

## Chapter 397

# GEE Tests for the Slope of Two Groups in a Repeated Measures Design (Count Outcome)

---

## Introduction

This module calculates the power for testing the difference between two slopes from correlated count data that are analyzed using the GEE method. Such data occur in two design types: clustered and longitudinal.

GEE does not require the full specification of the joint distribution of the repeated measurements, as long as the marginal model is correctly specified. Estimation consistency is achieved even if the correlation matrix is incorrect. Also, the correlation matrix of the responses is specified directly, rather than using an intermediate, random effects model as is the case in MM. For clustered designs, GEE often uses a *compound symmetric* (CS) correlation structure. For longitudinal data, an *autoregressive* (AR(1)) correlation structure is often used.

---

## Missing Values

This procedure allows you to specify various patterns of incomplete (or missing) data. Subjects may miss some appointments but attend others. This phenomenon of incomplete data can be accounted for in the sample size calculation which can greatly reduce the overall sample size from that calculated by just omitting subjects with incomplete observations.

---

## Technical Details

---

### Theory and Notation

The details of the calculation of sample size and power is given in Ahn, Heo, and Zhang (2015), chapter 4, section 4.7, pages 126 - 129. We will use their notation here.

Suppose we have  $n_1$  subjects in group 1 (treatment) and  $n_2$  subjects in group 2 (control) for a total of  $N$  subjects, each measured on  $M$  occasions at times  $t_j$  ( $j = 1, \dots, M$ ). For convenience, we normalize these time points to the proportion of total time so that  $t_1 = 0$  and  $t_M = 1$ . The mean event rate of the Poisson count responses  $\mu_{kij}$  is modeled by the log model

$$\log(\mu_{kij}) = a_k + \beta_k t_j$$

## GEE Tests for the Slope of Two Groups in a Repeated Measures Design (Count Outcome)

where

- $y_{kij}$  is the  $j^{\text{th}}$  response from subject  $i$  in group  $k$ .
- $\mu_{kij}$  is expectation of  $y_{kij}$  assuming the Poisson distribution,
- $a_1$  is the regression coefficient giving intercept of the treatment group,
- $a_2$  is the regression coefficient giving intercept of the control group,
- $\beta_1$  is the regression coefficient giving slope of the treatment group,
- $\beta_2$  is the regression coefficient giving slope of the control group.

This model can be expressed as

$$\mu_{kij} = \exp(a_k + \beta_k t_j)$$

GEE is used to estimate and test hypotheses about the equality of the slopes  $\beta_1$  and  $\beta_2$ .

## Correlation Patterns

In a longitudinal design with  $N$  subjects, each measured  $M$  times, observations from a single subject are correlated, and a pattern of those correlations through time needs to be specified. Several choices are available.

### Compound Symmetry

A compound symmetry correlation model assumes that all correlations are equal. That is

$$[\rho_{jj'}] = \begin{bmatrix} 1 & \rho & \rho & \rho & \cdots & \rho \\ \rho & 1 & \rho & \rho & \cdots & \rho \\ \rho & \rho & 1 & \rho & \cdots & \rho \\ \rho & \rho & \rho & 1 & \cdots & \rho \\ \vdots & \vdots & \vdots & \vdots & \ddots & \vdots \\ \rho & \rho & \rho & \rho & \cdots & 1 \end{bmatrix}_{M \times M}$$

where  $\rho$  is the baseline correlation.

### Banded(1)

A Banded(1) (banded order 1) correlation model assumes that correlations for observations one time period apart are equal to  $\rho$ , and correlations for measurements greater than one time period apart are equal to zero. That is

$$[\rho_{jj'}] = \begin{bmatrix} 1 & \rho & 0 & 0 & \cdots & 0 \\ \rho & 1 & \rho & 0 & \cdots & 0 \\ 0 & \rho & 1 & \rho & \cdots & 0 \\ 0 & 0 & \rho & 1 & \cdots & 0 \\ \vdots & \vdots & \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & 0 & 0 & \cdots & 1 \end{bmatrix}_{M \times M}$$

where  $\rho$  is the baseline correlation.

## Banded(2)

A Banded(2) (banded order 2) correlation model assumes that correlations for observations one time period or two periods apart are equal to  $\rho$ , and correlations for measurements greater than one time period apart are equal to zero. That is

$$[\rho_{jj'}] = \begin{bmatrix} 1 & \rho & \rho & 0 & \cdots & 0 \\ \rho & 1 & \rho & \rho & \cdots & 0 \\ \rho & \rho & 1 & \rho & \cdots & 0 \\ 0 & \rho & \rho & 1 & \cdots & 0 \\ \vdots & \vdots & \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & 0 & 0 & \cdots & 1 \end{bmatrix}_{M \times M}$$

where  $\rho$  is the baseline correlation.

## AR1 (Traditional)

This version of AR1 (autoregressive order 1) correlation model assumes that correlations  $t$  time periods apart are equal to  $\rho^t$ . That is

$$[\rho_{jj'}] = \begin{bmatrix} 1 & \rho & \rho^2 & \rho^3 & \cdots & \rho^{M-1} \\ \rho & 1 & \rho & \rho^2 & \cdots & \rho^{M-2} \\ \rho^2 & \rho & 1 & \rho & \cdots & \rho^{M-3} \\ \rho^3 & \rho^2 & \rho & 1 & \cdots & \rho^{M-4} \\ \vdots & \vdots & \vdots & \vdots & \ddots & \vdots \\ \rho^{M-1} & \rho^{M-2} & \rho^{M-3} & \rho^{M-4} & \cdots & 1 \end{bmatrix}_{M \times M}$$

where  $\rho$  is the baseline correlation.

## AR1 (Proportional)

This version of AR1 (autoregressive order 1) correlation model is described in the book by Ahn et al. (2015). It assumes that correlations  $|t_j - t_k|$  time periods apart are equal to  $\rho^{|t_j - t_k|}$ . That is

$$[\rho_{jk}] = [\rho^{|t_j - t_k|}]_{M \times M}$$

where  $\rho$  is the baseline correlation. Note that in this pattern, the value of  $\rho$  is shown in the final column since in this case  $t_j = 0$  and  $t_k = 1$ , so  $|t_j - t_k| = 1$ .

## Damped Exponential

A damped exponential is an extension of the AR(1) correlation model in which the exponents are raised to the power  $Dexp$  ( $\theta = Dexp$  in the diagram below). This causes the resulting correlations to be reduced (dampened). Here is an example

$$[\rho_{jj'}] = \begin{bmatrix} 1 & \rho & \rho^{2\theta} & \rho^{3\theta} & \dots & \rho^{(M-1)\theta} \\ \rho & 1 & \rho & \rho^{2\theta} & \dots & \rho^{(M-2)\theta} \\ \rho^{2\theta} & \rho & 1 & \rho & \dots & \rho^{(M-3)\theta} \\ \rho^{3\theta} & \rho^{2\theta} & \rho & 1 & \dots & \rho^{(M-4)\theta} \\ \vdots & \vdots & \vdots & \vdots & \ddots & \vdots \\ \rho^{(M-1)\theta} & \rho^{(M-2)\theta} & \rho^{(M-3)\theta} & \rho^{(M-4)\theta} & \dots & 1 \end{bmatrix}_{M \times M}$$

where  $\rho$  is the baseline correlation.

## Damped Exponential (Proportional)

This version of the damped exponential correlation model is described in the book by Ahn et al. (2015). It assumes that correlations  $|t_j - t_k|$  time periods apart are equal to  $\rho^{|t_j - t_k|^\theta}$ . That is

$$[\rho_{jk}] = \left[ \rho^{|t_j - t_k|^\theta} \right]_{M \times M}$$

where  $\rho$  is the baseline correlation. Note that in this pattern, the value of  $\rho^{|t_j - t_k|^\theta}$  turns up in the final column since in this case  $t_j = 0$  and  $t_k = 1$ , so  $|t_j - t_k| = 1$ .

## Linear Exponential Decay

A linear exponential decay correlation model is one in which the exponent of the correlation decays according to a linear equation from 1 at the *Base Time Proportion* to a final value, *Emax*. The resulting pattern looks similar to the damped exponential. Note that the exponents are applied to the absolute difference between the Measurement Time Proportions. This method allows you to easily construct comparable correlation matrices of different dimensions. Otherwise, differences in the resulting power would be more strongly due to differences in the correlation matrices.

Here is an example. Suppose  $m$  is 6,  $\rho = 0.5$ ,  $Emax = 3$ , the *Base Time Proportion* is 0.20, and the Measurement Time Proportions are (0, 0.2, 0.4, 0.6, 0.8, 1). The following correlation matrix would be obtained

$$[\rho_{jj'}] = \begin{bmatrix} 1 & 0.5 & 0.3536 & 0.25 & 0.1768 & 0.125 \\ 0.5 & 1 & 0.5 & 0.3536 & 0.25 & 0.1768 \\ 0.3536 & 0.5 & 1 & 0.5 & 0.3536 & 0.25 \\ 0.25 & 0.3536 & 0.5 & 1 & 0.5 & 0.3536 \\ 0.1768 & 0.25 & 0.3536 & 0.5 & 0.3536 & 0.5 \\ 0.125 & 0.1768 & 0.25 & 0.3536 & 0.5 & 1 \end{bmatrix}_{M \times M}$$

## GEE Tests for the Slope of Two Groups in a Repeated Measures Design (Count Outcome)

Note that in the top row, the correlation is 0.5 for the second (0.2 - 0) time point and 0.125 (0.5<sup>3</sup>) at the last (1 - 0) time points. The correlations are obtained by raising 0.5 to the appropriate exponent. The linear equation from 1 to 3 results in the exponents 1, 1.5, 2, 2.5, 3 correspondent to the time proportions 0, 0.2, 0.4, 0.6, 0.8, and 1.

