Chapter 398

GEE Tests for the Slope of Two Groups in a Repeated Measures Design (Binary Outcome)

Introduction

This module calculates the power for testing the difference between two slopes from binary, correlated data that are analyzed using the GEE method. Such data occur in two design types: clustered and longitudinal.

GEE is does not require the full specification of the joint distribution of the repeated measurements, as long as the marginal model is correctly specified. Estimation consistency is achieved even if the correlation matrix is incorrect. Also, the correlation matrix of the responses is specified directly, rather than using an intermediate, random effects model as is the case in MM. For clustered designs, GEE often uses a *compound symmetric* (CS) correlation structure. For longitudinal data, an *autoregressive* (AR(1)) correlation structure is often used.

Missing Values

This procedure allows you to specify various patterns of incomplete (or missing) data. Subjects may miss some appointments but attend others. This phenomenon of incomplete data can be accounted for in the sample size calculation which can greatly reduce the overall sample size from that calculated by just omitting subjects with incomplete observations.

Technical Details

Theory and Notation

The details of the calculation of sample size and power is given in Ahn, Heo, and Zhang (2015), section 4.5.1, page 119-123. See also Jung and Ahn (2005). We will use their notation here.

Suppose we have n_1 subjects in group 1 (treatment) and n_2 subjects in group 2 (control) for a total of N subjects, each measured on m occasions at times t_j (j = 1, …, m). For convenience, we normalize these time points to the proportion of total time so that t_1 = 0 and t_m = 1. The mean of the binary responses y_{kij} is modeled by the log-odds model

$$\operatorname{logit}(p_{kij}) = \log\left(\frac{p_{kij}}{1-p_{kij}}\right) = a_k + \beta_k t_j$$

where

y_{kij} is the j^{th} response from subject i in group k.

p_{kij} is expectation of y_{kij},

a_1 is the regression coefficient giving intercept of the treatment group,

a_2 is the regression coefficient giving intercept of the control group,

β_1 is the regression coefficient giving slope of the treatment group,

β_2 is the regression coefficient giving slope of the control group.

This model can be expressed as

$$p_{kij} = \frac{\exp\,(a_k + \beta_k t_j)}{1 + \exp\,(a_k + \beta_k t_j)}$$

GEE is used to estimate and test hypotheses about the equality of the slopes β_1 and β_2.

Odds Ratio of P1(1) and P2(1)

Notice that when $a_1 = a_2$ the odds ratio of P1(1) and P2(1) is related to the difference in the slopes as follows.

$$\begin{aligned} \log\big[OR\big(P1(1), P2(1)\big)\big] &= \text{logit}\big(P1(1)\big) - \text{logit}\big(P2(1)\big) \\ &= a_1 + \beta_1(1) - a_2 + \beta_2(1) \\ &= \beta_1 - \beta_2 \end{aligned}$$

Hence a hypothesis about the difference in slopes is a hypothesis about the odds ratio of P1(1) and P2(1).

Correlation Patterns

In a longitudinal design with *N* subjects, each measured *m* times, observations from a single subject are correlated, and a pattern of those correlations through time needs to be specified. Several choices are available.

Compound Symmetry

A compound symmetry correlation model assumes that all correlations are equal. That is

$$[\rho_{jj'}] = \begin{bmatrix} 1 & \rho & \rho & \rho & \cdots & \rho \\ \rho & 1 & \rho & \rho & \cdots & \rho \\ \rho & \rho & 1 & \rho & \cdots & \rho \\ \rho & \rho & \rho & 1 & \cdots & \rho \\ \vdots & \vdots & \vdots & \vdots & \ddots & \vdots \\ \rho & \rho & \rho & \rho & \cdots & 1 \end{bmatrix}_{M \times M}$$

where ρ is the baseline correlation.

Banded(1)

A Banded(1) (banded order 1) correlation model assumes that correlations for observations one time period apart are equal to ρ, and correlations for measurements greater than one time period apart are equal to zero. That is

$$[\rho_{jj'}] = \begin{bmatrix} 1 & \rho & 0 & 0 & \cdots & 0 \\ \rho & 1 & \rho & 0 & \cdots & 0 \\ 0 & \rho & 1 & \rho & \cdots & 0 \\ 0 & 0 & \rho & 1 & \cdots & 0 \\ \vdots & \vdots & \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & 0 & 0 & \cdots & 1 \end{bmatrix}_{M\times M}$$

where ρ is the baseline correlation.

Banded(2)

A Banded(2) (banded order 2) correlation model assumes that correlations for observations one time period or two periods apart are equal to ρ, and correlations for measurements greater than one time period apart are equal to zero. That is

$$[\rho_{jj'}] = \begin{bmatrix} 1 & \rho & \rho & 0 & \cdots & 0 \\ \rho & 1 & \rho & \rho & \cdots & 0 \\ \rho & \rho & 1 & \rho & \cdots & 0 \\ 0 & \rho & \rho & 1 & \cdots & 0 \\ \vdots & \vdots & \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & 0 & 0 & \cdots & 1 \end{bmatrix}_{M\times M}$$

where ρ is the baseline correlation.

AR1 (Traditional)

This version of AR1 (autoregressive order 1) correlation model assumes that correlations t time periods apart are equal to ρ^t. That is

$$[\rho_{jj'}] = \begin{bmatrix} 1 & \rho & \rho^2 & \rho^3 & \cdots & \rho^{M-1} \\ \rho & 1 & \rho & \rho^2 & \cdots & \rho^{M-2} \\ \rho^2 & \rho & 1 & \rho & \cdots & \rho^{M-3} \\ \rho^3 & \rho^2 & \rho & 1 & \cdots & \rho^{M-4} \\ \vdots & \vdots & \vdots & \vdots & \ddots & \vdots \\ \rho^{M-1} & \rho^{M-2} & \rho^{M-3} & \rho^{M-4} & \cdots & 1 \end{bmatrix}_{M\times M}$$

where ρ is the baseline correlation.

AR1 (Proportional)

This version of AR1 (autoregressive order 1) correlation model is described in the book by Ahn et al. (2015). It assumes that correlations $|t_j - t_k|$ time periods apart are equal to $\rho^{|t_j - t_k|}$. That is

$$[\rho_{jk}] = \left[\rho^{|t_j - t_k|}\right]_{M\times M}$$

where ρ is the baseline correlation. Note that in this pattern, the value of ρ is shown in the final column since in this case $t_j = 0$ and $t_k = 1$, so $|t_j - t_k| = 1$.

Damped Exponential

A damped exponential is an extension of the AR(1) correlation model in which the exponents are raised to the power *Dexp* (θ = *Dexp* in the diagram below). This causes the resulting correlations to be reduced (dampened). Here is an example

$$[\rho_{jj'}] = \begin{bmatrix} 1 & \rho & \rho^{2^\theta} & \rho^{3^\theta} & \cdots & \rho^{(M-1)^\theta} \\ \rho & 1 & \rho & \rho^{2^\theta} & \cdots & \rho^{(M-2)^\theta} \\ \rho^{2^\theta} & \rho & 1 & \rho & \cdots & \rho^{(M-3)^\theta} \\ \rho^{3^\theta} & \rho^{2^\theta} & \rho & 1 & \cdots & \rho^{(M-4)^\theta} \\ \vdots & \vdots & \vdots & \vdots & \ddots & \vdots \\ \rho^{(M-1)^\theta} & \rho^{(M-2)^\theta} & \rho^{(M-3)^\theta} & \rho^{(M-4)^\theta} & \cdots & 1 \end{bmatrix}_{M\times M}$$

where ρ is the baseline correlation.

Damped Exponential (Proportional)

This version of the damped exponential correlation model is described in the book by Ahn et al. (2015). It assumes that correlations $|t_j - t_k|$ time periods apart are equal to $\rho^{|t_j - t_k|^\theta}$. That is

$$[\rho_{jk}] = \left[\rho^{|t_j - t_k|^\theta}\right]_{M\times M}$$

where ρ is the baseline correlation. Note that in this pattern, the value of $\rho^{|t_j - t_k|^\theta}$ turns up in the final column since in this case $t_j = 0$ and $t_k = 1$, so $|t_j - t_k| = 1$.

Linear Exponential Decay

A linear exponential decay correlation model is one in which the exponent of the correlation decays according to a linear equation from 1 at the *Base Time Proportion* to a final value, *Emax*. The resulting pattern looks similar to the damped exponential. Note that the exponents are applied to the absolute difference between the Measurement Time Proportions. This method allows you to easily construct comparable correlation matrices of different dimensions. Otherwise, differences in the resulting power would be more strongly due to differences in the correlation matrices.

Here is an example. Suppose *m* is 6, *ρ* = 0.5, *Emax* = 3, the *Base Time Proportion* is 0.20, and the Measurement Time Proportions are (0, 0.2, 0.4, 0.6, 0.8, 1). The following correlation matrix would be obtained

$$[\rho_{jj'}] = \begin{bmatrix} 1 & 0.5 & 0.3536 & 0.25 & 0.1768 & 0.125 \\ 0.5 & 1 & 0.5 & 0.3536 & 0.25 & 0.1768 \\ 0.3536 & 0.5 & 1 & 0.5 & 0.3536 & 0.25 \\ 0.25 & 0.3536 & 0.5 & 1 & 0.5 & 0.3536 \\ 0.1768 & 0.25 & 0.3536 & 0.5 & 0.3536 & 0.5 \\ 0.125 & 0.1768 & 0.25 & 0.3536 & 0.5 & 1 \end{bmatrix}_{M\times M}$$

Note that in the top row, the correlation is 0.5 for the second (0.2 - 0) time point and 0.125 (0.5^3) at the last (1 - 0) time points. The correlations are obtained by raising 0.5 to the appropriate exponent. The linear equation from 1 to 3 results in the exponents 1, 1.5, 2, 2.5, 3 correspondent to the time proportions 0, 0.2, 0.4, 0.6, 0.8, and 1.

