_{ij}, as the sum of parameters called *effects*. A common linear model for a two-factor experiment is

$$\mu_{ij} = m + a_i + b_j + (ab)_{ij}$$

where i = 1, 2, ..., I and j = 1, 2, ..., J. This model expresses the value of a cell mean as the sum of four components:

m the grand mean.

a_i the effect of the i^{th} level of factor A. Note that $\sum a_i = 0$.

b_j the effect of the j^{th} level of factor B. Note that $\sum b_j = 0$.

ab_{ij} the combined effect of the i^{th} level of factor A and the j^{th} level of factor B. Note that $\sum (ab)_{ij} = 0$.

Another way of stating this model for the two factor case is

Cell Mean = Overall Effect + Row Effect + Column Effect + Interaction Effect.

Since this model is the sum of various constants, it is called a *linear model*.

Calculating the Effects

We will now calculate the effects for our example. We will let Drug Dose correspond to factor A and Diet correspond to factor B.

Step 1 – Remove the Grand Mean

Remove the grand mean from the table of means by subtracting 22.50 from each entry. The values in the margins are the *effects* of the corresponding factors.

Table of Mean Weight Losses After Subtracting the Grand Mean				
	Dietary Drug Dose Level			
Diet	Low	Medium	High	Overall
D1	-7.50	-6.00	3.00	-3.50
D2	-3.00	-2.50	16.00	3.50
Overall	-5.25	-4.25	9.50	22.50

Step 2 – Remove the Effects of Factor B (Diet)

Subtract the Diet effects (-3.50 and 3.50) from the entries in those rows.

Table of Mean Weight Losses After Subtracting the Diet Effects				
	Dietary Drug Dose Level			
Diet	Low	Medium	High	Overall
D1	-4.00	-2.50	6.50	-3.50
D2	-6.50	-6.00	12.50	3.50
Overall	-5.25	-4.25	9.50	22.50

Step 3 – Remove the Effects of Factor A (Drug Dose)

Subtract the Drug Dose effects (-5.25, -4.25, and 9.50) from the rest of the entries in those columns. This will result in a table of effects.

Table of Effects				
	Dietary Drug Dose Level			
Diet	Low	Medium	High	Overall
D1	1.25	1.75	-3.00	-3.50
D2	-1.25	-1.75	3.00	3.50
Overall	-5.25	-4.25	9.50	22.50

We have calculated a table of effects for the two-way linear model. Each cell mean can calculated by summing the appropriate entries from this table.

The estimated linear effects are:

m = 22.50

a1 = -5.25	a2 = -4.25	a3 = 9.50
b1 = -3.50	b2 = 3.50	
ab11 = 1.25	ab21 = 1.75	ab31 = -3.00
ab12 = -1.25	ab22 = -1.75	ab32 = 3.00.

The six cell means are calculated from these effects as follows:

15.00 = 22.50 - 5.25 - 3.50 + 1.25

19.50 = 22.50 - 5.25 + 3.50 - 1.25

16.50 = 22.50 - 4.25 - 3.50 + 1.75

20.00 = 22.50 - 4.25 + 3.50 - 1.75

25.50 = 22.50 + 9.50 - 3.50 - 3.00

38.50 = 22.50 + 9.50 + 3.50 + 3.00

Analysis of Variance Hypotheses

The hypotheses that are tested in an analysis of variance table concern the effects, so in order to conduct a power analysis you must have a firm grasp of their meaning. For example, we would usually test the following hypotheses:

1. Are there differences in weight loss among the three drug doses? That is, are the drug dose effects all zero? This hypothesis is tested by the *F* test for factor *A*, which tests whether the standard deviation of the a_i is zero.

2. Is there a difference in weight loss between the two diets? That is, are the diet effects all zero? This hypothesis is tested by the *F* test for factor *B*, which tests whether the standard deviation of the b_j is zero.

3. Are there any diet-dose combinations that exhibit a weight loss that cannot be explained by diet and/or drug dose singly? This hypothesis is tested by the *F* test for the *AB* interaction, which tests whether the standard deviation of the $(ab)_{ij}$ is zero.

Each of these hypotheses can be tested at a different alpha level and different precision. Hence each can have a different power. One of the tasks in planning such an experiment is to determine a sample size that yields necessary power values for each of these hypothesis tests. This is accomplished using this program module.

Definition of Terms

Factorial designs evaluate the effect of two or more categorical variables (called *factors*) on a response variable by testing hypotheses about various averages. These designs are popular because they allow experimentation across a wide variety of conditions and because they evaluate the *interaction* of two or more factors. Interaction is the effect that may be attributed to a combination of two or more factors, but not to one factor singly.