As a further example, note that the correlation for the 0.4 time point is,  $0.5^{1.5} = 0.35355339 \approx 0.3536$ .

This method allows you to compare various values of  $m$  while keeping the correlation matrix similar. To see what we mean, consider what the correlation matrix looks like when  $m$  is reduced to 4 and the measurement time proportions are set to (0, 0.2, 0.6, 1). It becomes

$$[\rho_{jj'}] = \begin{bmatrix} 1 & 0.5 & 0.25 & 0.125 \\ 0.5 & 1 & 0.5 & 0.25 \\ 0.25 & 0.5 & 1 & 0.5 \\ 0.125 & 0.25 & 0.5 & 1 \end{bmatrix}_{M \times M}$$

Note that the correlation at a measurement time difference of 0.6 is equal to 0.25 in both matrices.

## Missing Data Patterns

The problem of missing data occurs for several reasons. In longitudinal studies in which a subject is measured multiple times, missing data becomes more complicated to model because it is possible that a subject is measured only some of the time. In fact, it is probably more common for data to be incomplete than complete. The approach of omitting subjects with incomplete data during the planning phase is very inaccurate. Certainly, subjects with partial measurements are included in the analysis. This procedure provides several missing data patterns to choose from so that your sample size calculations are more realistic.

In the presentation to follow, we denote the percent of subjects with a missing response at time point  $t_j$  as  $\kappa_j$ . The proportion non-missing at a particular time point is  $\phi_j = 1 - \kappa_j$ . We will refer to  $\phi_j$  as the *marginal observant probability* at time  $t_j$  and  $\phi_{jj'}$  as a *joint observant probability* at times  $t_j$  and  $t_{j'}$ .

### Pairwise Missing Pattern

The program provides three options for how the pairwise (joint) observant probabilities  $\phi_{jj'}$  are calculated. These are

**Independent (Ind):**  $\phi_{jj'} = \phi_j \phi_{j'}$ ,  $\phi_{jj} = \phi_j$

**Monotonic (Mon):**  $\phi_{jj'} = \phi_k$  where  $k = \max(j, j')$

**Mixture:**  $\phi_{jj'} = W(\text{Ind}) + (1 - W)(\text{Mon})$  for weighting factor  $W$ .

## Missing Input Type

There are several ways in which the missing value pattern can be specified. Each missing value pattern is a list of missing proportions at each of the  $M$  time points. Each value in the list must be non-negative and less than 1. Possible input choices are

- **Constant = 0**

All missing proportions are set to 0. That is, there are no missing values.

- **Constant**

All missing proportions are set to constant value.

- **Piecewise Constant on Spreadsheet**

A set of missing proportions are defined for several time intervals using the spreadsheet. One column contains the missing proportions for the interval, going down the rows. Another column defines the corresponding upper limit of time proportion of the interval. The lower limit is implied by the limit given immediately above. The program assumes that the first-time interval starts at 0 percent.

- **Linear (Steady Change)**

The missing proportions fall along a straight-line between 0 and 1 elapsed time. Only the first and last proportions are entered.

- **Piecewise Linear on Spreadsheet**

The missing proportions fall along a set of connected straight-lines that are defined by two columns on the spreadsheet.

- **List**

Enter a list of  $M$  missing proportions, one for each time point.

- **Multiple Lists on Spreadsheet**

Select multiple columns containing vertical lists of missing proportions. Each column contains a set of missing proportions in rows, one for each time point.

- **Pairwise Observed Proportions on Spreadsheet**

Enter an  $M \times M$  matrix of observed probabilities by selecting  $M$  columns. These observed probabilities are the proportion of the responses for both the row and column time points that are observed.

## Sample Size Calculations

The details of the calculation of sample size and power is given in Ahn, Heo, and Zhang (2015), Chapter 4. These are summarized here.

GEE is used to estimate the regression coefficients  $\beta_1$  and  $\beta_2$ . The significance of  $\beta_1 - \beta_2$ , the coefficient associated with the difference between the treatment and control group slopes, is tested using a Wald statistic for which the following sample size formula is derived

$$n = \frac{\left(\frac{v_1^2}{r_1} + \frac{v_2^2}{r_2}\right) \left(z_{1-\alpha} + z_{1-\gamma}\right)^2}{\delta^2}$$

## GEE Tests for the Slope of Two Groups in a Repeated Measures Design (Count Outcome)

where

$h = 1$  (one-sided test) or  $2$  (two-sided test)

$\gamma = 1 - \text{power}$

$\alpha = \text{significance level}$

$$\nu_k^2 = \frac{s_k^2 + c_k^2}{s_k^4}$$

$$\delta = \beta_1 - \beta_2$$

$$s_k^2 = \sum_{j=1}^M \phi_j \mu_{kj} (t_j - \tau_k)^2$$

$$c_k^2 = \sum_{j \neq j'} \sum_{j'=1}^M \phi_{jj'} \rho_{jj'} \sqrt{\mu_{kj} \mu_{kj'}} (t_j - \tau_k)(t_{j'} - \tau_k)$$

$$\tau_k = \frac{\sum_{j=1}^M \phi_j \mu_{kj} t_j}{\sum_{j=1}^M \phi_j \mu_{kj}}$$

$$r_k = n_k/N$$

$\phi_j = 1 - \kappa_j$ , where  $\kappa_j$  = proportion missing at the  $j^{\text{th}}$  time point

$\rho_{jj'}$  is the corresponding element from within-subject correlation matrix

$\phi_{jj'}$  is the joint observant probability of observing both  $y_{ij}$  and  $y_{ij'}$  for every subject  $i$

Three possible choices are available to calculate  $\phi_{jj'}$ . These are

*Independent:*  $\phi_{jj'} = \phi_j \phi_{j'}$ ,  $\phi_{jj} = \phi_j$

*Monotonic:*  $\phi_{jj'} = \phi_k$  where  $k = \max(j, j')$

*Mixture:*  $\phi_{jj'} = W(\text{Independent}) + (1 - W)(\text{Monotonic})$  for weighting factor  $W$ .

The above formula is easily rearranged to obtain a formula for power.

## Example 1 – Determining Sample Size

Researchers are planning a study to assess the effectiveness of a new anti-epileptic drug which is administered with the standard drug. They want to compare subjects who take the new drug with subjects who only take the standard drug. Their experimental protocol calls for a baseline measurement, followed by administration of the drug therapies, followed by five additional measurements one week apart. They want to detect a difference of 1 in the final response rates. They also want a sensitivity analysis by considering a range of possible differences from 0.5 to 1.5.

Similar studies have found that the average baseline rate is 2.5 which is reduced to 1.75 at the end of the treatment period with the standard drug.

Previous studies showed an autocorrelation between adjacent measurements on the same individual of 0.7, so they want to try autocorrelations of 0.6, 0.7, and 0.8. The researchers assume that first-order autocorrelation adequately represents the autocorrelation pattern. A two-sided Wald test will be conducted at the 0.05 significance level and at 90% power. The subjects will be divided equally between the treatment and control groups.

The researchers anticipate that the missing data pattern across time will begin at 0% missing and increase steadily to 40% at the sixth measurement. They assume that the pairwise missing probabilities are *independent*.

What are the sample size requirements for this study?

### Setup

If the procedure window is not already open, use the PASS Home window to open it. The parameters for this example are listed below and are stored in the **Example 1** settings file. To load these settings to the procedure window, click **Open Example Settings File** in the Help Center or File menu.

#### Design Tab

Solve For .....	<b>Sample Size</b>
Alternative Hypothesis .....	<b>Two-Sided</b>
Power.....	<b>0.90</b>
Alpha.....	<b>0.05</b>
R (Group 1 Allocation %) .....	<b>50</b>
Measurement Time Input Type .....	<b>Equally Spaced Measurement Times</b>
M (Number of Measurements) .....	<b>6</b>
$\mu_1(0)$ and $\mu_2(0)$ Input Type .....	<b><math>\mu_1(0) = \mu_2(0)</math></b>
$\mu_1(0)$ and $\mu_2(0)$ .....	<b>2.5</b>
$\mu_1(1)$ Input Type .....	<b>Difference (<math>\mu_1(1) - \mu_2(1)</math>)</b>
Difference ( $\mu_1(1) - \mu_2(1)$ ) .....	<b>0.5, 1, 1.5</b>
$\mu_2(1)$ .....	<b>1.75</b>
Pattern of p's Across Time.....	<b>AR1 (Traditional)</b>
$\rho$ (Base Correlation).....	<b>0.6, 0.7, 0.8</b>
Missing Input Type.....	<b>Linear (Steady Change)</b>
Pairwise Missing Pattern.....	<b>Independent (Ind)</b>
First Missing Proportion (Ind).....	<b>0</b>
Last Missing Proportion (Ind) .....	<b>0.4</b>

## GEE Tests for the Slope of Two Groups in a Repeated Measures Design (Count Outcome)

## Output

Click the Calculate button to perform the calculations and generate the following output.