As a further example, note that the correlation for the 0.4 time point is, $0.5^{1.5} = 0.35355339 \approx 0.3536$.

This method allows you to compare various values of *m* while keeping the correlation matrix similar. To see what we mean, consider what the correlation matrix looks like when *m* is reduced to 4 and the measurement time proportions are set to (0, 0.2, 0.6, 1). It becomes

$$[\rho_{jj'}] = \begin{bmatrix} 1 & 0.5 & 0.25 & 0.125 \\ 0.5 & 1 & 0.5 & 0.25 \\ 0.25 & 0.5 & 1 & 0.5 \\ 0.125 & 0.25 & 0.5 & 1 \end{bmatrix}_{M\times M}$$

Note that the correlation at a measurement time difference of 0.6 is equal to 0.25 in both matrices.

Missing Data Patterns

The problem of missing data occurs for several reasons. In longitudinal studies in which a subject is measured multiple times, missing data becomes more complicated to model because it is possible that a subject is measured only some of the time. In fact, it is probably more common for data to be incomplete than complete. The approach of omitting subjects with incomplete data during the planning phase is very inaccurate. Certainly, subjects with partial measurements are included in the analysis. This procedure provides several missing data patterns to choose from so that your sample size calculations are more realistic.

In the presentation to following, we denote the percent of subjects with a missing response at time point t_j as κ_j. The proportion non-missing at a particular time point is $\phi_j = 1 - \kappa_j$. We will refer to ϕ_j as the *marginal observant probability* at time t_j and $\phi_{jj'}$ as a *joint observant probability* at times t_j and $t_{j'}$.

Pairwise Missing Pattern

The program provides three options for how the pairwise (joint) observant probabilities $\phi_{jj'}$ are calculated. These are

Independent (Ind): $\phi_{jj'} = \phi_j \phi_{j'}, \phi_{jj} = \phi_j$

Monotonic (Mon): $\phi_{jj'} = \phi_k \text{ where } k = \max(j, j')$

Mixture: $\phi_{jj'} = W(\text{Ind}) + (1 - W)(\text{Mon})$ for weighting factor *W*.

Missing Input Type

The are several ways in which the missing value pattern can be specified. Each missing value pattern is a list of missing proportions at each of the *M* time points. Each value in the list must be non-negative and less than 1. Possible input choices are

- **Constant = 0**

  All missing proportions are set to 0. That is, there are no missing values.

- **Constant**

  All missing proportions are set to constant value.

- **Piecewise Constant on Spreadsheet**

  A set of missing proportions are defined for several time intervals using the spreadsheet. One column contains the missing proportions for the interval, going down the rows. Another column defines the corresponding upper limit of time proportion of the interval. The lower limit is implied by the limit given immediately above. The program assumes that the first-time interval starts at 0 percent.

- **Linear (Steady Change)**

  The missing proportions fall along a straight-line between 0 and 1 elapsed time. Only the first and last proportions are entered.

- **Piecewise Linear on Spreadsheet**

  The missing proportions fall along a set of connected straight-lines that are defined by two columns on the spreadsheet.

- **List**

  Enter a list of M missing proportions, one for each time point.

- **Multiple Lists on Spreadsheet**

  Select multiple columns containing vertical lists of missing proportions. Each column contains a set of missing proportions in rows, one for each time point.

- **Pairwise Observed Proportions on Spreadsheet**

  Enter an $M \times M$ matrix of observant probabilities by selecting M columns. These observant probabilities are the proportion of the responses for both the row and column time points that are observed.

Sample Size Calculations

The details of the calculation of sample size and power is given in Ahn, Heo, and Zhang (2015), Chapter 4. These are summarized here.

GEE is used to estimate the regression coefficients β_1 and β_2. The significance of $\beta_1 - \beta_2$, the coefficient associated with the difference between the treatment and control group slopes, is tested using a Wald statistic for which the following sample size formula is derived

$$n = \frac{\left(\frac{v_1^2}{r_1} + \frac{v_2^2}{r_2}\right)\left(z_{1-\frac{\alpha}{h}} + z_{1-\gamma}\right)^2}{\delta^2}$$

where

h = 1 (one-sided test) or 2 (two-sided test)

γ = 1 – power

α = significance level

$$v_k^2 = \frac{s_k^2 + c_k^2}{s_k^4}$$

$$\delta = \beta_1 - \beta_2$$

$$s_k^2 = \sum_{j=1}^{M} \phi_j p_{kj} q_{kj} \left(t_j - \tau_k\right)^2$$

$$c_k^2 = \sum \sum_{j \neq j'}^{M} \phi_{jj'} \rho_{jj'} \sqrt{p_{kj} q_{kj} p_{kj'} q_{kj'}} \left(t_j - \tau_k\right)\left(t_{j'} - \tau_k\right)$$

$$\tau_k = \frac{\sum_{j=1}^{M} \phi_j p_{kj} q_{kj} t_j}{\sum_{j=1}^{M} \phi_j p_{kj} q_{kj}}$$

$$q_{kj} = 1 - p_{kj}$$

$$r_k = n_k / N$$

$\phi_j = 1 - \kappa_j$, where $\kappa_j =$ proportion missing at the j^{th} time point

$\rho_{jj'}$ is the corresponding element from within-subject correlation matrix

$\phi_{jj'}$ is the joint observant probability of observing both y_{ij} and $y_{ij'}$ for every subject i

Three possible choices are available to calculate $\phi_{jj'}$. These are

Independent: $\phi_{jj'} = \phi_j \phi_{j'}, \phi_{jj} = \phi_j$

Monotonic: $\phi_{jj'} = \phi_k$ where $k = \max(j, j')$

Mixture: $\phi_{jj'} = W(\text{Independent}) + (1 - W)(\text{Monotonic})$ for weighting factor *W*.

The above formula is easily rearranged to obtain a formula for power.

Example 1 – Determining Sample Size

Researchers are planning a study to assess whether of a new drug will change the response rate to a particular event. They want to compare subjects who take the new drug with subjects who take a placebo. Their experimental protocol calls for a baseline measurement, followed by administration of the new drug or the placebo, followed by the event of interest, followed by three additional measurements one day apart. They want to detect a difference of 0.2 in the final response probabilities. They also want a sensitivity analysis by considering a range of possible differences from 0.1 to 0.3.

Similar studies have found that 80% of subjects do not show a response before the event. Three hours after the event, the response rate is 50%. Using these results, the basic parameters are P1(0) = 0.80, P2(0) = 0.80, and P2(1) = 0.50. A difference of 0.2 implies that P1(1) = 0.5 + 0.2 = 0.7. This difference is equivalent to an odds ratio of 2.333.

These studies also showed an autocorrelation between adjacent measurements on the same individual of 0.7, so they want to try autocorrelations of 0.6, 0.7, and 0.8. The researchers assume that first-order autocorrelation adequately represents the autocorrelation pattern. A two-sided Wald test will be conducted at the 0.05 significance level and at 90% power. The subjects will be divided equally between the treatment and control groups.

The researchers anticipate that the missing data pattern across time will begin at 0% missing and increase steadily to 30% at the fourth measurement. They assume that the pairwise missing probabilities are *independent*.

What are the sample size requirements for this study?

Setup

If the procedure window is not already open, use the PASS Home window to open it. The parameters for this example are listed below and are stored in the **Example 1** settings file. To load these settings to the procedure window, click **Open Example Settings File** in the Help Center or File menu.

Design Tab

Solve For	**Sample Size**
Alternative Hypothesis	**Two-Sided**
Power	**0.9**
Alpha	**0.05**
R (Group 1 Allocation %)	**50**
Measurement Time Input Type	**Equally Spaced Measurement Times**
M (Number of Measurements)	**4**
P1(0) and P2(0) Input Type	**P1(0) = P2(0)**
P1(0) and P2(0)	**0.8**
P1(1) Input Type	**Difference (P1(1) - P2(1))**
Difference (P1(1) - P2(1))	**0.1 0.2 0.3**
P2(1)	**0.5**
Pattern of ρ's Across Time	**AR1 (Traditional)**
ρ (Base Correlation)	**0.6 0.7 0.8**

Missing Input Type..**Linear (Steady Change)**
Pairwise Missing Pattern................................**Independent (Ind)**
First Missing Proportion (Ind).........................**0**
Last Missing Proportion (Ind).........................**0.3**

Output

Click the Calculate button to perform the calculations and generate the following output.