A *factor* is a variable that relates to the response. Either the factor is discrete by nature (as in location or gender) or has been made discrete by collapsing a continuous variable (as in income level or age group). The term *factorial* implies that all possible combinations of the factors being studied are included in the design.

A *fixed* factor is one in which all possible *levels* (categories) are considered. Examples of fixed factors are gender, dose level, and country of origin. They are different from *random* factors which represent a random selection of individuals from the population described by the factor. Examples of random factors are people living within a region, a sample of schools in a state, or a selection of labs. Again, a fixed factor includes the range of interest while a random factor includes only a sample of all possible levels.

A factorial design is analyzed using the analysis of variance. When only fixed factors are used in the design, the analysis is said to be a *fixed-effects analysis of variance*.

Suppose a group of individuals have agreed to be in a study involving six treatments. In a *completely randomized factorial design*, each individual is assigned at random to one of the six groups and then the treatments are applied. In some situations, the randomization occurs by randomly selecting individuals from the populations defined by the treatment groups. The designs analyzed by this module are completely randomized factorial designs.

Power Calculations

The calculation of the power of a particular test proceeds as follows

1. Determine the critical value, $F_{df1,df2,\alpha}$ where *df1* is the numerator degrees of freedom, *df2* is the denominator degrees of freedom, and α is the probability of a type-I error (significance level). Note that the *F* test is a two-tailed test as no logical direction is assigned in the alternative hypothesis.
2. Calculate the standard deviation of the hypothesized effects, using the formula:

$$\sigma_m = \sqrt{\frac{\sum_{i=1}^{k}\left(e_i - \bar{e}\right)^2}{k}}$$

   where the e_i are effect values and k is the number of effects. Note that the average effect will be zero by construction, so this formula reduces to

$$\sigma_m = \sqrt{\frac{\sum_{i=1}^{k}\left(e_i\right)^2}{k}}$$

3. Compute the noncentrality parameter λ using the relationship:

$$\lambda = N\frac{\sigma_m^2}{\sigma^2}$$

   where N is the total number of subjects.
4. Compute the power as the probability of being greater than $F_{df1,df2,\alpha}$ on a noncentral-F distribution with noncentrality parameter λ.

Example

In the example discussed earlier, the standard deviation of the dose effects is

$$\sigma_m(A) = \sqrt{\frac{(-5.25)^2 + (-4.25)^2 + 9.50^2}{3}}$$
$$= 6.729908$$

the standard deviation of the diet effects is

$$\sigma_m(B) = \sqrt{\frac{(-3.5)^2 + 3.5^2}{2}}$$
$$= 3.5$$

and the standard deviation of the interaction effects is

$$\sigma_m(AB) = \sqrt{\frac{1.25^2 + (-1.25)^2 + 1.75^2 + (-1.75)^2 + (-3.00)^2 + 3.00^2}{6}}$$
$$= 2.131119$$

Standard Deviation of Effects (of Means)

In the two-sample t-test case, the alternative hypothesis was represented as the difference between two group means. Unfortunately, for three or more groups, there is no simple extension of the two group difference. Instead, you must hypothesize a set of effects and calculate the value of σ_m.

Some might wish to specify the alternative hypothesis as the effect size, *f*, which is defined as

$$f = \frac{\sigma_m}{\sigma}$$

where σ is the standard deviation of values within a cell (see Sigma below). If you want to use *f*, set $\sigma = 1$ and then f is always equal to σ_m so that the values you enter for σ_m will be the values of *f*. Cohen (1988) has designated values of *f* less than 0.1 as *small*, values around 0.25 to be *medium*, and values over 0.4 to be *large*. You should come up with your own cutoff values for low, medium, and high.

When you are analyzing the power of an existing analysis of variance table, you can compute the values of σ_m for each term from its mean square or *F* ratio using the following formulas:

$$\sigma_m = \sqrt{\frac{df_{numerator} MS_{numerator}}{N}}$$

or

$$\sigma_m = \sqrt{\frac{df_{numerator} (F)(MSE)}{N}}$$

where *N* is the total number of observations, *MSE* is the mean square error, *df* is the numerator degrees of freedom, *MS* is the mean square of the term, and *F* is the *F* ratio of the term. If you do this, you are setting the sample effects equal to the population effects for the purpose of computing the power.

Procedure Options

This section describes the options that are specific to this procedure. These are located on the Design tab. For more information about the options of other tabs, go to the Procedure Window chapter.

Design Tab

The Design tab contains most of the parameters and options that you will be concerned with.

Solve For

Solve For

This option specifies whether you want to solve for power or sample size. This choice controls which options are displayed in the Sample Size section.

If you select sample size, another option appears.