### Numeric Reports

#### Numeric Results for the Test of Slope-Difference in Count Data using GEE

Solve For: **Sample Size**  
 Measurement Times: Equally spaced  
 Correlation: AR1:  $\rho_{(j,k)} = \rho^{|j-k|}$   
 Missing Pattern: Range of missing proportions  
 Observant Proportions: Assume independence  
 $\mu_k(t)$ : Event rate of group k (1 = treatment, 2 = control) at time proportion t

Power	Total Sample Size	Group 1 Allocation Percent	Number of Meas. Times	Event Rate						Base Corr. $\rho$	First Row of Corr. Matrix	Missing Data Proportions	Measurement Times	Alpha					
				at First Measurement			at Last Measurement												
				$\mu_1(0)$	$\mu_2(0)$	$\mu_1(1)$	$\mu_2(1)$	Difference											
0.9001	703	50	6	2.5	2.5	2.25	1.75	0.5	0.6	$\rho_1(1)$	$Ms1(1)$	T(1)	0.05						
0.9003	654	50	6	2.5	2.5	2.25	1.75	0.5	0.7	$\rho_2(1)$	$Ms1(1)$	T(1)	0.05						
0.9002	553	50	6	2.5	2.5	2.25	1.75	0.5	0.8	$\rho_3(1)$	$Ms1(1)$	T(1)	0.05						
0.9006	208	50	6	2.5	2.5	2.75	1.75	1.0	0.6	$\rho_1(1)$	$Ms1(1)$	T(1)	0.05						
0.9000	193	50	6	2.5	2.5	2.75	1.75	1.0	0.7	$\rho_2(1)$	$Ms1(1)$	T(1)	0.05						
0.9014	164	50	6	2.5	2.5	2.75	1.75	1.0	0.8	$\rho_3(1)$	$Ms1(1)$	T(1)	0.05						
0.9004	107	50	6	2.5	2.5	3.25	1.75	1.5	0.6	$\rho_1(1)$	$Ms1(1)$	T(1)	0.05						
0.9018	100	50	6	2.5	2.5	3.25	1.75	1.5	0.7	$\rho_2(1)$	$Ms1(1)$	T(1)	0.05						
0.9030	85	50	6	2.5	2.5	3.25	1.75	1.5	0.8	$\rho_3(1)$	$Ms1(1)$	T(1)	0.05						

Item	Values
$p_1(1)$	1, 0.6, 0.36, 0.216, 0.13, 0.078
$p_2(1)$	1, 0.7, 0.49, 0.343, 0.24, 0.168
$p_3(1)$	1, 0.8, 0.64, 0.512, 0.41, 0.328
$Ms1(1)$	0, 0.08, 0.16, 0.24, 0.32, 0.4
T(1)	0, 0.2, 0.4, 0.6, 0.8, 1

Power	The probability of rejecting a false null hypothesis when the alternative hypothesis is true.
N	The total number of subjects in the study.
R	The treatment group allocation proportion. It is the proportion of subjects that are in the treatment group.
M	The number of time points at which each subject is measured.
Difference	The difference between the treatment and control event rates at the final measurement time. Difference = $\mu_1(1) - \mu_2(1)$ .
$\mu_k(t)$	The mean event rate of group k (1 = treatment, 2 = control) at the measurement time proportion t.
$\rho$	The base correlation between two responses on the same subject. It may be transformed based on the correlation pattern.
First Row of Correlation Matrix	Presents the top row of the correlation matrix.
Missing Data Proportions	Gives the name of the set containing the missing data proportions across time.
Measurement Times	Gives the name of the set containing the measurement time proportions. These measurement times represent the proportion of the total study time that has elapsed just before the measurement.
Alpha	The probability of rejecting a true null hypothesis.

## GEE Tests for the Slope of Two Groups in a Repeated Measures Design (Count Outcome)

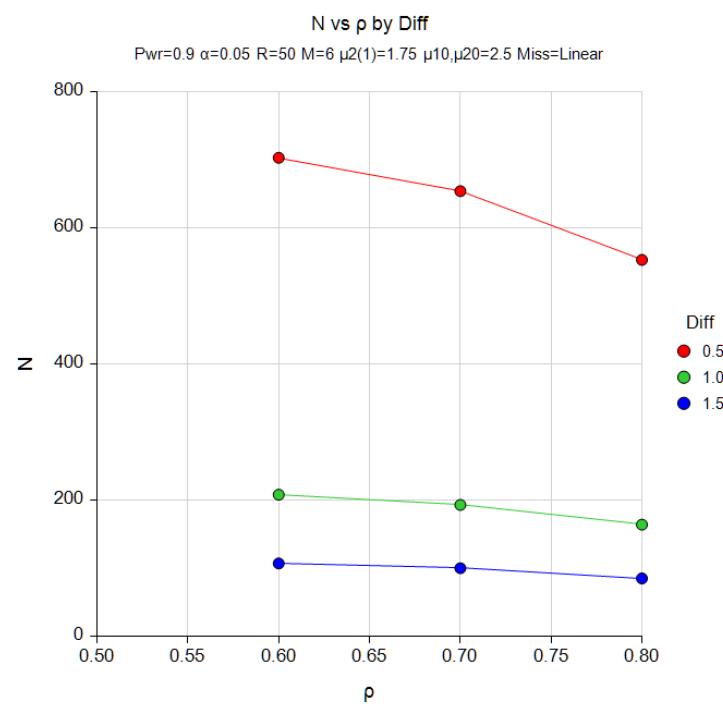
**Summary Statements**

A two-group repeated measures design (with a count response and with 6 measurements for each subject) will be used to test whether there is a group difference in slopes. The comparison will be made using a two-sided Wald test using GEE methods, with a Type I error rate ( $\alpha$ ) of 0.05. The (repeated) measurements of each subject will be made at the following 6 times, expressed as proportions of the total study time: 0, 0.2, 0.4, 0.6, 0.8, 1. Missing values are assumed to occur completely at random (MCAR). The missing value proportions will be combined to form the pairwise observant probabilities using an independent pairwise missing pattern. The anticipated proportions missing at each measurement time are 0, 0.08, 0.16, 0.24, 0.32, 0.4. The first row of the autocorrelation matrix of the responses within a subject is assumed to be 1, 0.6, 0.36, 0.216, 0.13, 0.078, with subsequent rows following the same pattern (AR1:  $\rho(j,k) = \rho^{|j-k|}$ ). The initial event rates  $\mu_1(0)$  and  $\mu_2(0)$  are both assumed to be 2.5. To detect a final Group 1 event rate  $\mu_1(1)$  of 2.25 (and the corresponding slope) and a final Group 2 event rate  $\mu_2(1)$  of 1.75 (and the corresponding slope) (for a final time event rate difference of 0.5) with 90% power, the total number of needed subjects is 703 (with 50% of the subjects in the treatment group (Group 1)).