Numeric Reports

Numeric Results for the Test of Slope-Difference in Binary Data using GEE

Solve For: Sample Size
Measurement Times: Equally spaced
Correlation: AR1: ρ(j,k) = ρ^|j-k|
Missing Pattern: Range of missing proportions
Observant Proportions: Assume independence
Pk(t): Event probability of group k (1 = treatment, 2 = control) at time proportion t

				Event Probability									
				at First Measurement		at Last Measurement							
Power	Total Sample Size N	Group 1 Allocation Percent R	Number of Meas. Times M	P1(0)	P2(0)	P1(1)	P2(1)	Difference	Base Corr. ρ	First Row of Corr. Matrix	Missing Data Proportions	Measurement Times	Alpha
0.9000	2518	50	4	0.8	0.8	0.6	0.5	0.1	0.6	ρ1(1)	Ms1(1)	T(1)	0.05
0.9001	2191	50	4	0.8	0.8	0.6	0.5	0.1	0.7	ρ2(1)	Ms1(1)	T(1)	0.05
0.9000	1748	50	4	0.8	0.8	0.6	0.5	0.1	0.8	ρ3(1)	Ms1(1)	T(1)	0.05
0.9000	596	50	4	0.8	0.8	0.7	0.5	0.2	0.6	ρ1(1)	Ms1(1)	T(1)	0.05
0.9000	518	50	4	0.8	0.8	0.7	0.5	0.2	0.7	ρ2(1)	Ms1(1)	T(1)	0.05
0.9002	413	50	4	0.8	0.8	0.7	0.5	0.2	0.8	ρ3(1)	Ms1(1)	T(1)	0.05
0.9007	240	50	4	0.8	0.8	0.8	0.5	0.3	0.6	ρ1(1)	Ms1(1)	T(1)	0.05
0.9001	208	50	4	0.8	0.8	0.8	0.5	0.3	0.7	ρ2(1)	Ms1(1)	T(1)	0.05
0.9010	166	50	4	0.8	0.8	0.8	0.5	0.3	0.8	ρ3(1)	Ms1(1)	T(1)	0.05

Item	Values
ρ1(1)	1, 0.6, 0.36, 0.216
ρ2(1)	1, 0.7, 0.49, 0.343
ρ3(1)	1, 0.8, 0.64, 0.512
Ms1(1)	0, 0.1, 0.2, 0.3
T(1)	0, 0.33, 0.67, 1

Power: The probability of rejecting a false null hypothesis when the alternative hypothesis is true.
N: The total number of subjects in the study.
R: The treatment group allocation proportion. It is the proportion of subjects that are in the treatment group.
M: The number of time points at which each subject is measured.
Difference: The difference between the treatment and control event probabilities at the final measurement time. Difference = P1(1) - P2(1).
Pk(t): The event probability of group k (1 = treatment, 2 = control) at the measurement time proportion t.
ρ: The base correlation between two responses on the same subject. It may be transformed based on the correlation pattern.
First Row of Correlation Matrix: Presents the top row of the correlation matrix.
Missing Data Proportions: Gives the name of the set containing the missing data proportions across time.
Measurement Times: Gives the name of the set containing the measurement time proportions. These measurement times represent the proportion of the total study time that has elapsed just before the measurement.
Alpha: The probability of rejecting a true null hypothesis.

Summary Statements

A two-group repeated measures design (with a binary response and with 4 measurements for each subject) will be used to test whether there is a group difference in slopes. The comparison will be made using a two-sided Wald Z-test using GEE methods, with a Type I error rate (α) of 0.05. The (repeated) measurements of each subject will be made at the following 4 times, expressed as proportions of the total study time: 0, 0.33, 0.67, 1. Missing values are assumed to occur completely at random (MCAR). The missing value proportions will be combined to form the pairwise observant probabilities using an independent pairwise missing pattern. The anticipated proportions missing at each measurement time are 0, 0.1, 0.2, 0.3. The first row of the autocorrelation matrix of the responses within a subject is assumed to be 1, 0.6, 0.36, 0.216, with subsequent rows following the same pattern (AR1: ρ(j,k) = ρ^|j-k|). The initial proportions P1(0) and P2(0) are both assumed to be 0.8. To detect a final Group 1 proportion P1(1) of 0.6 (and the corresponding slope) and a final Group 2 proportion P2(1) of 0.5 (and the corresponding slope) (for a final time proportion difference of 0.1) with 90% power, the total number of needed subjects is 2518 (with 50% of the subjects in the treatment group (Group 1)).

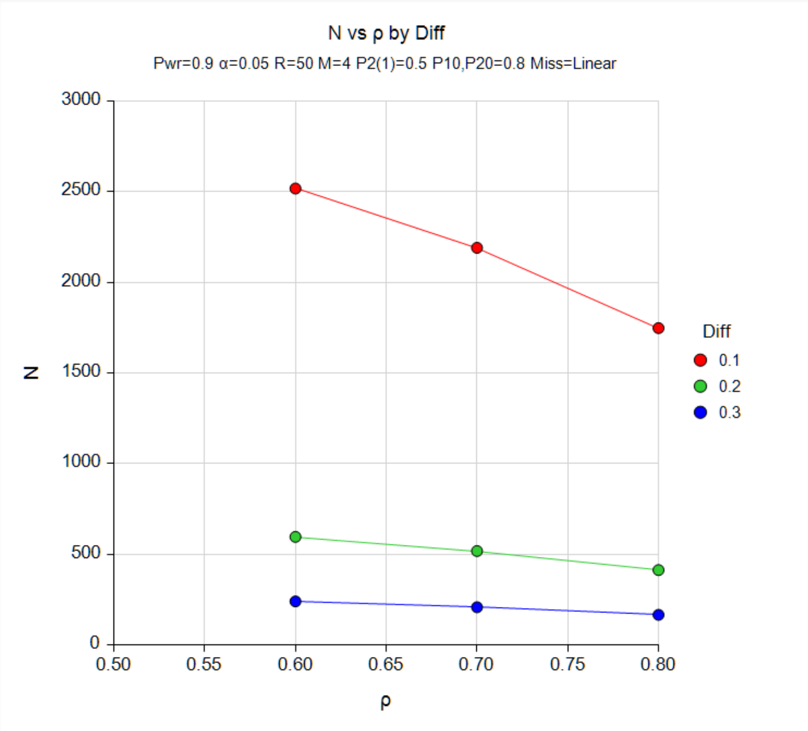
References

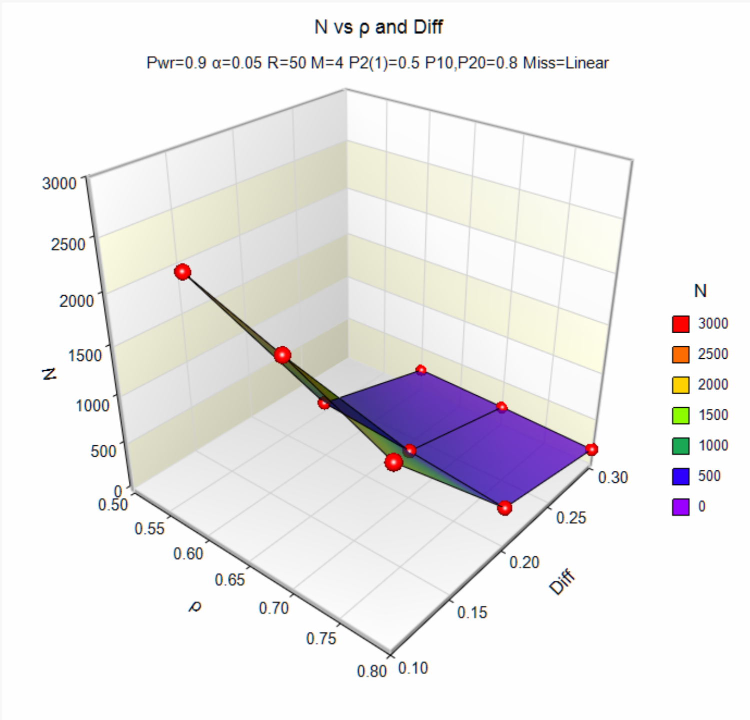
Ahn, C., Heo, M., and Zhang, S. 2015. Sample Size Calculations for Clustered and Longitudinal Outcomes in Clinical Research. CRC Press. New York.
Jung, S.H. and Ahn, C. 2005. 'Sample size for a two-group comparison of repeated binary measurements using GEE'. Statistics in Medicine, Volume 24, pages 2583-2596.

This report gives the sample size for each value of the other parameters. The definitions of each of the items is given in the Reports Definitions section at the end of the report.

Plots Section

Plots





These charts show the relationship between sample size, difference, and ρ when the other parameters in the design are held constant.

Autocorrelation Matrices

Autocorrelation Matrix for Report Row 1

Time	T(0)	T(0.33)	T(0.67)	T(1)
T(0)	1.000	0.60	0.36	0.216
T(0.33)	0.600	1.00	0.60	0.360
T(0.67)	0.360	0.60	1.00	0.600
T(1)	0.216	0.36	0.60	1.000

Autocorrelation Matrix for Report Row 2

Time	T(0)	T(0.33)	T(0.67)	T(1)
T(0)	1.000	0.70	0.49	0.343
T(0.33)	0.700	1.00	0.70	0.490
T(0.67)	0.490	0.70	1.00	0.700
T(1)	0.343	0.49	0.70	1.000

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(More Reports Follow)

These reports show the autocorrelation matrix for the indicated row of the report.

Example 2 – Finding the Power

Continuing with Example 1, the researchers want to determine the power corresponding to sample sizes ranging from 100 to 1000 for the main cases of the other parameters.

Setup

If the procedure window is not already open, use the PASS Home window to open it. The parameters for this example are listed below and are stored in the **Example 2** settings file. To load these settings to the procedure window, click **Open Example Settings File** in the Help Center or File menu.

Design Tab

Solve For	**Power**
Alternative Hypothesis	**Two-Sided**
Alpha	**0.05**
N (Subjects)	**100 to 1000 by 100**
R (Group 1 Allocation %)	**50**
Measurement Time Input Type	**Equally Spaced Measurement Times**
M (Number of Measurements)	**4**
P1(0) and P2(0) Input Type	**P1(0) = P2(0)**
P1(0) and P2(0	**0.80**
P1(1) Input Type	**Difference (P1(1) - P2(1))**
Difference (P1(1) - P2(1))	**0.2**
P2(1)	**0.5**
Pattern of ρ's Across Time	**AR1 (Traditional)**
ρ (Base Correlation)	**0.7**
Missing Input Type	**Linear (Steady Change)**
Pairwise Missing Pattern	**Independent (Ind)**
First Missing Proportion (Ind)	**0**
Last Missing Proportion (Ind)	**0.3**

Output

Click the Calculate button to perform the calculations and generate the following output.