Solve For Sample Size 'Based On Term'

Specify which term to use when searching for a sample size. The power of this term's test will be evaluated as the search is conducted.

- **All**

  If you want to use all terms, select 'All.' Note that only terms that are active may be selected. If you select a term that is not active, you will be prompted to select another term.

Power and Alpha

Minimum Power

Enter a value for the minimum power to be achieved when solving for the sample size. The resulting sample size is large enough so that the power is greater than this amount. This search is conducted using either all terms or a specific term.

Power is the probability of rejecting the null hypothesis when it is false. Beta, the probability of obtaining a false negative on the statistical test, is equal to 1-power, so specifying power implicitly specifies beta.

Range

Since power is a probability, its valid range is between 0 and 1.

Recommended

Different disciplines have different standards for power. A popular value is 0.9, but 0.8 is also popular.

Alpha for All Terms

Enter a single value of alpha to be used in all statistical tests.

Alpha is the probability of obtaining a false positive on a statistical test. That is, it is the probability of rejecting a true null hypothesis. The null hypothesis is usually that the variance of the means (or effects is) zero.

Range

Since Alpha is a probability, it is bounded by 0 and 1. Commonly, it is between 0.001 and 0.250.

Recommended

Alpha is usually set to 0.05 for two-sided tests such as those considered here.

Design and Effects

Number of Factors

Select the number of factors included in the design. Up to three factors may be selected.

Main Effects

Levels (A, B, and C)

These options specify the number of levels (categories) contained in each active factor. A factor must have at least 2 levels. We have arbitrarily set the maximum at 100, but realistically, the maximum is about 10.

Since the total sample size is equal to the product of the number of levels in each factor and the number of observations per group (n), increasing the number of levels of a factor increases the total sample size of the experiment.

Means

The standard deviation of the means is calculated using the formula

$$\sigma_m = \sqrt{\sum_{i=1}^{k} \frac{\left(\mu_i - \overline{\mu}\right)^2}{k}}$$

where k is the number of means. This option lets you select the method used to enter σ_m: either σ_m directly or as a list of means.

As a Std Dev

Enter value of σ_m in the box to the right. The value entered represents the magnitude of the variation among the means that you want to detect.

You can use the Standard Deviation Estimator window to calculate a value of σ_m for various sets of means. This window is obtained by pressing the 'Sm' icon to the right.

List of Means

Enter a list of means in the box to the right. The standard deviation of these means is calculated using the formula

$$\sigma_m = \sqrt{\sum_{i=1}^{k} \frac{\left(\mu_i - \overline{\mu}\right)^2}{k}}$$

where k is the number of means. Note that the standard deviation will be the same whether you enter means or effects.

Enter a set of means that give the pattern of differences you expect or the pattern that you wish to detect. For example, in a particular study involving a factor with three categories, your research might be meaningful if either of two treatment means is 50% larger than the control mean. If the control mean is 50, then you would enter *50,75,75* as the three means.

Note that it is not the values of the means themselves that is important, but their differences. Thus, the mean values *0,1,2* produce the same results as the values *100,101,102*.

If not enough means are entered to match the number of groups, the last mean is repeated. For example, suppose that four means are needed and you enter *1,2* (only two means). ***PASS*** will treat this as *1,2,2,2*. If too many values are entered, ***PASS*** will truncate the list to the number of means needed.

Interactions

A*B, …, A*B*C

These check boxes indicate whether the term is included in the analysis of variance model. You cannot include an interaction term without including all shorter terms that make up that interaction. Hence, when any of the two way interactions are not selected, the three-way interaction will be unavailable.

Effects

Select the method used to enter the effects.

Std Dev of Effects

Enter the value of σ_m directly. You can use the 'Sm' tool to help you calculate an appropriate value.

Multiple of Another Term

Enter the value of σ_m as a multiple of another term. You will enter a multiplier and select a term.

Multiplier

Enter the value of the multiplier. Usually, this will be a value near one. If you enter ‘1,’ the σ_m value of the other term will be used for this term.

Discussion

The general formula for the calculation of the standard deviation of the interaction effects is

$$\sigma_m = \sqrt{\frac{\sum_{i=1}^{k}(e_i)^2}{k}}$$

where k is the number of effects. In the case of a two-way interaction, the standard deviation is calculated using the formula:

$$\sigma_m(AB) = \sqrt{\frac{\sum_{i=1}^{I}\sum_{j=1}^{J}\left(\mu_{ij} - \mu_{i\bullet} - \mu_{\bullet j} + \overline{\mu}\right)^2}{IJ}}$$

where i is the factor A index (from 1 to I), j is the factor B index (from 1 to J), μ_{ij} is the mean in the ij^{th} *cell,* $\mu_{i\bullet}$ is the i^{th} mean of factor A across all levels of other factors, $\mu_{\bullet j}$ is the j^{th} mean when factor B across all levels of other factors, and $\overline{\mu}$ is the overall mean of the means.