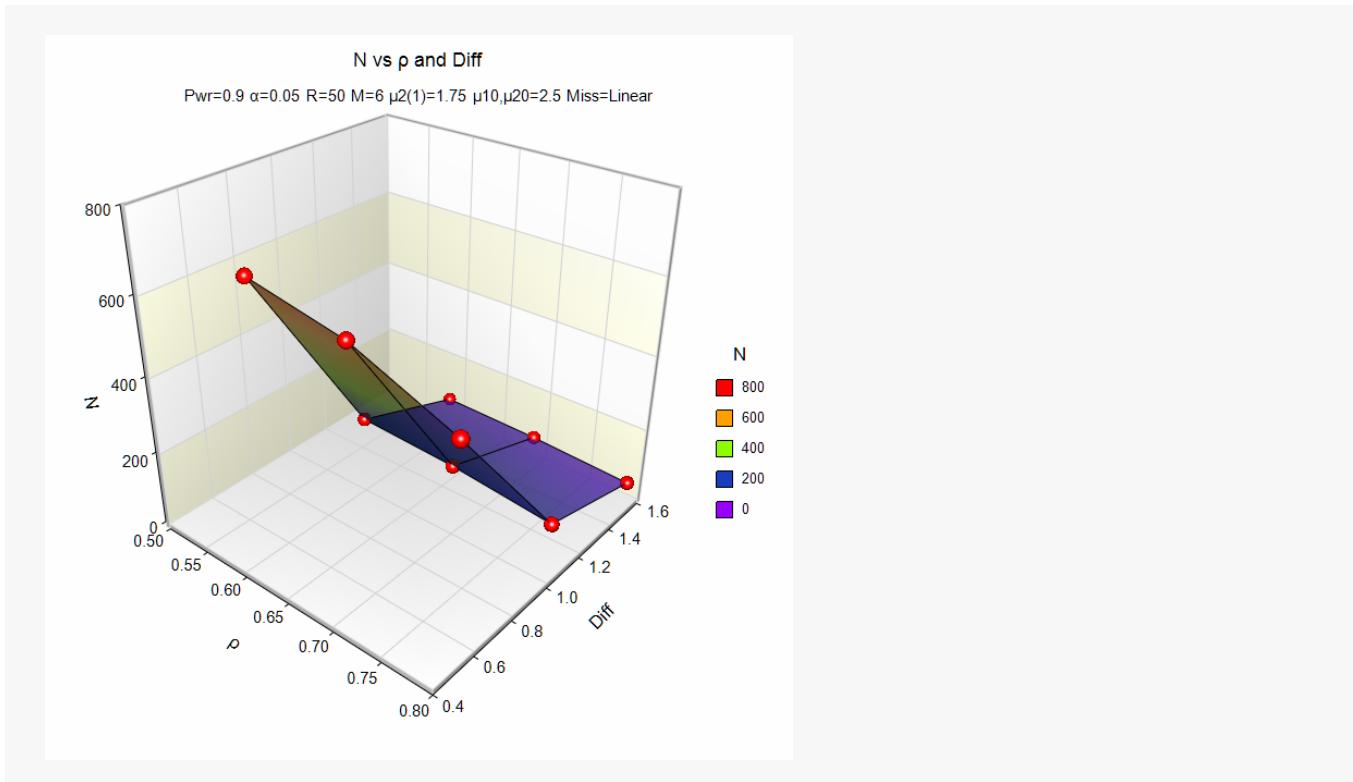
**References**

Ahn, C., Heo, M., and Zhang, S. 2015. Sample Size Calculations for Clustered and Longitudinal Outcomes in Clinical Research. CRC Press. New York.

This report gives the sample size for each value of the other parameters. The definitions of each of the items is given in the Reports Definitions section at the end of the report.

**Plots Section****Plots**

## GEE Tests for the Slope of Two Groups in a Repeated Measures Design (Count Outcome)



These plots show the relationship between sample size, difference, and  $p$  when the other parameters in the design are held constant.

## Autocorrelation Matrices

**Autocorrelation Matrix for Report Row 1**

Time	T(0)	T(0.2)	T(0.4)	T(0.6)	T(0.8)	T(1)
T(0)	1.000	0.600	0.360	0.216	0.130	0.078
T(0.2)	0.600	1.000	0.600	0.360	0.216	0.130
T(0.4)	0.360	0.600	1.000	0.600	0.360	0.216
T(0.6)	0.216	0.360	0.600	1.000	0.600	0.360
T(0.8)	0.130	0.216	0.360	0.600	1.000	0.600
T(1)	0.078	0.130	0.216	0.360	0.600	1.000

**Autocorrelation Matrix for Report Row 2**

Time	T(0)	T(0.2)	T(0.4)	T(0.6)	T(0.8)	T(1)
T(0)	1.000	0.700	0.490	0.343	0.240	0.168
T(0.2)	0.700	1.000	0.700	0.490	0.343	0.240
T(0.4)	0.490	0.700	1.000	0.700	0.490	0.343
T(0.6)	0.343	0.490	0.700	1.000	0.700	0.490
T(0.8)	0.240	0.343	0.490	0.700	1.000	0.700
T(1)	0.168	0.240	0.343	0.490	0.700	1.000

(More Reports Follow)

These reports show the autocorrelation matrix for the indicated row of the report.

## Example 2 – Finding the Power

Continuing with Example 1, the researchers want to determine the power corresponding to sample sizes ranging from 50 to 300 for the main cases of the other parameters.

### Setup

If the procedure window is not already open, use the PASS Home window to open it. The parameters for this example are listed below and are stored in the **Example 2** settings file. To load these settings to the procedure window, click **Open Example Settings File** in the Help Center or File menu.

#### Design Tab

Solve For .....	<b>Power</b>
Alternative Hypothesis .....	<b>Two-Sided</b>
Alpha.....	<b>0.05</b>
N (Subjects) .....	<b>50 to 300 by 50</b>
R (Group 1 Allocation %) .....	<b>50</b>
Measurement Time Input Type .....	<b>Equally Spaced Measurement Times</b>
M (Number of Measurements) .....	<b>6</b>
$\mu_1(0)$ and $\mu_2(0)$ Input Type .....	$\mu_1(0) = \mu_2(0)$
$\mu_1(0)$ and $\mu_2(0)$ .....	<b>2.5</b>
$\mu_1(1)$ Input Type .....	<b>Difference (<math>\mu_1(1) - \mu_2(1)</math>)</b>
Difference ( $\mu_1(1) - \mu_2(1)$ ) .....	<b>1</b>
$\mu_2(1)$ .....	<b>1.75</b>
Pattern of $\rho$ 's Across Time .....	<b>AR1 (Traditional)</b>
$\rho$ (Base Correlation).....	<b>0.7</b>
Missing Input Type.....	<b>Linear (Steady Change)</b>
Pairwise Missing Pattern.....	<b>Independent (Ind)</b>
First Missing Proportion (Ind).....	<b>0</b>
Last Missing Proportion (Ind).....	<b>0.4</b>

## GEE Tests for the Slope of Two Groups in a Repeated Measures Design (Count Outcome)

## Output

Click the Calculate button to perform the calculations and generate the following output.

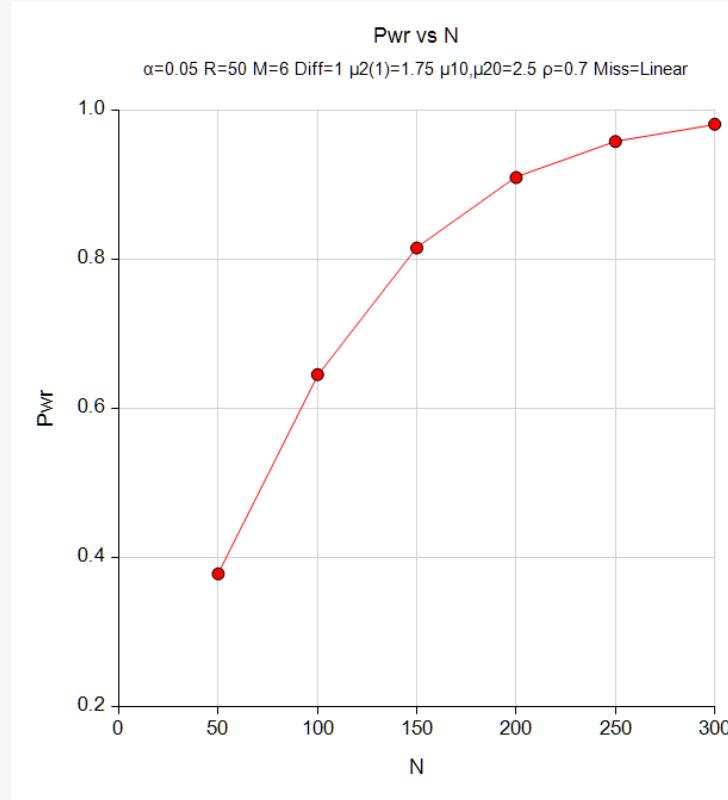
**Numeric Results for the Test of Slope-Difference in Count Data using GEE**

Solve For: Power  
 Measurement Times: Equally spaced  
 Correlation: AR1:  $\rho_{(j,k)} = \rho^{|j-k|}$   
 Missing Pattern: Range of missing proportions  
 Observant Proportions: Assume independence  
 $\mu_k(t)$ : Event rate of group k (1 = treatment, 2 = control) at time proportion t

Power	Total Sample Size N	Group 1 Allocation Percent R	Number of Meas. Times M	Event Rate				Base Corr. $\rho$	First Row of Corr. Matrix	Missing Data Proportions	Measurement Times	Alpha				
				at First Measurement		at Last Measurement										
				$\mu_1(0)$	$\mu_2(0)$	$\mu_1(1)$	$\mu_2(1)$	Difference								
0.3783	50	50	6	2.5	2.5	2.75	1.75	1	0.7	$\rho_1(1)$	$Ms1(1)$	T(1) 0.05				
0.6456	100	50	6	2.5	2.5	2.75	1.75	1	0.7	$\rho_1(1)$	$Ms1(1)$	T(1) 0.05				
0.8153	150	50	6	2.5	2.5	2.75	1.75	1	0.7	$\rho_1(1)$	$Ms1(1)$	T(1) 0.05				
0.9099	200	50	6	2.5	2.5	2.75	1.75	1	0.7	$\rho_1(1)$	$Ms1(1)$	T(1) 0.05				
0.9581	250	50	6	2.5	2.5	2.75	1.75	1	0.7	$\rho_1(1)$	$Ms1(1)$	T(1) 0.05				
0.9813	300	50	6	2.5	2.5	2.75	1.75	1	0.7	$\rho_1(1)$	$Ms1(1)$	T(1) 0.05				

Item	Values
$\rho_1(1)$	1, 0.7, 0.49, 0.343, 0.24, 0.168
$Ms1(1)$	0, 0.08, 0.16, 0.24, 0.32, 0.4
T(1)	0, 0.2, 0.4, 0.6, 0.8, 1

## Plots



The reports and plot indicate the power for each value of N.

## Example 3 – Impact of the Number of Repeated Measurements

Continuing with Examples 1 and 2, the researchers want to study the impact on the sample size of changing the number of measurements made on each individual. Their experimental protocol calls for 6 measurements. They want to see the impact of reducing the number of measurements to 3.