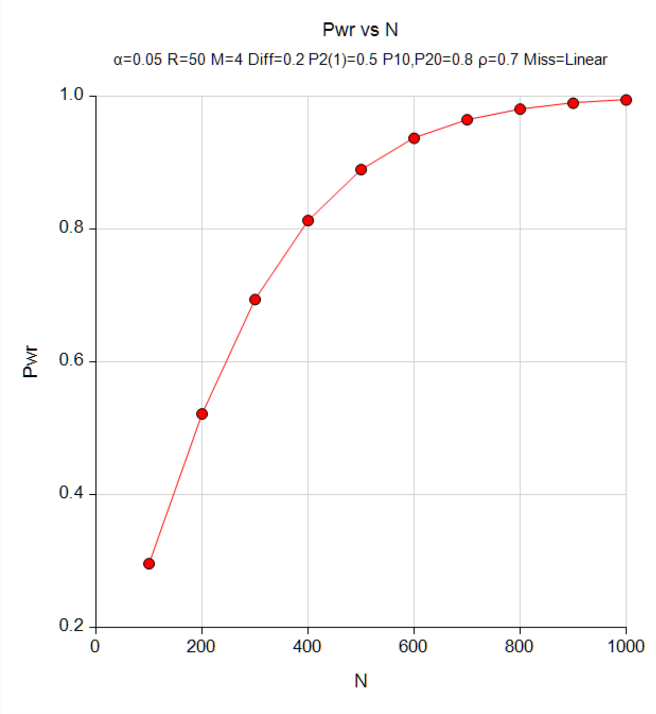
Numeric Results for the Test of Slope-Difference in Binary Data using GEE

Solve For: Power
Measurement Times: Equally spaced
Correlation: AR1: ρ(j,k) = ρ^|j-k|
Missing Pattern: Range of missing proportions
Observant Proportions: Assume independence
Pk(t): Event probability of group k (1 = treatment, 2 = control) at time proportion t

				Event Probability									
				at First Measurement		at Last Measurement							
Power	Total Sample Size N	Group 1 Allocation Percent R	Number of Meas. Times M	P1(0)	P2(0)	P1(1)	P2(1)	Difference	Base Corr. ρ	First Row of Corr. Matrix	Missing Data Proportions	Measurement Times	Alpha
0.2961	100	50	4	0.8	0.8	0.7	0.5	0.2	0.7	ρ1(1)	Ms1(1)	T(1)	0.05
0.5216	200	50	4	0.8	0.8	0.7	0.5	0.2	0.7	ρ1(1)	Ms1(1)	T(1)	0.05
0.6939	300	50	4	0.8	0.8	0.7	0.5	0.2	0.7	ρ1(1)	Ms1(1)	T(1)	0.05
0.8129	400	50	4	0.8	0.8	0.7	0.5	0.2	0.7	ρ1(1)	Ms1(1)	T(1)	0.05
0.8897	500	50	4	0.8	0.8	0.7	0.5	0.2	0.7	ρ1(1)	Ms1(1)	T(1)	0.05
0.9368	600	50	4	0.8	0.8	0.7	0.5	0.2	0.7	ρ1(1)	Ms1(1)	T(1)	0.05
0.9647	700	50	4	0.8	0.8	0.7	0.5	0.2	0.7	ρ1(1)	Ms1(1)	T(1)	0.05
0.9807	800	50	4	0.8	0.8	0.7	0.5	0.2	0.7	ρ1(1)	Ms1(1)	T(1)	0.05
0.9896	900	50	4	0.8	0.8	0.7	0.5	0.2	0.7	ρ1(1)	Ms1(1)	T(1)	0.05
0.9945	1000	50	4	0.8	0.8	0.7	0.5	0.2	0.7	ρ1(1)	Ms1(1)	T(1)	0.05

Item	Values
ρ1(1)	1, 0.7, 0.49, 0.343
Ms1(1)	0, 0.1, 0.2, 0.3
T(1)	0, 0.33, 0.67, 1

Plots



The reports and plot indicate the power for each value of N.

Example 3 – Impact of the Number of Repeated Measurements

Continuing with Examples 1 and 2, the researchers want to study the impact on the sample size of changing the number of measurements made on each individual. Their experimental protocol calls for 4 measurements. They want to see the impact of increasing the number of measurements to 7.

Setup

If the procedure window is not already open, use the PASS Home window to open it. The parameters for this example are listed below and are stored in the **Example 3** settings file. To load these settings to the procedure window, click **Open Example Settings File** in the Help Center or File menu.

Design Tab

Solve For	**Power**
Alternative Hypothesis	**Two-Sided**
Alpha	**0.05**
N (Subjects)	**100 to 1000 by 100**
R (Group 1 Allocation %)	**50**
Measurement Time Input Type	**Equally Spaced Measurement Times**
M (Number of Measurements)	**4 7**
P1(0) and P2(0) Input Type	**P1(0) = P2(0)**
P1(0) and P2(0)	**0.8**
P1(1) Input Type	**Difference (P1(1) - P2(1))**
Difference (P1(1) - P2(1))	**0.2**
P2(1)	**0.5**
Pattern of ρ's Across Time	**AR1 (Traditional)**
ρ (Base Correlation)	**0.7**
Missing Input Type	**Linear (Steady Change)**
Pairwise Missing Pattern	**Independent (Ind)**
First Missing Proportion (Ind)	**0**
Last Missing Proportion (Ind)	**0.3**

Output

Click the Calculate button to perform the calculations and generate the following output.

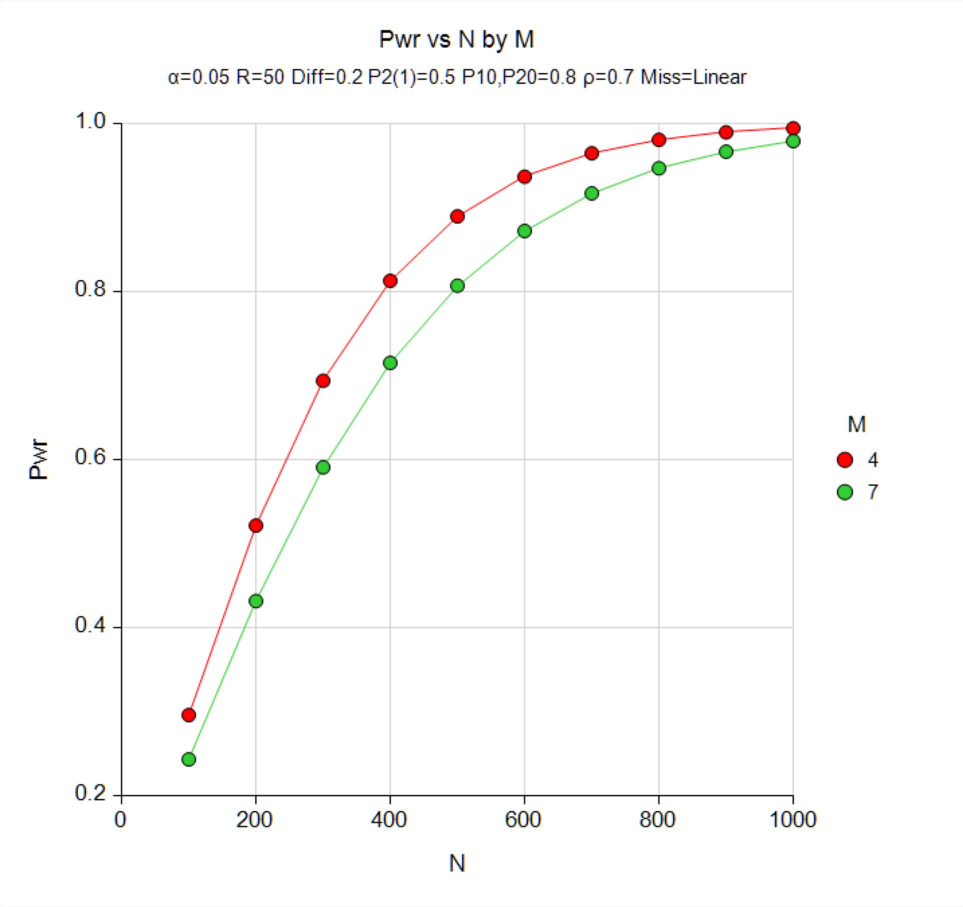
Numeric Results for the Test of Slope-Difference in Binary Data using GEE

Solve For: Power
Measurement Times: Equally spaced
Correlation: AR1: ρ(j,k) = ρ^|j-k|
Missing Pattern: Range of missing proportions
Observant Proportions: Assume independence
Pk(t): Event probability of group k (1 = treatment, 2 = control) at time proportion t

				Event Probability									
				at First Measurement		at Last Measurement							
Power	Total Sample Size N	Group 1 Allocation Percent R	Number of Meas. Times M	P1(0)	P2(0)	P1(1)	P2(1)	Difference	Base Corr. ρ	First Row of Corr. Matrix	Missing Data Proportions	Measurement Times	Alpha
0.2961	100	50	4	0.8	0.8	0.7	0.5	0.2	0.7	ρ1(1)	Ms1(1)	T(1)	0.05
0.2434	100	50	7	0.8	0.8	0.7	0.5	0.2	0.7	ρ1(2)	Ms1(2)	T(2)	0.05
0.5216	200	50	4	0.8	0.8	0.7	0.5	0.2	0.7	ρ1(1)	Ms1(1)	T(1)	0.05
0.4318	200	50	7	0.8	0.8	0.7	0.5	0.2	0.7	ρ1(2)	Ms1(2)	T(2)	0.05
0.6939	300	50	4	0.8	0.8	0.7	0.5	0.2	0.7	ρ1(1)	Ms1(1)	T(1)	0.05
0.5910	300	50	7	0.8	0.8	0.7	0.5	0.2	0.7	ρ1(2)	Ms1(2)	T(2)	0.05
0.8129	400	50	4	0.8	0.8	0.7	0.5	0.2	0.7	ρ1(1)	Ms1(1)	T(1)	0.05
0.7153	400	50	7	0.8	0.8	0.7	0.5	0.2	0.7	ρ1(2)	Ms1(2)	T(2)	0.05
0.8897	500	50	4	0.8	0.8	0.7	0.5	0.2	0.7	ρ1(1)	Ms1(1)	T(1)	0.05
0.8071	500	50	7	0.8	0.8	0.7	0.5	0.2	0.7	ρ1(2)	Ms1(2)	T(2)	0.05
0.9368	600	50	4	0.8	0.8	0.7	0.5	0.2	0.7	ρ1(1)	Ms1(1)	T(1)	0.05
0.8723	600	50	7	0.8	0.8	0.7	0.5	0.2	0.7	ρ1(2)	Ms1(2)	T(2)	0.05
0.9647	700	50	4	0.8	0.8	0.7	0.5	0.2	0.7	ρ1(1)	Ms1(1)	T(1)	0.05
0.9170	700	50	7	0.8	0.8	0.7	0.5	0.2	0.7	ρ1(2)	Ms1(2)	T(2)	0.05
0.9807	800	50	4	0.8	0.8	0.7	0.5	0.2	0.7	ρ1(1)	Ms1(1)	T(1)	0.05
0.9470	800	50	7	0.8	0.8	0.7	0.5	0.2	0.7	ρ1(2)	Ms1(2)	T(2)	0.05
0.9896	900	50	4	0.8	0.8	0.7	0.5	0.2	0.7	ρ1(1)	Ms1(1)	T(1)	0.05
0.9666	900	50	7	0.8	0.8	0.7	0.5	0.2	0.7	ρ1(2)	Ms1(2)	T(2)	0.05
0.9945	1000	50	4	0.8	0.8	0.7	0.5	0.2	0.7	ρ1(1)	Ms1(1)	T(1)	0.05
0.9792	1000	50	7	0.8	0.8	0.7	0.5	0.2	0.7	ρ1(2)	Ms1(2)	T(2)	0.05