To see how this works, consider the following table of means from an experiment with I = 2 and J = 3:

		i		
		1	2	
	1	2.0	4.0	| 3.0
j	2	4.0	6.0	| 5.0
	3	6.0	11.0	| 8.5
	---	---	---	| ---
	Total	4.0	7.0	| 5.5

Now, if we subtract the factor *A* means, subtract the factor *B* means, and add the overall mean, we get the interaction effects:

0.5	-0.5
0.5	-0.5
-1.0	1.0

Next, we sum the squares of these six values:

$$(0.5)^2 + (-0.5)^2 + (0.5)^2 + (-0.5)^2 + (-1.0)^2 + (1.0)^2 = 3$$

Next we divide this value by (2)(3) = 6:

$$3/6 = 0.5$$

Finally, we take the square root of this value:

$$\sqrt{0.5} = 0.7071$$

Hence, for this configuration of means,

$$\sigma_m(AB) = 0.7071\,.$$

Notice that the average of the absolute values of the interaction effects is:

[0.5 + 0.5 + 0.5 + 0.5 + 1.0 + 1.0]/6 = 0.6667

We see that SD(interaction) is close to the average absolute interaction effect. That is, 0.7071 is close to 0.6667. This will usually be the case. Hence, one way to interpret the interaction-effects standard deviation is as a number a little larger than the average absolute interaction effect.

Standard Deviation

σ (Standard Deviation of Subjects)

This option specifies the value of the standard deviation (σ) within a group (the analysis of variance assumes that σ is constant across all groups). Since they are positive square roots, the numbers must be strictly greater than zero. You can press the σ button to the right to obtain further help on estimating the standard deviation.

This value may be estimated from a previous analysis of variance table as the square root of the mean square error.

Sample Size

n (Size Per Group)

This is the average sample size per group. Fractional values are allowed. When you have an unequal number of observations per group, enter the average group size here.

If you enter more than one value, a separate analysis will be generated for each value.

Example 1 – Power after a Study

This example will explain how to calculate the power of *F* tests from data that have already been collected and analyzed.

Analyze the power of the experiment that was given at the beginning of this chapter. These data were analyzed using the analysis of variance procedure in ***NCSS*** and the following results were obtained.

Analysis of Variance Table

Source Term	DF	Sum of Squares	Mean Square	F-Ratio	Prob Level	Power (Alpha=0.05)
A (Dose)	2	543.5	271.75	50.95	0.000172*	1.000000
B (Diet)	1	147	147	27.56	0.001920*	0.990499
AB	2	54.5	27.25	5.11	0.050629	0.588884
S	6	32	5.333333			
Total (Adjusted)	11	777				
Total	12					

* Term significant at alpha = 0.05

Means and Effects Section

Term	Count	Mean	Standard Error	Effect
All	12	22.50		22.50
A: Dose				
High	4	32.00	1.154701	9.50
Medium	4	18.25	1.154701	-4.25
Low	4	17.25	1.154701	-5.25
B: Diet				
D1	6	19.00	0.942809	-3.50
D2	6	26.00	0.942809	3.50
AB: Dose,Diet				
High,D1	2	25.50	1.632993	-3.00
High,D2	2	38.50	1.632993	3.00
Low,D1	2	15.00	1.632993	1.25
Low,D2	2	19.50	1.632993	-1.25
Medium,D1	2	16.50	1.632993	1.75
Medium,D2	2	20.00	1.632993	-1.75

Setup

To analyze these data, we can enter the means for factors *A* and *B* as well as the *AB* interaction effects.

Alternatively, we could have calculated the standard deviation of the interaction. This can be done in either of two ways.

Using mean square for *AB* (27.25), the degrees of freedom for *AB* (2), and the total sample size (12), the standard deviation of the *AB*-interaction effects is calculated as follows

$$\sigma_m(AB) = \sqrt{\frac{2(27.25)}{12}} = 2.1311$$

Using the formula based on the effects, the standard deviation of the *AB*-interaction effects is calculated as follows

$$\sigma_m(AB) = \sqrt{\frac{3^2 + 3^2 + 1.25^2 + 1.25^2 + 1.75^2 + 1.75^2}{6}} = 2.1311$$

The value of σ is estimated from the square root of the mean square error:

$$\sigma = \sqrt{5.333333} = 2.3094$$

This section presents the values of each of the parameters needed to run this example. First, from the PASS Home window, load the **Factorial Analysis of Variance** procedure window by expanding **Means**, then clicking on **Multi-Factor Designs (ANOVA)**, and then clicking on **Factorial Analysis of Variance**. You may then make the appropriate entries as listed below, or open **Example 1** by going to the **File** menu and choosing **Open Example Template**.