### Setup

If the procedure window is not already open, use the PASS Home window to open it. The parameters for this example are listed below and are stored in the **Example 3** settings file. To load these settings to the procedure window, click **Open Example Settings File** in the Help Center or File menu.

#### Design Tab

Solve For .....	<b>Power</b>
Alternative Hypothesis .....	<b>Two-Sided</b>
Alpha.....	<b>0.05</b>
N (Subjects).....	<b>50 to 300 by 50</b>
R (Group 1 Allocation %) .....	<b>50</b>
Measurement Time Input Type .....	<b>Equally Spaced Measurement Times</b>
M (Number of Measurements) .....	<b>3 6</b>
$\mu_1(0)$ and $\mu_2(0)$ Input Type .....	<b><math>\mu_1(0) = \mu_2(0)</math></b>
$\mu_1(0)$ and $\mu_2(0)$ .....	<b>2.5</b>
$\mu_1(1)$ Input Type .....	<b>Difference (<math>\mu_1(1) - \mu_2(1)</math>)</b>
Difference ( $\mu_1(1) - \mu_2(1)$ ) .....	<b>1</b>
$\mu_2(1)$ .....	<b>1.75</b>
Pattern of $\rho$ 's Across Time.....	<b>AR1 (Traditional)</b>
$\rho$ (Base Correlation).....	<b>0.7</b>
Missing Input Type.....	<b>Linear (Steady Change)</b>
Pairwise Missing Pattern.....	<b>Independent (Ind)</b>
First Missing Proportion (Ind).....	<b>0</b>
Last Missing Proportion (Ind) .....	<b>0.4</b>

## GEE Tests for the Slope of Two Groups in a Repeated Measures Design (Count Outcome)

## Output

Click the Calculate button to perform the calculations and generate the following output.

**Numeric Results for the Test of Slope-Difference in Count Data using GEE**

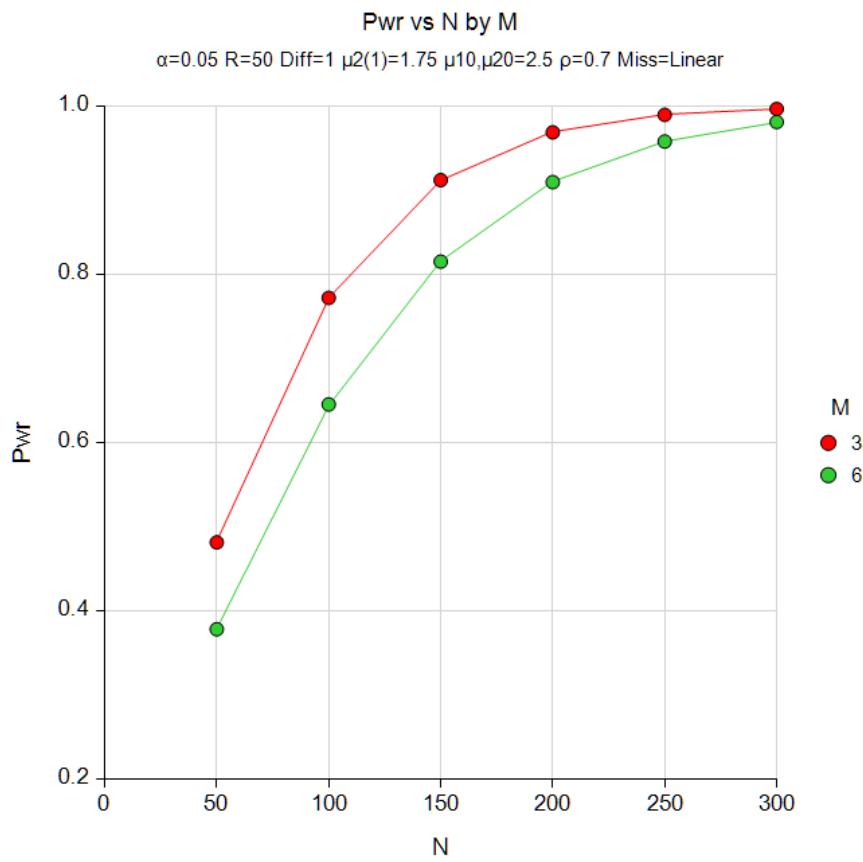
Solve For: Power  
 Measurement Times: Equally spaced  
 Correlation: AR1:  $\rho_{(j,k)} = \rho^{|j-k|}$   
 Missing Pattern: Range of missing proportions  
 Observant Proportions: Assume independence  
 $\mu_k(t)$ : Event rate of group k (1 = treatment, 2 = control) at time proportion t

Power	Total Sample Size N	Group 1 Allocation Percent R	Number of Meas. Times M	Event Rate								Alpha	
				at First Measurement			at Last Measurement			Base Corr. $\rho$	First Row of Corr. Matrix	Missing Data Proportions	Measurement Times
				$\mu_1(0)$	$\mu_2(0)$	$\mu_1(1)$	$\mu_2(1)$	Difference					
0.4815	50	50	3	2.5	2.5	2.75	1.75	1	0.7	$\rho_1(1)$	$Ms1(1)$	T(1)	0.05
0.3783	50	50	6	2.5	2.5	2.75	1.75	1	0.7	$\rho_1(2)$	$Ms1(2)$	T(2)	0.05
0.7723	100	50	3	2.5	2.5	2.75	1.75	1	0.7	$\rho_1(1)$	$Ms1(1)$	T(1)	0.05
0.6456	100	50	6	2.5	2.5	2.75	1.75	1	0.7	$\rho_1(2)$	$Ms1(2)$	T(2)	0.05
0.9122	150	50	3	2.5	2.5	2.75	1.75	1	0.7	$\rho_1(1)$	$Ms1(1)$	T(1)	0.05
0.8153	150	50	6	2.5	2.5	2.75	1.75	1	0.7	$\rho_1(2)$	$Ms1(2)$	T(2)	0.05
0.9691	200	50	3	2.5	2.5	2.75	1.75	1	0.7	$\rho_1(1)$	$Ms1(1)$	T(1)	0.05
0.9099	200	50	6	2.5	2.5	2.75	1.75	1	0.7	$\rho_1(2)$	$Ms1(2)$	T(2)	0.05
0.9898	250	50	3	2.5	2.5	2.75	1.75	1	0.7	$\rho_1(1)$	$Ms1(1)$	T(1)	0.05
0.9581	250	50	6	2.5	2.5	2.75	1.75	1	0.7	$\rho_1(2)$	$Ms1(2)$	T(2)	0.05
0.9968	300	50	3	2.5	2.5	2.75	1.75	1	0.7	$\rho_1(1)$	$Ms1(1)$	T(1)	0.05
0.9813	300	50	6	2.5	2.5	2.75	1.75	1	0.7	$\rho_1(2)$	$Ms1(2)$	T(2)	0.05

Item	Values
$\rho_1(1)$	1, 0.7, 0.49
$\rho_1(2)$	1, 0.7, 0.49, 0.343, 0.24, 0.168
$Ms1(1)$	0, 0.2, 0.4
$Ms1(2)$	0, 0.08, 0.16, 0.24, 0.32, 0.4
T(1)	0, 0.5, 1
T(2)	0, 0.2, 0.4, 0.6, 0.8, 1

## GEE Tests for the Slope of Two Groups in a Repeated Measures Design (Count Outcome)

## Plots



Note that decreasing the number of measurements has had the surprising result of decreasing the power, probably because the assumption of the AR(1) model for the autocorrelation has changed the way in which the correlations are formed. Note from the footnotes that the final autocorrelation increases from 0.168 when  $M = 6$  to 0.490 when  $M = 3$ . Look at the next example to see how the autocorrelations can be put on a more equal footing.

## Example 4 – Impact of Changing M with Linear Exponential Decay

We saw in Example 3 that the decreasing the number of measurements from 6 to 3 had the counter-intuitive result of increasing the power when the sample size was held constant. We surmised that this was partially due to the differing autocorrelation matrices that were used when the AR(1) model was assumed. In this example, we will leave all parameters the same, except that we will use a Linear Exponential Decay model for the autocorrelation. This will keep the autocorrelation matrices more comparable.

### Setup

If the procedure window is not already open, use the PASS Home window to open it. The parameters for this example are listed below and are stored in the **Example 4** settings file. To load these settings to the procedure window, click **Open Example Settings File** in the Help Center or File menu.