Item	Values
ρ1(1)	1, 0.7, 0.49, 0.343
ρ1(2)	1, 0.7, 0.49, 0.343, 0.24, 0.168, 0.118
Ms1(1)	0, 0.1, 0.2, 0.3
Ms1(2)	0, 0.05, 0.1, 0.15, 0.2, 0.25, 0.3
T(1)	0, 0.33, 0.67, 1
T(2)	0, 0.17, 0.33, 0.5, 0.67, 0.83, 1

Plots



Note that increasing the number of measurements has had the surprising result of decreasing the power, probably because the assumption of the AR(1) model for the autocorrelation has changed the way in which the correlations are formed. Note from the footnotes that the final autocorrelation drops from 0.343 when M = 4 to 0.118 when M = 7. Look at the next example to see how the autocorrelations can be put on a more equal footing.

Example 4 – Impact of Changing M with Linear Exponential Decay

We saw in Example 3 that the increasing the number of measurements from 4 to 7 had the counter-intuitive result of reducing the power when the sample size was held constant. We surmised that this was partially due to the differing autocorrelation matrices that were used when the AR(1) model as assumed. In this example, we will leave all parameters the same, except that we will use a Linear Exponential Decay model for the autocorrelation. This will keep the autocorrelation matrices more comparable.

Setup

If the procedure window is not already open, use the PASS Home window to open it. The parameters for this example are listed below and are stored in the **Example 4** settings file. To load these settings to the procedure window, click **Open Example Settings File** in the Help Center or File menu.

Design Tab

Solve For .... **Power**
Alternative Hypothesis .... **Two-Sided**
Alpha.. **0.05**
N (Subjects) .... **100 to 1000 by 100**
R (Group 1 Allocation %) **50**
Measurement Time Input Type .... **Equally Spaced Measurement Times**
M (Number of Measurements) **4 7**
P1(0) and P2(0) Input Type .... **P1(0) = P2(0)**
P1(0) and P2(0) **0.8**
P1(1) Input Type .... **Difference (P1(1) - P2(1))**
Difference (P1(1) - P2(1)) **0.2**
P2(1) **0.5**
Pattern of ρ's Across Time .... **Linear Exponential Decay**
ρ (Base Correlation)...................................... **0.7**
Base Time Proportion **0.1666666**
Emax (Max Decay Exponent) **3**
Missing Input Type.. **Linear (Steady Change)**
Pairwise Missing Pattern............................... **Independent (Ind)**
First Missing Proportion (Ind)......................... **0**
Last Missing Proportion (Ind) **0.3**

Output

Click the Calculate button to perform the calculations and generate the following output.

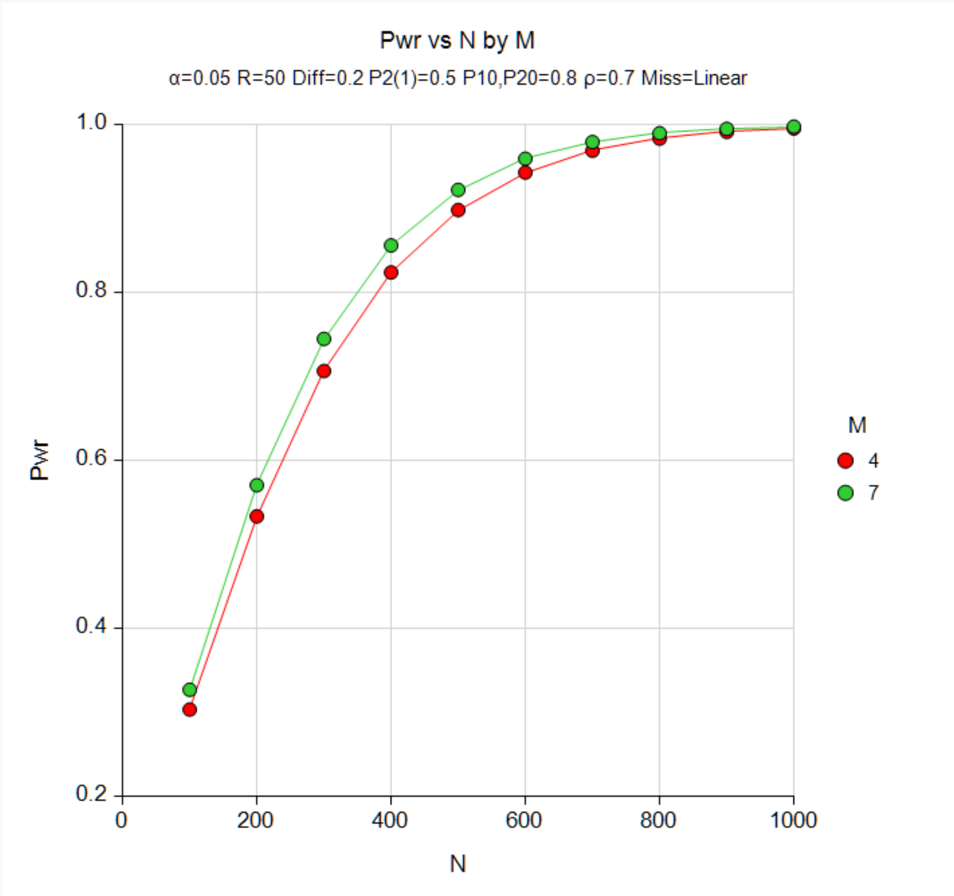
Numeric Results for the Test of Slope-Difference in Binary Data using GEE

Solve For: Power
Measurement Times: Equally spaced
Correlation: Linear exponential decay, with Emax = 3 and Base Time Prop = 0.1666666
Missing Pattern: Range of missing proportions
Observant Proportions: Assume independence
Pk(t): Event probability of group k (1 = treatment, 2 = control) at time proportion t

				Event Probability									
				at First Measurement		at Last Measurement							
Power	Total Sample Size N	Group 1 Allocation Percent R	Number of Meas. Times M	P1(0)	P2(0)	P1(1)	P2(1)	Difference	Base Corr. ρ	First Row of Corr. Matrix	Missing Data Proportions	Measurement Times	Alpha
0.3034	100	50	4	0.8	0.8	0.7	0.5	0.2	0.7	ρ1(1)	Ms1(1)	T(1)	0.05
0.3269	100	50	7	0.8	0.8	0.7	0.5	0.2	0.7	ρ1(2)	Ms1(2)	T(2)	0.05
0.5334	200	50	4	0.8	0.8	0.7	0.5	0.2	0.7	ρ1(1)	Ms1(1)	T(1)	0.05
0.5705	200	50	7	0.8	0.8	0.7	0.5	0.2	0.7	ρ1(2)	Ms1(2)	T(2)	0.05
0.7065	300	50	4	0.8	0.8	0.7	0.5	0.2	0.7	ρ1(1)	Ms1(1)	T(1)	0.05
0.7448	300	50	7	0.8	0.8	0.7	0.5	0.2	0.7	ρ1(2)	Ms1(2)	T(2)	0.05
0.8239	400	50	4	0.8	0.8	0.7	0.5	0.2	0.7	ρ1(1)	Ms1(1)	T(1)	0.05
0.8562	400	50	7	0.8	0.8	0.7	0.5	0.2	0.7	ρ1(2)	Ms1(2)	T(2)	0.05
0.8982	500	50	4	0.8	0.8	0.7	0.5	0.2	0.7	ρ1(1)	Ms1(1)	T(1)	0.05
0.9222	500	50	7	0.8	0.8	0.7	0.5	0.2	0.7	ρ1(2)	Ms1(2)	T(2)	0.05
0.9429	600	50	4	0.8	0.8	0.7	0.5	0.2	0.7	ρ1(1)	Ms1(1)	T(1)	0.05
0.9593	600	50	7	0.8	0.8	0.7	0.5	0.2	0.7	ρ1(2)	Ms1(2)	T(2)	0.05
0.9688	700	50	4	0.8	0.8	0.7	0.5	0.2	0.7	ρ1(1)	Ms1(1)	T(1)	0.05
0.9793	700	50	7	0.8	0.8	0.7	0.5	0.2	0.7	ρ1(2)	Ms1(2)	T(2)	0.05
0.9833	800	50	4	0.8	0.8	0.7	0.5	0.2	0.7	ρ1(1)	Ms1(1)	T(1)	0.05
0.9897	800	50	7	0.8	0.8	0.7	0.5	0.2	0.7	ρ1(2)	Ms1(2)	T(2)	0.05
0.9912	900	50	4	0.8	0.8	0.7	0.5	0.2	0.7	ρ1(1)	Ms1(1)	T(1)	0.05
0.9950	900	50	7	0.8	0.8	0.7	0.5	0.2	0.7	ρ1(2)	Ms1(2)	T(2)	0.05
0.9955	1000	50	4	0.8	0.8	0.7	0.5	0.2	0.7	ρ1(1)	Ms1(1)	T(1)	0.05
0.9976	1000	50	7	0.8	0.8	0.7	0.5	0.2	0.7	ρ1(2)	Ms1(2)	T(2)	0.05