Option	Value
Design Tab	
Solve For	**Power**
Alpha for All Terms	**0.05**
Number of Factors	**2**
A - Levels	**3**
A - Means	**List of Means**
A - List of Means	**17.25 18.25 32**
B - Levels	**2**
B - Means	**List of Means**
B - List of Means	**19 26**
A*B	**Checked**
Effects	**Std Dev of Effects**
Std Dev of Effects	**2.1311**
σ (Std Dev of Subjects)	**2.3094**
n (Size Per Group)	**2**

Annotated Output

Click the Calculate button to perform the calculations and generate the following output.

Numeric Results

Term	Power	Grp Size (n)	Total Size (N)	df1	df2	Standard Deviation of Means (σm)	Subject Standard Deviation (σ)	Effect Size (σm/σ)	Alpha	Beta
A(3)	1.0000	2	12	2	6	6.730	2.309	2.914	0.05	0.00
B(2)	0.9905	2	12	1	6	3.500	2.309	1.516	0.05	0.01
A*B	0.5889	2	12	2	6	2.131	2.309	0.923	0.05	0.41

References

Neter, J., Kutner, M., Nachtsheim, C., and Wasserman, W. 1996. Applied Linear Statistical Models. Richard D. Irwin, Inc. Chicago, Illinois.
Winer, B.J. 1991. Statistical Principles in Experimental Design. Third Edition. McGraw-Hill. New York, NY.

Report Definitions

Power is the probability of rejecting a false null hypothesis.
n is the average sample size of per group (treatment combination).
Total N is the total sample size of all groups combined.
df1 is the numerator degrees of freedom.
df2 is the denominator degrees of freedom.
σm is the standard deviation of the group means.
σ is the subject-to-subject standard deviation.
Effect Size is the ratio σm/σ.
Alpha is the probability of rejecting a true null hypothesis.
Beta is the probability of accepting a false null hypothesis.

Summary Statements
A factorial design with two factors at 3 and 2 levels has 6 cells (treatment combinations). A total of 12 subjects are required to provide 2 subjects per cell. The within-cell standard deviation is 2.309. This design achieves 100% power when an F test is used to test factor A at a 5% significance level and the actual standard deviation among the appropriate means is 6.730 (an effect size of 2.914), achieves 99% power when an F test is used to test factor B at a 5% significance level and the actual standard deviation among the appropriate means is 3.500 (an effect size of 1.516), and achieves 59% power when an F test is used to test the A*B interaction at a 5% significance level and the actual standard deviation among the appropriate means is 2.131 (an effect size of 0.923).

This report shows the power for each of the three factors.

It is important to emphasize that these power values are for the case when the effects associated with the alternative hypotheses are equal to those given by the data. It will usually be more informative to calculate the power for other values as well.

Term

This is the term (main effect or interaction) from the analysis of variance model being displayed on this line.

Power

This is the power of the *F* test for this term. Note that since adding and removing terms changes the denominator degrees of freedom (*df2*), the power depends on which other terms are included in the model.

n

This is the sample size per group (treatment combination). Fractional values indicate an unequal allocation among the cells.

Total N

This is the total sample size for the complete design.

df1

This is the numerator degrees of freedom of the *F* test.

df2

This is the denominator degrees of freedom of the *F* test. This value depends on which terms are included in the AOV model.

Std Dev of Means (σm)

This is the standard deviation of the means (or effects). It represents the size of the differences among the effects that is to be detected by the analysis. If you have entered hypothesized means, only their standard deviation is displayed here.

Standard Deviation(σ)

This is the standard deviation of the means (or effects). It represents the size of the differences among the effects that is to be detected by the analysis. If you have entered hypothesized means, only their standard deviation is displayed here.

Effect Size (σm/σ)

This is the standard deviation of the means divided by the standard deviation of subjects. It provides an index of the magnitude of the difference among the means that can be detected by this design.

Alpha

This is the significance level of the *F* test. This is the probability of a type-I error given the null hypothesis of equal means and zero effects.

Beta

This is the probability of the type-II error for this test given the sample size, significance level, and effect size.

Example 2 – Finding the Sample Size

In this example, we will investigate the impact of increasing the sample size on the power of each of the seven tests in the analysis of variance table of a three factor experiment. The first factor (*A*) has two levels, the second factor (*B*) has three levels, and the third factor (*C*) has four levels. This creates a design with 2 x 3 x 4 = 24 treatment combinations.