#### Design Tab

Solve For .....	<b>Power</b>
Alternative Hypothesis .....	<b>Two-Sided</b>
Alpha.....	<b>0.05</b>
N (Subjects) .....	<b>50 to 300 by 50</b>
R (Group 1 Allocation %) .....	<b>50</b>
Measurement Time Input Type .....	<b>Equally Spaced Measurement Times</b>
M (Number of Measurements) .....	<b>3 6</b>
$\mu_1(0)$ and $\mu_2(0)$ Input Type .....	<b><math>\mu_1(0) = \mu_2(0)</math></b>
$\mu_1(0)$ and $\mu_2(0)$ .....	<b>2.5</b>
$\mu_1(1)$ Input Type .....	<b>Difference (<math>\mu_1(1) - \mu_2(1)</math>)</b>
Difference ( $\mu_1(1) - \mu_2(1)$ ) .....	<b>1</b>
$\mu_2(1)$ .....	<b>1.75</b>
Pattern of $\rho$ 's Across Time.....	<b>Linear Exponential Decay</b>
$\rho$ (Base Correlation).....	<b>0.7</b>
Base Time Proportion .....	<b>0.166666666</b>
Emax (Max Decay Exponent) .....	<b>3</b>
Missing Input Type.....	<b>Linear (Steady Change)</b>
Pairwise Missing Pattern.....	<b>Independent (Ind)</b>
First Missing Proportion (Ind).....	<b>0</b>
Last Missing Proportion (Ind) .....	<b>0.4</b>

## GEE Tests for the Slope of Two Groups in a Repeated Measures Design (Count Outcome)

## Output

Click the Calculate button to perform the calculations and generate the following output.

**Numeric Results for the Test of Slope-Difference in Count Data using GEE**

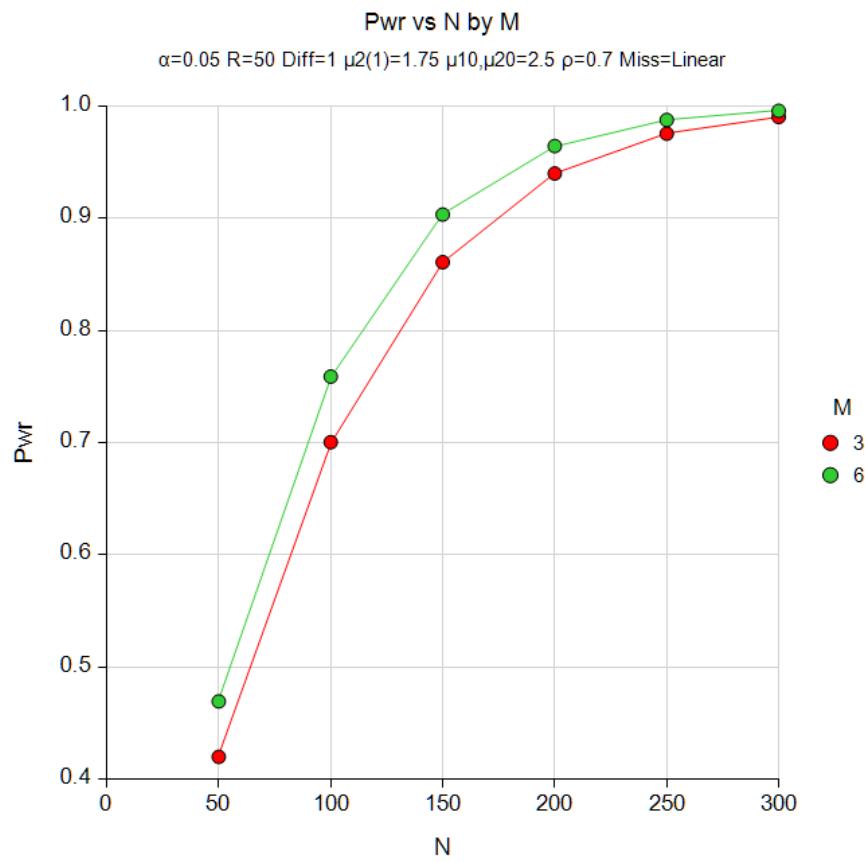
Solve For: Power  
 Measurement Times: Equally spaced  
 Correlation: Linear exponential decay, with Emax = 3 and Base Time Prop = 0.16666666  
 Missing Pattern: Range of missing proportions  
 Observant Proportions: Assume independence  
 $\mu_k(t)$ : Event rate of group k (1 = treatment, 2 = control) at time proportion t

Power	Total Sample Size N	Group 1 Allocation Percent R	Number of Meas. Times M	Event Rate								Measurement Times	Alpha	
				at First Measurement			at Last Measurement			Base Corr. $\rho$	First Row of Corr. Matrix	Missing Data Proportions		
				$\mu_1(0)$	$\mu_2(0)$	$\mu_1(1)$	$\mu_2(1)$	Difference						
0.4199	50	50	3	2.5	2.5	2.75	1.75	1	0.7	$\rho_1(1)$	$Ms1(1)$	T(1)	0.05	
0.4694	50	50	6	2.5	2.5	2.75	1.75	1	0.7	$\rho_1(2)$	$Ms1(2)$	T(2)	0.05	
0.7005	100	50	3	2.5	2.5	2.75	1.75	1	0.7	$\rho_1(1)$	$Ms1(1)$	T(1)	0.05	
0.7590	100	50	6	2.5	2.5	2.75	1.75	1	0.7	$\rho_1(2)$	$Ms1(2)$	T(2)	0.05	
0.8609	150	50	3	2.5	2.5	2.75	1.75	1	0.7	$\rho_1(1)$	$Ms1(1)$	T(1)	0.05	
0.9035	150	50	6	2.5	2.5	2.75	1.75	1	0.7	$\rho_1(2)$	$Ms1(2)$	T(2)	0.05	
0.9401	200	50	3	2.5	2.5	2.75	1.75	1	0.7	$\rho_1(1)$	$Ms1(1)$	T(1)	0.05	
0.9646	200	50	6	2.5	2.5	2.75	1.75	1	0.7	$\rho_1(2)$	$Ms1(2)$	T(2)	0.05	
0.9756	250	50	3	2.5	2.5	2.75	1.75	1	0.7	$\rho_1(1)$	$Ms1(1)$	T(1)	0.05	
0.9878	250	50	6	2.5	2.5	2.75	1.75	1	0.7	$\rho_1(2)$	$Ms1(2)$	T(2)	0.05	
0.9905	300	50	3	2.5	2.5	2.75	1.75	1	0.7	$\rho_1(1)$	$Ms1(1)$	T(1)	0.05	
0.9960	300	50	6	2.5	2.5	2.75	1.75	1	0.7	$\rho_1(2)$	$Ms1(2)$	T(2)	0.05	

Item	Values
$\rho_1(1)$	1, 0.526, 0.343
$\rho_1(2)$	1, 0.68, 0.573, 0.483, 0.407, 0.343
$Ms1(1)$	0, 0.2, 0.4
$Ms1(2)$	0, 0.08, 0.16, 0.24, 0.32, 0.4
T(1)	0, 0.5, 1
T(2)	0, 0.2, 0.4, 0.6, 0.8, 1

## GEE Tests for the Slope of Two Groups in a Repeated Measures Design (Count Outcome)

## Plots



Note from the footnotes that the final autocorrelation between the two models is now identical at 0.3430 ( $= 0.7^3$ ) when M is changed from 6 to 3. Now that the autocorrelation matrices are more comparable, the power values have increased in all cases, although only slightly. We see that increasing M has not had a huge impact on power.

## Example 5 – Validation using Hand Calculations

We could not find a published validation example, so we will create an example by hand.

In this example we set  $\mu_1(0) = 2.5$ ,  $\mu_2(0) = 2.5$ ,  $\mu_1(1) = 2.75$ ,  $\mu_2(1) = 1.75$ , the significance level = 0.05, the power = 0.90, R = 50%, M = 3, and the correlation matrix is compound symmetric with  $\rho = 0.7$ . The proportions missing at each time point are 0.0, 0.2, and 0.4.

The time values are 0, 0.5, and 1.

Taking the log values of the event rates yields

**Log( $\mu_1(0)$ ) = 0.916291**

**Log( $\mu_2(0)$ ) = 0.916291**

**Log( $\mu_1(1)$ ) = 1.011601**

**Log( $\mu_2(1)$ ) = 0.559616**

The slope and intercept for group 1 is 0.09531 and 0.916291.

The slope and intercept for group 2 is -0.35667 and 0.916291.

Using these values  $\mu_1(0.5) = 2.622022$  and  $\mu_2(0.5) = 2.09165$ .

Subtracting the missing proportions from one gives the observant proportions,  $\phi_j$ , for both groups as 1.0, 0.8, and 0.6. Using these values, we calculate  $\tau_1 = 0.431974$  and  $\tau_2 = 0.361199$ .

The observant proportions matrix,  $\phi_{jj'}$ , is

<b>1.0</b>	<b>0.8</b>	<b>0.6</b>
<b>0.8</b>	<b>0.8</b>	<b>0.48</b>
<b>0.6</b>	<b>0.48</b>	<b>0.6</b>

Using these values and  $\rho = 0.7$ , we find  $s_1^2 = 1.008589$ ,  $s_2^2 = 0.78687$ ,  $c_1^2 = -0.55497$ , and  $c_2^2 = -0.41980$ .

These calculations lead to  $v_1^2 = 0.445926$  and  $v_2^2 = 0.592843$ .

The result is N = 106.8555 which is rounded up to 107.

## Setup

If the procedure window is not already open, use the PASS Home window to open it. The parameters for this example are listed below and are stored in the **Example 5** settings file. To load these settings to the procedure window, click **Open Example Settings File** in the Help Center or File menu.