Item	Values
ρ1(1)	1, 0.607, 0.456, 0.343
ρ1(2)	1, 0.7, 0.607, 0.526, 0.456, 0.396, 0.343
Ms1(1)	0, 0.1, 0.2, 0.3
Ms1(2)	0, 0.05, 0.1, 0.15, 0.2, 0.25, 0.3
T(1)	0, 0.33, 0.67, 1
T(2)	0, 0.17, 0.33, 0.5, 0.67, 0.83, 1

Plots



Note from the footnotes that the final autocorrelation between the two models is now identical at 0.3430 $(= 0.7^3)$ when M is increased from 4 to 7. Now that the autocorrelation matrices are more comparable, the power values have increased in all cases, although only slightly. We see that increasing M has not had a huge impact on power.

Example 5 – Validation of Sample Size Calculation using Ahn, Heo, and Zhang (2015)

Ahn, Heo, and Zhang (2015) page 123 present an example that we will use to validate this procedure. In their example, P1(0) = 0.75, P2(0) = 0.75, P1(1) = 0.75, P2(1) = 0.50, the significance level is 0.05, the power is 0.80, R is 50%, M is 6, and the correlation matrix is AR1 with ρ = 0.8. The proportions missing at each time point are 0.0, 0.05, 0.1, 0.15, 0.2, and 0.25. They calculate the sample size to be 215.

Setup

If the procedure window is not already open, use the PASS Home window to open it. The parameters for this example are listed below and are stored in the **Example 5** settings file. To load these settings to the procedure window, click **Open Example Settings File** in the Help Center or File menu.

Design Tab

Solve For .... **Sample Size**
Alternative Hypothesis .... **Two-Sided**
Power.. **0.80**
Alpha... **0.05**
R (Group 1 Allocation %) **50**
Measurement Time Input Type .... **Equally Spaced Measurement Times**
M (Number of Measurements) **6**
P1(0) and P2(0) Input Type .... **P1(0) = P2(0)**
P1(0) and P2(0) **0.75**
P1(1) Input Type .... **P1(1)**
P1(1) **0.75**
P2(1) **0.50**
Pattern of ρ's Across Time............................ **AR1(Traditional)**
ρ (Base Correlation)...................................... **0.8**
Missing Input Type.. **Linear (Steady Change)**
Pairwise Missing Pattern................................ **Independent (Ind)**
First Missing Proportion (Ind)......................... **0**
Last Missing Proportion (Ind) **0.25**

Output

Click the Calculate button to perform the calculations and generate the following output.

Numeric Results for the Test of Slope-Difference in Binary Data using GEE

Solve For:	Sample Size
Measurement Times:	Equally spaced
Correlation:	AR1: ρ(j,k) = ρ^|j-k|
Missing Pattern:	Range of missing proportions
Observant Proportions:	Assume independence
Pk(t):	Event probability of group k (1 = treatment, 2 = control) at time proportion t

				Event Probability									
				at First Measurement		at Last Measurement							
Power	Total Sample Size N	Group 1 Allocation Percent R	Number of Meas. Times M	P1(0)	P2(0)	P1(1)	P2(1)	Difference	Base Corr. ρ	First Row of Corr. Matrix	Missing Data Proportions	Measurement Times	Alpha
0.8017	215	50	6	0.75	0.75	0.75	0.5	0.25	0.8	ρ1(1)	Ms1(1)	T(1)	0.05

Item	Values
ρ1(1)	1, 0.8, 0.64, 0.512, 0.41, 0.328
Ms1(1)	0, 0.05, 0.1, 0.15, 0.2, 0.25
T(1)	0, 0.2, 0.4, 0.6, 0.8, 1

Note that the sample size is 215, which matches that of Ahn, Heo, and Zhang (2015) exactly.

Example 6 – Impact of Measurement Time Distribution

This example will investigate the impact of measurement time on power. It will compare the power of studies that are evenly spaced with those that take more measurements at the beginning of the study, near the middle of the study, and at the end of the study.

In this example the basic parameters are P1(0) = 0.72, P2(0) = 0.72, P1(1) = 0.75, and P2(1) = 0.55. The significance level is 0.05, the sample size ranges from 100 to 500, and R is 50%. The correlation pattern will be Linear Exponential Decay with a base correlation of 0.6, Base Time Proportion of 0.10, and Emax set to 3. The missing input type will be set to Linear from 0 to 0.30 and the pairwise missing assumption will be independent.

The measurement times for five scenarios are given in the following table.

Table of Measurement Times in Proportion of Total Study Time

Tm1	Tm2	Tm3	Tm4	Tm5
0	0	0	0	0
0.20	0.60	0.10	0.10	0.45
0.40	0.70	0.20	0.20	0.50
0.60	0.80	0.30	0.80	0.55
0.80	0.90	0.40	0.90	0.60
1.00	1.00	1.00	1.00	1.00

Note that the measurements in Tm1 are evenly spaced, those in Tm2 are loaded near the end, those of Tm3 occur at the beginning, those of Tm4 occur only at the beginning and the end, and those of Tm5 occur mostly near the middle.

Setup

If the procedure window is not already open, use the PASS Home window to open it. The parameters for this example are listed below and are stored in the **Example 6** settings file. To load these settings to the procedure window, click **Open Example Settings File** in the Help Center or File menu.

Design Tab

Solve For .... **Power**
Alternative Hypothesis .... **Two-Sided**
Alpha.. **0.05**
N (Subjects) .... **100 to 500 by 100**
R (Group 1 Allocation %) **50**
Measurement Time Input Type .... **Columns of Measurement Time Proportions**
Column(s) of Measurement Proportions **Tm1-Tm5**
P1(0) and P2(0) Input Type .... **P1(0) = P2(0)**
P1(0) and P2(0) **0.72**
P1(1) Input Type .... **P1(1)**
P1(1) **0.75**
P2(1) **0.55**
Pattern of ρ's Across Time............................ **Linear Exponential Decay**

ρ (Base Correlation)....................................**0.6**
Base Time Proportion **0.10**
Emax (Max Decay Exponent) **3**
Missing Input Type.......................................**Linear (Steady Change)**
Pairwise Missing Pattern...............................**Independent (Ind)**
First Missing Proportion (Ind)........................**0**
Last Missing Proportion (Ind)........................**0.3**

Input Spreadsheet Data

Row	Tm1	Tm2	Tm3	Tm4	Tm5
1	0.0	0.0	0.0	0.0	0.00
2	0.2	0.6	0.1	0.1	0.45
3	0.4	0.7	0.2	0.2	0.50
4	0.6	0.8	0.3	0.8	0.55
5	0.8	0.9	0.4	0.9	0.60
6	1.0	1.0	1.0	1.0	1.00

Output

Click the Calculate button to perform the calculations and generate the following output.

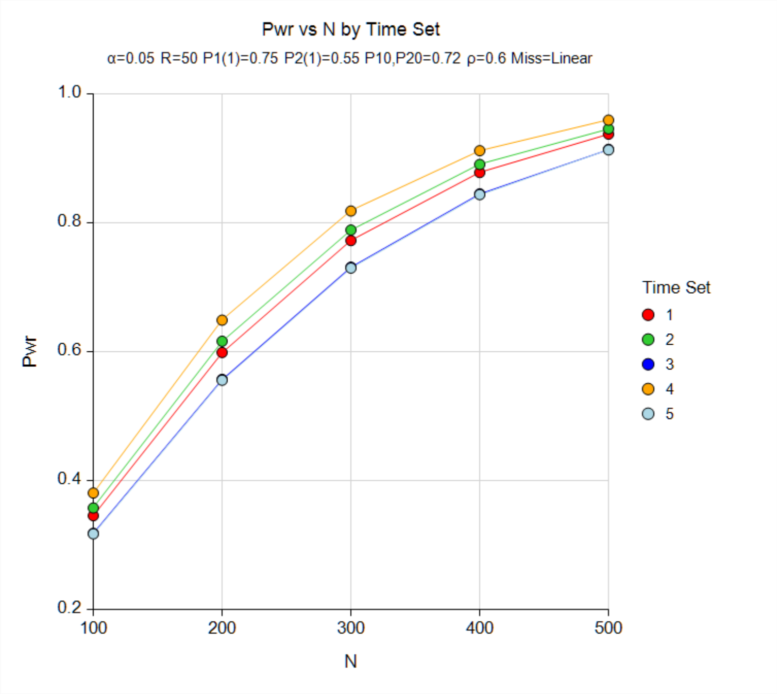
Numeric Results for the Test of Slope-Difference in Binary Data using GEE

Solve For: Power
Measurement Times: Lists in spreadsheet columns: {TM1-TM5}
Correlation: Linear exponential decay, with Emax = 3 and Base Time Prop = 0.10
Missing Pattern: Range of missing proportions
Observant Proportions: Assume independence
Pk(t): Event probability of group k (1 = treatment, 2 = control) at time proportion t