All values of σ_m will be set equal to 0.2, σ is set equal to 1.0, alpha is set to 0.05, and minimum power is 0.90. Determine the required sample size so that the F-tests of all terms meet the minimum power requirement.

Setup

This section presents the values of each of the parameters needed to run this example. First, from the PASS Home window, load the **Factorial Analysis of Variance** procedure window by expanding **Means**, then clicking on **Multi-Factor Designs (ANOVA)**, and then clicking on **Factorial Analysis of Variance**. You may then make the appropriate entries as listed below, or open **Example 2** by going to the **File** menu and choosing **Open Example Template**.

Option	Value
Design Tab	
Solve For	**Sample Size**
Based on Term	**All**
Minimum Power	**0.90**
Alpha for All Terms	**0.05**
Number of Factors	**3**
A - Levels	**2**
A - Means	**As a Std Dev**
A - Std Dev	**0.2**
B - Levels	**3**
B - Means	**As a Std Dev**
B - Std Dev	**0.2**
C - Levels	**4**
C - Means	**As a Std Dev**
C - Std Dev	**0.2**
A*B	**Checked**
Effects	**Std Dev of Effects**
Std Dev of Effects	**0.2**
*All other interactions are set the same as A*B*	
σ (Std Dev of Subjects)	**1**

Output

Click the Calculate button to perform the calculations and generate the following output.

Numeric Results

Term	Power	Grp Size (n)	Total Size (N)	df1	df2	Standard Deviation of Means (σm)	Subject Standard Deviation (σ)	Effect Size (σm/σ)	Alpha	Beta
A(2)	0.9893	19	456	1	432	0.200	1.000	0.200	0.05	0.01
B(3)	0.9752	19	456	2	432	0.200	1.000	0.200	0.05	0.02
C(4)	0.9598	19	456	3	432	0.200	1.000	0.200	0.05	0.04
A*B	0.9752	19	456	2	432	0.200	1.000	0.200	0.05	0.02
A*C	0.9598	19	456	3	432	0.200	1.000	0.200	0.05	0.04
B*C	0.9103	19	456	6	432	0.200	1.000	0.200	0.05	0.09
A*B*C	0.9103	19	456	6	432	0.200	1.000	0.200	0.05	0.09

The necessary sample size is 19 per group for a total of 456 subjects.

Example 3 – Latin Square Design

This example shows how to study the power of a complicated experimental design like a Latin square. Suppose you want to run a five-level Latin square design. This design consists of three factors each at five levels. One factor is associated with the columns of the square, a second factor is associated with the rows of the square, and a third factor is associated with the letters of the square. There are only 5 x 5 = 25 observations used instead of the 5 x 5 x 5 = 125 that would normally be required. The Latin square design has reduced the number of observations by 80%.

The 80% decrease in observations comes at a price—the interaction terms must be ignored. If you can legitimately assume that the interactions are zero, the Latin square (or some other design which reduces the number of observations) is an efficient design to use.

We will now show you how to analyze the power of the *F* tests from such a design. The key is to enter 0.2 (which is 25/125) for *n* and set all the interaction indicators off.

Since all three factors have five levels, the power of the three *F* tests will be the same if σ_m is the same. Hence, we can try three different sets of hypothesized means. The first set will be five means 0.1 units apart. The second set will be five means 0.5 units apart. The third set will be five means 1.0 unit apart. The standard deviation will be set to 1.0. All alpha levels will be set at 0.05.

The sample size per cell is set at 0.2 and 0.4. This will result in total sample sizes of 25 (one replication) and 50 (two replications).

Setup

This section presents the values of each of the parameters needed to run this example. First, from the PASS Home window, load the **Factorial Analysis of Variance** procedure window by expanding **Means**, then clicking on **Multi-Factor Designs (ANOVA)**, and then clicking on **Factorial Analysis of Variance**. You may then make the appropriate entries as listed below, or open **Example 3** by going to the **File** menu and choosing **Open Example Template**.

Option	Value
Design Tab	
Solve For	Power
Alpha for All Terms	0.05
Number of Factors	3
A - Levels	5
A - Means	List of Means
A - List of Means	1.0 1.1 1.2 1.3 1.4
B - Levels	5
B - Means	List of Means
B - List of Means	1.0 1.5 2.0 2.5 3.0
C - Levels	5
C - Means	List of Means
C - List of Means	1 2 3 4 5
All interactions are unchecked	
σ (Std Dev of Subjects)	1
n (Size Per Group)	0.2 0.4
Reports Tab	
Decimal Places - n	2

Output

Click the Calculate button to perform the calculations and generate the following output.