### Design Tab

Solve For .....	<b>Sample Size</b>
Alternative Hypothesis .....	<b>Two-Sided</b>
Power.....	<b>0.90</b>
Alpha.....	<b>0.05</b>
R (Group 1 Allocation %) .....	<b>50</b>
Measurement Time Input Type .....	<b>Equally Spaced Measurement Times</b>

## GEE Tests for the Slope of Two Groups in a Repeated Measures Design (Count Outcome)

M (Number of Measurements) .....	<b>3</b>
$\mu_1(0)$ and $\mu_2(0)$ Input Type .....	<b><math>\mu_1(0) = \mu_2(0)</math></b>
$\mu_1(0)$ and $\mu_2(0)$ .....	<b>2.5</b>
$\mu_1(1)$ Input Type .....	<b><math>\mu_1(1)</math></b>
$\mu_1(1)$ .....	<b>2.75</b>
$\mu_2(1)$ .....	<b>1.75</b>
Pattern of $\rho$ 's Across Time .....	<b>Compound Symmetry (All <math>\rho</math>'s Equal)</b>
$\rho$ (Base Correlation).....	<b>0.7</b>
Missing Input Type.....	<b>Linear (Steady Change)</b>
Pairwise Missing Pattern.....	<b>Independent (Ind)</b>
First Missing Proportion (Ind).....	<b>0</b>
Last Missing Proportion (Ind).....	<b>0.4</b>

## Output

Click the Calculate button to perform the calculations and generate the following output.

### Numeric Results for the Test of Slope-Difference in Count Data using GEE

Solve For: **Sample Size**  
 Measurement Times: Equally spaced  
 Correlation: Compound symmetry (all  $\rho$ 's equal)  
 Missing Pattern: Range of missing proportions  
 Observant Proportions: Assume independence  
 $\mu_k(t)$ : Event rate of group k (1 = treatment, 2 = control) at time proportion t

Power	Total Sample Size N	Group 1 Allocation Percent R	Number of Meas. Times M	Event Rate						Base Corr. $\rho$	First Row of Corr. Matrix	Missing Data Proportions	Measurement Times	Alpha					
				at First Measurement			at Last Measurement												
				$\mu_1(0)$	$\mu_2(0)$	$\mu_1(1)$	$\mu_2(1)$	Difference											
0.9004	107	50	3	2.5	2.5	2.75	1.75		1	0.7	$\rho_{1(1)}$	$Ms_{1(1)}$	T(1)	0.05					

Item	Values
$\rho_{1(1)}$	1, 0.7, 0.7
$Ms_{1(1)}$	0, 0.2, 0.4
T(1)	0, 0.5, 1

Note that the sample size is 107 which matches our hand calculations exactly.

## Example 6 – Impact of Measurement Time Distribution

This example will investigate the impact of measurement time on power. It will compare the power of studies that are evenly spaced with those that take more measurements at the beginning of the study, near the middle of the study, and at the end of the study.

In this example we set  $\mu_1(0) = 2.5$ ,  $\mu_2(0) = 2.5$ ,  $\mu_1(1) = 2.75$ ,  $\mu_2(1) = 1.75$ . The significance level is 0.05, the sample size ranges from 50 to 250, and R is 50%. The correlation pattern will be Linear Exponential Decay with a base correlation of 0.6, Base Time Proportion of 0.10, and Emax set to 3. The missing input type will be set to Linear from 0 to 0.30 and the pairwise missing assumption will be independent.

The measurement times for five scenarios are given in the following table.

**Table of Measurement Times in Proportion of Total Study Time**

Tm1	Tm2	Tm3	Tm4	Tm5
0	0	0	0	0
0.20	0.60	0.10	0.10	0.45
0.40	0.70	0.20	0.20	0.50
0.60	0.80	0.30	0.80	0.55
0.80	0.90	0.40	0.90	0.60
1.00	1.00	1.00	1.00	1.00

Note that the measurements in Tm1 are evenly spaced, those in Tm2 are loaded near the end, those of Tm3 occur at the beginning, those of Tm4 occur only at the beginning and the end, and those of Tm5 occur mostly near the middle.

## Setup

If the procedure window is not already open, use the PASS Home window to open it. The parameters for this example are listed below and are stored in the **Example 6** settings file. To load these settings to the procedure window, click **Open Example Settings File** in the Help Center or File menu.

### Design Tab

Solve For .....	<b>Power</b>
Alternative Hypothesis .....	<b>Two-Sided</b>
Alpha.....	<b>0.05</b>
N (Subjects) .....	<b>50 to 250 by 50</b>
R (Group 1 Allocation %) .....	<b>50</b>
Measurement Time Input Type .....	<b>Columns of Measurement Time Proportions</b>
Column(s) of Time Proportions .....	<b>Tm1-Tm5</b>
$\mu_1(0)$ and $\mu_2(0)$ Input Type .....	<b><math>\mu_1(0) = \mu_2(0)</math></b>
$\mu_1(0)$ and $\mu_2(0)$ .....	<b>2.5</b>
$\mu_1(1)$ Input Type .....	<b><math>\mu_1(1)</math></b>
$\mu_1(1)$ .....	<b>2.75</b>
$\mu_2(1)$ .....	<b>1.75</b>
Pattern of $\rho$ 's Across Time.....	<b>Linear Exponential Decay</b>
$\rho$ (Base Correlation).....	<b>0.6</b>

## GEE Tests for the Slope of Two Groups in a Repeated Measures Design (Count Outcome)

Base Time Proportion ..... **0.1**  
 Emax (Max Decay Exponent) ..... **3**  
 Missing Input Type ..... **Linear (Steady Change)**  
 Pairwise Missing Pattern ..... **Independent (Ind)**  
 First Missing Proportion (Ind) ..... **0**  
 Last Missing Proportion (Ind) ..... **0.3**

**Input Spreadsheet Data**

Row	Tm1	Tm2	Tm3	Tm4	Tm5
1	0.0	0.0	0.0	0.0	0.00
2	0.2	0.6	0.1	0.1	0.45
3	0.4	0.7	0.2	0.2	0.50
4	0.6	0.8	0.3	0.8	0.55
5	0.8	0.9	0.4	0.9	0.60
6	1.0	1.0	1.0	1.0	1.00

**Output**

Click the Calculate button to perform the calculations and generate the following output.

**Numeric Results for the Test of Slope-Difference in Count Data using GEE**

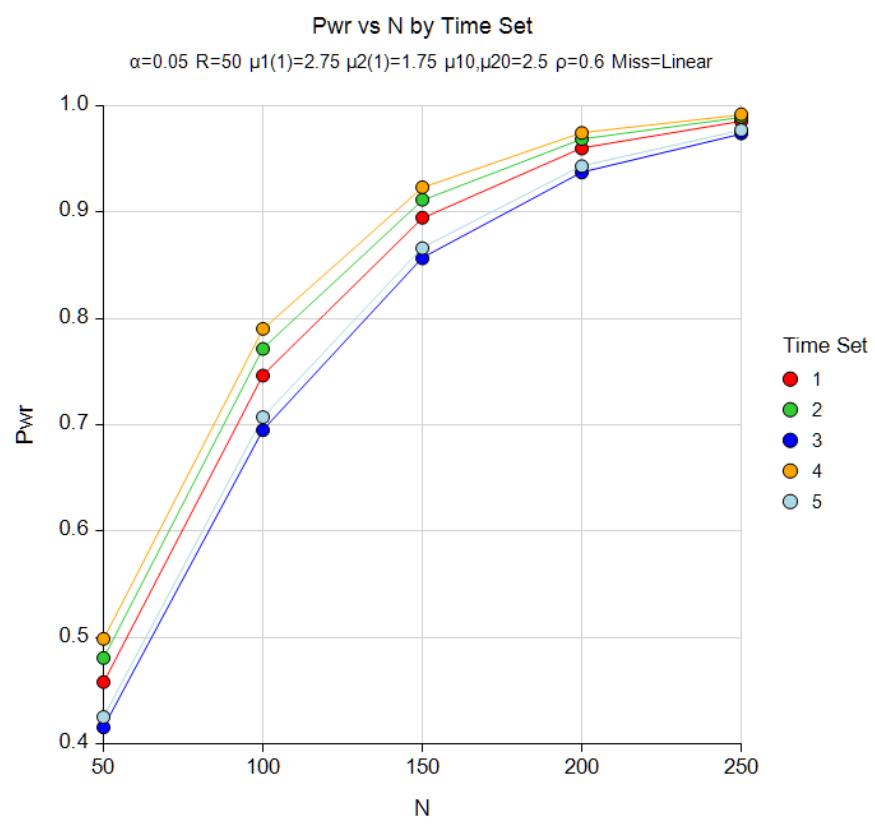
Solve For: **Power**  
 Measurement Times: Lists in spreadsheet columns: {TM1-TM5}  
 Correlation: Linear exponential decay, with Emax = 3 and Base Time Prop = 0.1  
 Missing Pattern: Range of missing proportions  
 Observant Proportions: Assume independence  
 $\mu_k(t)$ : Event rate of group k (1 = treatment, 2 = control) at time proportion t