				Event Probability									
	Total	Group 1	Number	at First		at Last			Base	First Row			
	Sample	Allocation	of Meas.	Measurement		Measurement			Corr.	of Corr.	Missing Data	Measurement	
Power	Size N	Percent R	Times M	P1(0)	P2(0)	P1(1)	P2(1)	Difference	ρ	Matrix	Proportions	Times	Alpha
0.3457	100	50	6	0.72	0.72	0.75	0.55	0.2	0.6	ρ1(Tm1)	Ms1(Tm1)	Tm1(1)	0.05
0.3576	100	50	6	0.72	0.72	0.75	0.55	0.2	0.6	ρ1(Tm2)	Ms1(Tm2)	Tm2(2)	0.05
0.3184	100	50	6	0.72	0.72	0.75	0.55	0.2	0.6	ρ1(Tm3)	Ms1(Tm3)	Tm3(3)	0.05
0.3808	100	50	6	0.72	0.72	0.75	0.55	0.2	0.6	ρ1(Tm4)	Ms1(Tm4)	Tm4(4)	0.05
0.3175	100	50	6	0.72	0.72	0.75	0.55	0.2	0.6	ρ1(Tm5)	Ms1(Tm5)	Tm5(5)	0.05
0.5989	200	50	6	0.72	0.72	0.75	0.55	0.2	0.6	ρ1(Tm1)	Ms1(Tm1)	Tm1(1)	0.05
0.6163	200	50	6	0.72	0.72	0.75	0.55	0.2	0.6	ρ1(Tm2)	Ms1(Tm2)	Tm2(2)	0.05
0.5572	200	50	6	0.72	0.72	0.75	0.55	0.2	0.6	ρ1(Tm3)	Ms1(Tm3)	Tm3(3)	0.05
0.6491	200	50	6	0.72	0.72	0.75	0.55	0.2	0.6	ρ1(Tm4)	Ms1(Tm4)	Tm4(4)	0.05
0.5558	200	50	6	0.72	0.72	0.75	0.55	0.2	0.6	ρ1(Tm5)	Ms1(Tm5)	Tm5(5)	0.05
0.7725	300	50	6	0.72	0.72	0.75	0.55	0.2	0.6	ρ1(Tm1)	Ms1(Tm1)	Tm1(1)	0.05
0.7890	300	50	6	0.72	0.72	0.75	0.55	0.2	0.6	ρ1(Tm2)	Ms1(Tm2)	Tm2(2)	0.05
0.7313	300	50	6	0.72	0.72	0.75	0.55	0.2	0.6	ρ1(Tm3)	Ms1(Tm3)	Tm3(3)	0.05
0.8184	300	50	6	0.72	0.72	0.75	0.55	0.2	0.6	ρ1(Tm4)	Ms1(Tm4)	Tm4(4)	0.05
0.7298	300	50	6	0.72	0.72	0.75	0.55	0.2	0.6	ρ1(Tm5)	Ms1(Tm5)	Tm5(5)	0.05
0.8782	400	50	6	0.72	0.72	0.75	0.55	0.2	0.6	ρ1(Tm1)	Ms1(Tm1)	Tm1(1)	0.05
0.8907	400	50	6	0.72	0.72	0.75	0.55	0.2	0.6	ρ1(Tm2)	Ms1(Tm2)	Tm2(2)	0.05
0.8451	400	50	6	0.72	0.72	0.75	0.55	0.2	0.6	ρ1(Tm3)	Ms1(Tm3)	Tm3(3)	0.05
0.9120	400	50	6	0.72	0.72	0.75	0.55	0.2	0.6	ρ1(Tm4)	Ms1(Tm4)	Tm4(4)	0.05
0.8438	400	50	6	0.72	0.72	0.75	0.55	0.2	0.6	ρ1(Tm5)	Ms1(Tm5)	Tm5(5)	0.05
0.9376	500	50	6	0.72	0.72	0.75	0.55	0.2	0.6	ρ1(Tm1)	Ms1(Tm1)	Tm1(1)	0.05
0.9459	500	50	6	0.72	0.72	0.75	0.55	0.2	0.6	ρ1(Tm2)	Ms1(Tm2)	Tm2(2)	0.05
0.9141	500	50	6	0.72	0.72	0.75	0.55	0.2	0.6	ρ1(Tm3)	Ms1(Tm3)	Tm3(3)	0.05
0.9594	500	50	6	0.72	0.72	0.75	0.55	0.2	0.6	ρ1(Tm4)	Ms1(Tm4)	Tm4(4)	0.05
0.9132	500	50	6	0.72	0.72	0.75	0.55	0.2	0.6	ρ1(Tm5)	Ms1(Tm5)	Tm5(5)	0.05

Item	Values
ρ1(Tm1)	1, 0.536, 0.427, 0.34, 0.271, 0.216
ρ1(Tm2)	1, 0.34, 0.304, 0.271, 0.242, 0.216
ρ1(Tm3)	1, 0.6, 0.536, 0.478, 0.427, 0.216
ρ1(Tm4)	1, 0.6, 0.536, 0.271, 0.242, 0.216
ρ1(Tm5)	1, 0.403, 0.381, 0.36, 0.34, 0.216
Ms1(Tm1)	0, 0.06, 0.12, 0.18, 0.24, 0.3
Ms1(Tm2)	0, 0.18, 0.21, 0.24, 0.27, 0.3
Ms1(Tm3)	0, 0.03, 0.06, 0.09, 0.12, 0.3
Ms1(Tm4)	0, 0.03, 0.06, 0.24, 0.27, 0.3
Ms1(Tm5)	0, 0.14, 0.15, 0.17, 0.18, 0.3
Tm1(1)	0, 0.2, 0.4, 0.6, 0.8, 1
Tm2(2)	0, 0.6, 0.7, 0.8, 0.9, 1
Tm3(3)	0, 0.1, 0.2, 0.3, 0.4, 1
Tm4(4)	0, 0.1, 0.2, 0.8, 0.9, 1
Tm5(5)	0, 0.45, 0.5, 0.55, 0.6, 1

Plots



The legend, *Time Set*, gives the sequence number of the measurement columns. Thus, 1.0 is Tm1, 2.0 is Tm2, and so on.

The pattern Tm4 consistently produces the highest power across all sample sizes. Remember that Tm4 put the measurements at the beginning and the end, but none in the middle.

Patterns Tm3 and Tm5 are nearly tied for achieving the lowest powers. Tm3 put most of the measurements at the beginning of the study. Tm5 put most of the measurements during the middle of the study.

Note that Tm1, the equally spaced times, is in the middle of the pack.

Example 7 – Entering a Correlation Matrix

This example will show how a correlation matrix can be loaded directly.

In this example the basic parameters are P1(0) = 0.72, P2(0) = 0.72, P1(1) = 0.75, and P2(1) = 0.55. The significance level is 0.05, the sample size ranges from 100 to 500, and R is 50%. There are to be 4 equally spaced time measurements. A correlation matrix (shown below) is available from a previous study. The missing input type will be set to Linear from 0 to 30% and the pairwise missing assumption is independent.

Table of Correlations

C1	C2	C3	C4
1.0000	0.7000	0.4900	0.3430
0.7000	1.0000	0.7000	0.4900
0.4900	0.7000	1.0000	0.7000
0.3430	0.4900	0.7000	1.0000

Setup

If the procedure window is not already open, use the PASS Home window to open it. The parameters for this example are listed below and are stored in the **Example 7** settings file. To load these settings to the procedure window, click **Open Example Settings File** in the Help Center or File menu.

Design Tab

Solve For ....**Power**
Alternative Hypothesis ....**Two-Sided**
Alpha...**0.05**
N (Subjects)...**100 to 500 by 100**
R (Group 1 Allocation %) **50**
Measurement Time Input Type.....................**Equally Spaced Measurement Times**
M (Number of Measurements)......................**4**
P1(0) and P2(0) Input Type ....**P1(0) = P2(0)**
P1(0) and P2(0) **0.72**
P1(1) Input Type ....**P1(1)**
P1(1)..**0.75**
P2(1)..**0.55**
Pattern of ρ's Across Time...........................**Matrix on Spreadsheet**
Columns Containing the ρjk's **C1-C4**
Missing Input Type.......................................**Linear (Steady Change)**
Pairwise Missing Pattern...............................**Independent (Ind)**
First Missing Proportion (Ind)........................**0**
Last Missing Proportion (Ind).........................**0.3**

Input Spreadsheet Data

Row	C1	C2	C3	C4
1	1.000	0.70	0.49	0.343
2	0.700	1.00	0.70	0.490
3	0.490	0.70	1.00	0.700
4	0.343	0.49	0.70	1.000

Output

Click the Calculate button to perform the calculations and generate the following output.