Numeric Results

Term	Power	Grp Size (n)	Total Size (N)	df1	df2	Standard Deviation of Means (σm)	Subject Standard Deviation (σ)	Effect Size (σm/σ)	Alpha	Beta
A(5)	0.0681	0.20	25	4	12	0.141	1.000	0.141	0.05	0.93
B(5)	0.6367	0.20	25	4	12	0.707	1.000	0.707	0.05	0.36
C(5)	0.9987	0.20	25	4	12	1.414	1.000	1.414	0.05	0.00
A(5)	0.0984	0.40	50	4	37	0.141	1.000	0.141	0.05	0.90
B(5)	0.9774	0.40	50	4	37	0.707	1.000	0.707	0.05	0.02
C(5)	1.0000	0.40	50	4	37	1.414	1.000	1.414	0.05	0.00

In the first design in which N = 25, only the power of the test for C is greater than 0.9. Of course, this power value also depends on the value of the standard deviation of subjects.

It is interesting to note that doubling the sample size did not double the power!

Example 4 – Validation using Winer

Winer (1991) pages 428-429 presents the power calculations for a two-way design in which factor *A* has two levels and factor *B* has three levels. Winer provides estimates of the sum of squared *A* effects (1.0189), sum of squared *B* effects (5.06), and sum of squared interaction effects (42.11). The mean square error is 8.83 and the per cell sample size is 3. All alpha levels are set to 0.05.

Winer's results are approximate because he has to interpolate in the tables that he is using. He finds the power of the *F* test for factor *A* to be between 0.10 and 0.26. He estimates it as 0.17. The exact power of the *F* test for factor *B* is not given. Instead, the range is found to be between 0.26 and 0.36. The power of the *F* test for the *A***B* interaction is "approximately" 0.86.

Setup

This section presents the values of each of the parameters needed to run this example. First, from the PASS Home window, load the **Factorial Analysis of Variance** procedure window by expanding **Means**, then clicking on **Multi-Factor Designs (ANOVA)**, and then clicking on **Factorial Analysis of Variance**. You may then make the appropriate entries as listed below, or open **Example 4** by going to the **File** menu and choosing **Open Example Template**.

Option	Value
Design Tab	
Solve For	**Power**
Alpha for All Terms	**0.05**
Number of Factors	**2**
A - Levels	**2**
A - Means	**As a Std Dev**
A - Std Dev of Effects	**0.714**
B - Levels	**3**
B - Means	**As a Std Dev**
B - Standard Deviation of Effects	**1.3**
A*B	**Checked**
Effects	**Std Dev of Effects**
Std Dev of Effects	**2.65**
σ (Std Dev of Subjects)	**2.97**
n (Size Per Group)	**3**

Output

Click the Calculate button to perform the calculations and generate the following output.

Numeric Results

Term	Power	Grp Size (n)	Total Size (N)	df1	df2	Standard Deviation of Means (σm)	Subject Standard Deviation (σ)	Effect Size (σm/σ)	Alpha	Beta
A(2)	0.1558	3	18	1	12	0.714	2.970	0.240	0.05	0.84
B(3)	0.2918	3	18	2	12	1.300	2.970	0.438	0.05	0.71
A*B	0.8534	3	18	2	12	2.650	2.970	0.892	0.05	0.15

The power of the test for factor A is 0.16 which is between 0.10 and 0.26. It is close to the interpolated 0.17 that Winer obtained from his tables. The power of the test for factor B is 0.29 which is between 0.26 and 0.36. The power of the test for the A*B interaction is 0.85 which is close to the interpolated 0.86 that Winer obtained from his tables.

Example 5 – Validation using Prihoda

Prihoda (1983) pages 7-8 presents the power calculations for a two-way design with the following pattern of means:

		Factor B				
		1	**2**	**3**	**4**	**All**
Factor A	**1**	41	34	30	27	33
	2	33	24	22	29	27
All		37	29	26	28	30

The means may be manipulated to show the overall mean, the main effects, and the interaction effects:

		Factor B				
		1	**2**	**3**	**4**	**All**
Factor A	**1**	1	2	1	-4	3
	2	-1	-2	-1	4	-3
All		7	-1	-4	-2	30

Based on the above effects, Prihoda calculates the power of the interaction test when the sample size per group is 6, 8, 10, 12, and 14 to be 0.34, 0.45, 0.56, 0.65, and 0.73. The σ_m for the interaction is 2.345208. The mean square error is 64 and the alpha level is 0.05.