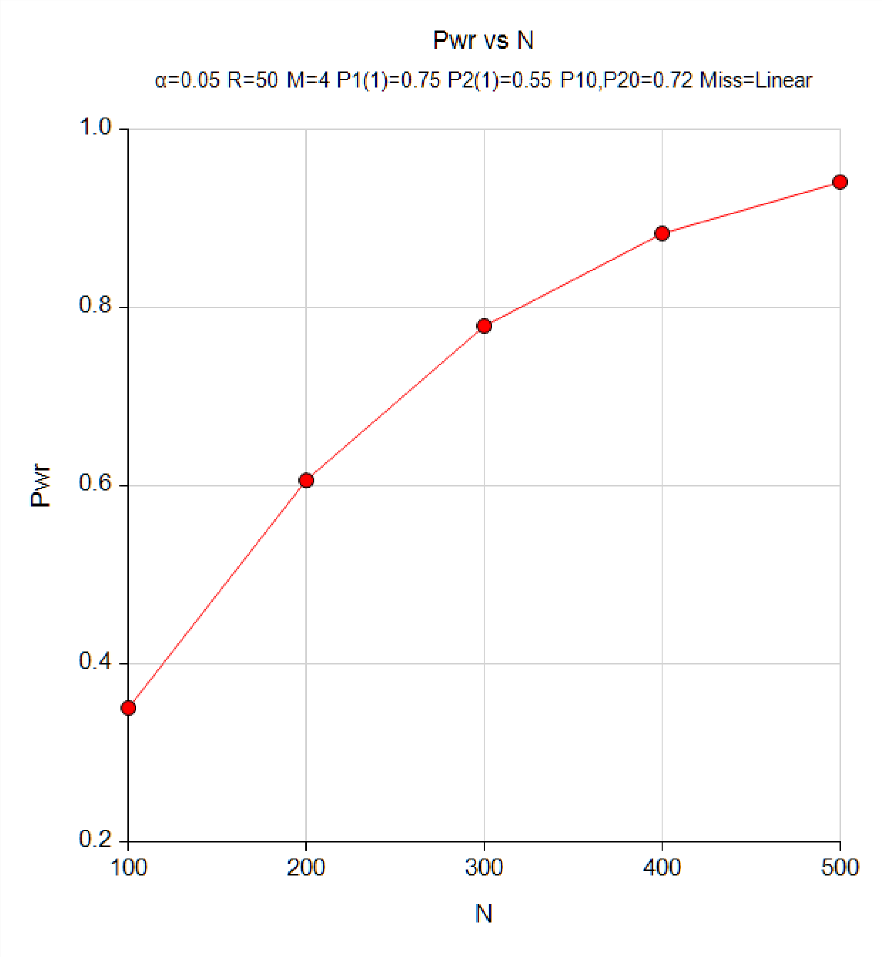
Numeric Results for the Test of Slope-Difference in Binary Data using GEE

Solve For: Power
Measurement Times: Equally spaced
Correlation: Matrix stored on spreadsheet in columns C1-C4
Missing Pattern: Range of missing proportions
Observant Proportions: Assume independence
Pk(t): Event probability of group k (1 = treatment, 2 = control) at time proportion t

				Event Probability									
	Total Sample Size	Group 1 Allocation Percent	Number of Meas. Times	at First Measurement		at Last Measurement			Base Corr.	First Row of Corr.	Missing Data	Measurement	
Power	N	R	M	P1(0)	P2(0)	P1(1)	P2(1)	Difference	ρ	Matrix	Proportions	Times	Alpha
0.3506	100	50	4	0.72	0.72	0.75	0.55	0.2	N/A	ρ1(1)	Ms1(1)	T(1)	0.05
0.6061	200	50	4	0.72	0.72	0.75	0.55	0.2	N/A	ρ1(1)	Ms1(1)	T(1)	0.05
0.7794	300	50	4	0.72	0.72	0.75	0.55	0.2	N/A	ρ1(1)	Ms1(1)	T(1)	0.05
0.8835	400	50	4	0.72	0.72	0.75	0.55	0.2	N/A	ρ1(1)	Ms1(1)	T(1)	0.05
0.9412	500	50	4	0.72	0.72	0.75	0.55	0.2	N/A	ρ1(1)	Ms1(1)	T(1)	0.05

Item	Values
ρ1(1)	1, 0.7, 0.49, 0.343
Ms1(1)	0, 0.1, 0.2, 0.3
T(1)	0, 0.33, 0.67, 1

Plots



The standard reports are displayed.

Example 8 – Entering an Observant Probabilities Matrix

This example will show how an observant probabilities matrix can be loaded directly.

In this example the basic parameters are P1(0) = 0.72, P2(0) = 0.72, P1(1) = 0.75, and P2(1) = 0.55. The significance level is 0.05, the sample size ranges from 100 to 500, and R is 50%. There are to be 4 equally spaced time measurements. The correlation pattern will be Linear Exponential Decay with a base correlation of 0.4, Base Time Proportion of 0.1, and Emax set to 4. The missing input type will be set to Matrix of Pairwise Missing.

Table of Observant Probabilities

Row	C1	C2	C3	C4
1	1.00	0.90	0.80	0.70
2	0.90	0.90	0.72	0.63
3	0.80	0.72	0.80	0.56
4	0.70	0.63	0.56	0.70

This table gives the pairwise observant probabilities. That is, each entry gives the probability of obtaining a response for both the row and column time points. For example, 0.63 is the probability of observing both the second response and the fourth response.

Setup

If the procedure window is not already open, use the PASS Home window to open it. The parameters for this example are listed below and are stored in the **Example 8** settings file. To load these settings to the procedure window, click **Open Example Settings File** in the Help Center or File menu.

Design Tab

Solve For .... **Power**
Alternative Hypothesis .... **Two-Sided**
Alpha.. **0.05**
N (Subjects) .... **100 to 500 by 100**
R (Group 1 Allocation %) **50**
Measurement Time Input Type .... **Equally Spaced Measurement Times**
M (Number of Measurements) **4**
P1(0) and P2(0) Input Type .... **P1(0) = P2(0)**
P1(0) and P2(0) **0.72**
P1(1) Input Type .... **P1(1)**
P1(1) **0.75**
P2(1) **0.55**
Pattern of ρ's Across Time............................ **Linear Exponential Decay**
ρ (Base Correlation)...................................... **0.4**
Base Time Proportion **0.10**
Emax (Max Decay Exponent) **4**
Missing Input Type.. **Pairwise Observed Proportions on Spreadsheet**
Columns of Pairwise Observed..................... **C1-C4**

Input Spreadsheet Data

Row	C1	C2	C3	C4
1	1.0	0.90	0.80	0.70
2	0.9	0.90	0.72	0.63
3	0.8	0.72	0.80	0.56
4	0.7	0.63	0.56	0.70

Output

Click the Calculate button to perform the calculations and generate the following output.

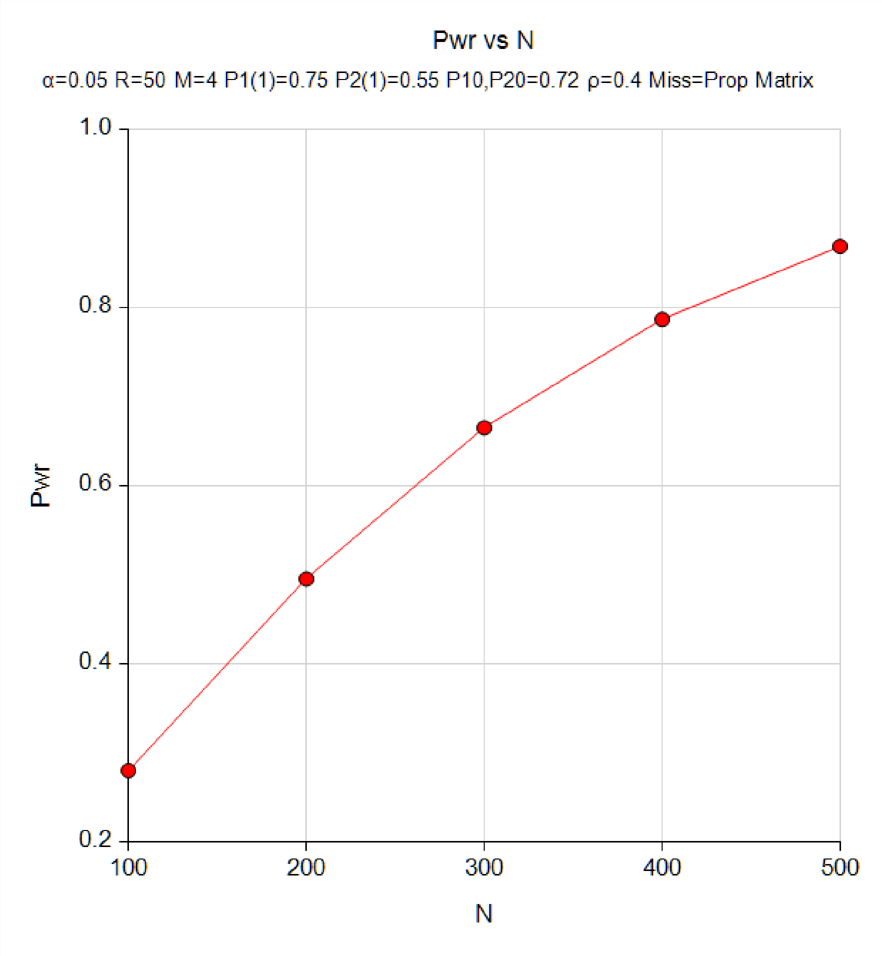
Numeric Results for the Test of Slope-Difference in Binary Data using GEE

Solve For: Power
Measurement Times: Equally spaced
Correlation: Linear exponential decay, with Emax = 4 and Base Time Prop = 0.10
Missing Pattern: N/A. Matrix of observant probabilities entered in columns C1-C4.
Pk(t): Event probability of group k (1 = treatment, 2 = control) at time proportion t

				Event Probability									
				at First Measurement		at Last Measurement							
Power	Total Sample Size N	Group 1 Allocation Percent R	Number of Meas. Times M	P1(0)	P2(0)	P1(1)	P2(1)	Difference	Base Corr. ρ	First Row of Corr. Matrix	Missing Data Proportions	Measurement Times	Alpha
0.2802	100	50	4	0.72	0.72	0.75	0.55	0.2	0.4	ρ1(1)	N/A	T(1)	0.05
0.4955	200	50	4	0.72	0.72	0.75	0.55	0.2	0.4	ρ1(1)	N/A	T(1)	0.05
0.6651	300	50	4	0.72	0.72	0.75	0.55	0.2	0.4	ρ1(1)	N/A	T(1)	0.05
0.7869	400	50	4	0.72	0.72	0.75	0.55	0.2	0.4	ρ1(1)	N/A	T(1)	0.05
0.8689	500	50	4	0.72	0.72	0.75	0.55	0.2	0.4	ρ1(1)	N/A	T(1)	0.05

Item	Values
ρ1(1)	1, 0.196, 0.071, 0.026
T(1)	0, 0.33, 0.67, 1

Plots



The standard reports are displayed.