Setup

This section presents the values of each of the parameters needed to run this example. First, from the PASS Home window, load the **Factorial Analysis of Variance** procedure window by expanding **Means**, then clicking on **Multi-Factor Designs (ANOVA)**, and then clicking on **Factorial Analysis of Variance**. You may then make the appropriate entries as listed below, or open **Example 5** by going to the **File** menu and choosing **Open Example Template**.

Option	Value
Design Tab	
Solve For	**Power**
Alpha for All Terms	**0.05**
Number of Factors	**2**
A - Levels	**2**
A - Means	**List of Means**
A - Std Dev of Effects	**33 27**
B - Levels	**4**
B - Means	**List of Means**
B - Standard Deviation of Effects	**37 29 26 28**
A*B	**Checked**
Effects	**Std Dev of Effects**
Std Dev of Effects	**2.345208**
σ (Std Dev of Subjects)	**8**
n (Size Per Group)	**6 8 10 12 14**

Output

Click the Calculate button to perform the calculations and generate the following output.

Numeric Results

Term	Power	Grp Size (n)	Total Size (N)	df1	df2	Standard Deviation of Means (σm)	Subject Standard Deviation (σ)	Effect Size (σm/σ)	Alpha	Beta
A(2)	0.7175	6	48	1	40	3.000	8.000	0.375	0.05	0.28
B(4)	0.8368	6	48	3	40	4.183	8.000	0.523	0.05	0.16
A*B	0.**3372**	6	48	3	40	2.345	8.000	0.293	0.05	0.66
A(2)	0.8385	8	64	1	56	3.000	8.000	0.375	0.05	0.16
B(4)	0.9387	8	64	3	56	4.183	8.000	0.523	0.05	0.06
A*B	0.**4510**	8	64	3	56	2.345	8.000	0.293	0.05	0.55
A(2)	0.9113	10	80	1	72	3.000	8.000	0.375	0.05	0.09
B(4)	0.9792	10	80	3	72	4.183	8.000	0.523	0.05	0.02
A*B	0.**5556**	10	80	3	72	2.345	8.000	0.293	0.05	0.44
A(2)	0.9529	12	96	1	88	3.000	8.000	0.375	0.05	0.05
B(4)	0.9935	12	96	3	88	4.183	8.000	0.523	0.05	0.01
A*B	0.**6475**	12	96	3	88	2.345	8.000	0.293	0.05	0.35
A(2)	0.9757	14	112	1	104	3.000	8.000	0.375	0.05	0.02
B(4)	0.9981	14	112	3	104	4.183	8.000	0.523	0.05	0.00
A*B	0.**7254**	14	112	3	104	2.345	8.000	0.293	0.05	0.27

Prihoda only presents the power for the interaction test at each sample size. You can check to see that the results match Prihoda's exactly.

Example 6 – Validation using Neter, Kutner, Nachtsheim, and Wasserman

Neter, Kutner, Nachtsheim, and Wasserman (1996) page 1057 presents a power analysis of a two-factor experiment in which factor *A* has three levels and factor *B* has two levels. The significance level is 0.05, the standard deviation is 3.0, and *n* is 2. They calculate a power of about 0.89 for the test of factor *A* when the three means are 50, 55, and 45.

Setup

This section presents the values of each of the parameters needed to run this example. First, from the PASS Home window, load the **Factorial Analysis of Variance** procedure window by expanding **Means**, then clicking on **Multi-Factor Designs (ANOVA)**, and then clicking on **Factorial Analysis of Variance**. You may then make the appropriate entries as listed below, or open **Example 6** by going to the **File** menu and choosing **Open Example Template**.

Option	Value
Design Tab	
Solve For	**Power**
Alpha for All Terms	**0.05**
Number of Factors	**2**
A - Levels	**3**
A - Means	**List of Means**
A - List of Means	**50 55 45**
B - Levels	**2**
B - Means	**As a Std Dev**
B - Standard Deviation of Effects	**1**
A*B	*Checked*
Effects	**Std Dev of Effects**
Std Dev of Effects	**1**
σ (Std Dev of Subjects)	**3**
n (Size Per Group)	**2**

Output

Click the Calculate button to perform the calculations and generate the following output.

Numeric Results

Term	Power	Grp Size (n)	Total Size (N)	df1	df2	Standard Deviation of Means (σm)	Subject Standard Deviation (σ)	Effect Size (σm/σ)	Alpha	Beta
A(3)	0.9016	2	12	2	6	4.082	3.000	1.361	0.05	0.10
B(2)	0.1648	2	12	1	6	1.000	3.000	0.333	0.05	0.84
A*B	0.1178	2	12	2	6	1.000	3.000	0.333	0.05	0.88

Note that the power of 0.90 that PASS has calculated is within rounding of the 0.89 that Neter *et al.* calculated.