Power	Total Sample Size	Group 1 Allocation Percent	Number of Meas. Times	Event Rate								First Row of Corr. Matrix	Missing Data Proportions	Measurement Times	Alpha				
				at First Measurement				at Last Measurement											
				$\mu_1(0)$	$\mu_2(0)$	$\mu_1(1)$	$\mu_2(1)$	Difference											
0.4582	50	50	6	2.5	2.5	2.75	1.75	1	0.6	$\rho_1(Tm1)$	$Ms1(Tm1)$	Tm1(1)	0.05						
0.4808	50	50	6	2.5	2.5	2.75	1.75	1	0.6	$\rho_1(Tm2)$	$Ms1(Tm2)$	Tm2(2)	0.05						
0.4155	50	50	6	2.5	2.5	2.75	1.75	1	0.6	$\rho_1(Tm3)$	$Ms1(Tm3)$	Tm3(3)	0.05						
0.4988	50	50	6	2.5	2.5	2.75	1.75	1	0.6	$\rho_1(Tm4)$	$Ms1(Tm4)$	Tm4(4)	0.05						
0.4253	50	50	6	2.5	2.5	2.75	1.75	1	0.6	$\rho_1(Tm5)$	$Ms1(Tm5)$	Tm5(5)	0.05						
0.7464	100	50	6	2.5	2.5	2.75	1.75	1	0.6	$\rho_1(Tm1)$	$Ms1(Tm1)$	Tm1(1)	0.05						
0.7715	100	50	6	2.5	2.5	2.75	1.75	1	0.6	$\rho_1(Tm2)$	$Ms1(Tm2)$	Tm2(2)	0.05						
0.6950	100	50	6	2.5	2.5	2.75	1.75	1	0.6	$\rho_1(Tm3)$	$Ms1(Tm3)$	Tm3(3)	0.05						
0.7903	100	50	6	2.5	2.5	2.75	1.75	1	0.6	$\rho_1(Tm4)$	$Ms1(Tm4)$	Tm4(4)	0.05						
0.7073	100	50	6	2.5	2.5	2.75	1.75	1	0.6	$\rho_1(Tm5)$	$Ms1(Tm5)$	Tm5(5)	0.05						
0.8949	150	50	6	2.5	2.5	2.75	1.75	1	0.6	$\rho_1(Tm1)$	$Ms1(Tm1)$	Tm1(1)	0.05						
0.9117	150	50	6	2.5	2.5	2.75	1.75	1	0.6	$\rho_1(Tm2)$	$Ms1(Tm2)$	Tm2(2)	0.05						
0.8566	150	50	6	2.5	2.5	2.75	1.75	1	0.6	$\rho_1(Tm3)$	$Ms1(Tm3)$	Tm3(3)	0.05						
0.9236	150	50	6	2.5	2.5	2.75	1.75	1	0.6	$\rho_1(Tm4)$	$Ms1(Tm4)$	Tm4(4)	0.05						
0.8662	150	50	6	2.5	2.5	2.75	1.75	1	0.6	$\rho_1(Tm5)$	$Ms1(Tm5)$	Tm5(5)	0.05						
0.9599	200	50	6	2.5	2.5	2.75	1.75	1	0.6	$\rho_1(Tm1)$	$Ms1(Tm1)$	Tm1(1)	0.05						
0.9688	200	50	6	2.5	2.5	2.75	1.75	1	0.6	$\rho_1(Tm2)$	$Ms1(Tm2)$	Tm2(2)	0.05						
0.9374	200	50	6	2.5	2.5	2.75	1.75	1	0.6	$\rho_1(Tm3)$	$Ms1(Tm3)$	Tm3(3)	0.05						
0.9746	200	50	6	2.5	2.5	2.75	1.75	1	0.6	$\rho_1(Tm4)$	$Ms1(Tm4)$	Tm4(4)	0.05						
0.9433	200	50	6	2.5	2.5	2.75	1.75	1	0.6	$\rho_1(Tm5)$	$Ms1(Tm5)$	Tm5(5)	0.05						
0.9857	250	50	6	2.5	2.5	2.75	1.75	1	0.6	$\rho_1(Tm1)$	$Ms1(Tm1)$	Tm1(1)	0.05						
0.9897	250	50	6	2.5	2.5	2.75	1.75	1	0.6	$\rho_1(Tm2)$	$Ms1(Tm2)$	Tm2(2)	0.05						
0.9742	250	50	6	2.5	2.5	2.75	1.75	1	0.6	$\rho_1(Tm3)$	$Ms1(Tm3)$	Tm3(3)	0.05						
0.9921	250	50	6	2.5	2.5	2.75	1.75	1	0.6	$\rho_1(Tm4)$	$Ms1(Tm4)$	Tm4(4)	0.05						
0.9773	250	50	6	2.5	2.5	2.75	1.75	1	0.6	$\rho_1(Tm5)$	$Ms1(Tm5)$	Tm5(5)	0.05						

## GEE Tests for the Slope of Two Groups in a Repeated Measures Design (Count Outcome)

Item	Values
p1(Tm1)	1, 0.536, 0.427, 0.34, 0.271, 0.216
p1(Tm2)	1, 0.34, 0.304, 0.271, 0.242, 0.216
p1(Tm3)	1, 0.6, 0.536, 0.478, 0.427, 0.216
p1(Tm4)	1, 0.6, 0.536, 0.271, 0.242, 0.216
p1(Tm5)	1, 0.403, 0.381, 0.36, 0.34, 0.216
Ms1(Tm1)	0, 0.06, 0.12, 0.18, 0.24, 0.3
Ms1(Tm2)	0, 0.18, 0.21, 0.24, 0.27, 0.3
Ms1(Tm3)	0, 0.03, 0.06, 0.09, 0.12, 0.3
Ms1(Tm4)	0, 0.03, 0.06, 0.24, 0.27, 0.3
Ms1(Tm5)	0, 0.14, 0.15, 0.17, 0.18, 0.3
Tm1(1)	0, 0.2, 0.4, 0.6, 0.8, 1
Tm2(2)	0, 0.6, 0.7, 0.8, 0.9, 1
Tm3(3)	0, 0.1, 0.2, 0.3, 0.4, 1
Tm4(4)	0, 0.1, 0.2, 0.8, 0.9, 1
Tm5(5)	0, 0.45, 0.5, 0.55, 0.6, 1

## Plots



The legend, *Time Set*, gives the sequence number of the measurement columns. Thus, 1.0 is Tm1, 2.0 is Tm2, and so on.

The pattern Tm4 consistently produces the highest power across all sample sizes. Remember that Tm4 put the measurements at the beginning and the end, but none in the middle.

Patterns Tm3 and Tm5 are nearly tied for achieving the lowest powers. Tm3 put most of the measurements at the beginning of the study. Tm5 put most of the measurements during the middle of the study.

Note that Tm1, the equally spaced times, is in the middle.

## Example 7 – Entering a Correlation Matrix

This example will show how a correlation matrix can be loaded directly.

In this example we set  $\mu_1(0) = 2.5$ ,  $\mu_2(0) = 2.5$ ,  $\mu_1(1) = 2.75$ ,  $\mu_2(1) = 1.75$ . The significance level is 0.05, the sample size ranges from 50 to 250, M = 4, and R is 50%. The missing input type will be set to Linear from 0 to 0.30 and the pairwise missing assumption will be independent. A correlation matrix (shown below) is available from a previous study and is entered into the spreadsheet.

**Table of Correlations**

C1	C2	C3	C4
1.0000	0.7000	0.4900	0.3430
0.7000	1.0000	0.7000	0.4900
0.4900	0.7000	1.0000	0.7000
0.3430	0.4900	0.7000	1.0000

## Setup

If the procedure window is not already open, use the PASS Home window to open it. The parameters for this example are listed below and are stored in the **Example 7** settings file. To load these settings to the procedure window, click **Open Example Settings File** in the Help Center or File menu.

### Design Tab

Solve For .....	<b>Power</b>
Alternative Hypothesis .....	<b>Two-Sided</b>
Alpha.....	<b>0.05</b>
N (Subjects).....	<b>50 to 250 by 50</b>
R (Group 1 Allocation %).....	<b>50</b>
Measurement Time Input Type .....	<b>Equally Spaced Measurement Times</b>
M (Number of Measurements) .....	<b>4</b>
$\mu_1(0)$ and $\mu_2(0)$ Input Type .....	<b><math>\mu_1(0) = \mu_2(0)</math></b>
$\mu_1(0)$ and $\mu_2(0)$ .....	<b>2.5</b>
$\mu_1(1)$ Input Type .....	<b><math>\mu_1(1)</math></b>
$\mu_1(1)$ .....	<b>2.75</b>
$\mu_2(1)$ .....	<b>1.75</b>
Pattern of p's Across Time.....	<b>Matrix on Spreadsheet</b>
Columns Containing the pjk's .....	<b>C1-C4</b>
Missing Input Type.....	<b>Linear (Steady Change)</b>
Pairwise Missing Pattern.....	<b>Independent (Ind)</b>
First Missing Proportion (Ind).....	<b>0</b>
Last Missing Proportion (Ind) .....	<b>0.30</b>

## GEE Tests for the Slope of Two Groups in a Repeated Measures Design (Count Outcome)

**Input Spreadsheet Data**

Row	C1	C2	C3	C4
1	1.000	0.70	0.49	0.343
2	0.700	1.00	0.70	0.490
3	0.490	0.70	1.00	0.700
4	0.343	0.49	0.70	1.000

**Output**

Click the Calculate button to perform the calculations and generate the following output.

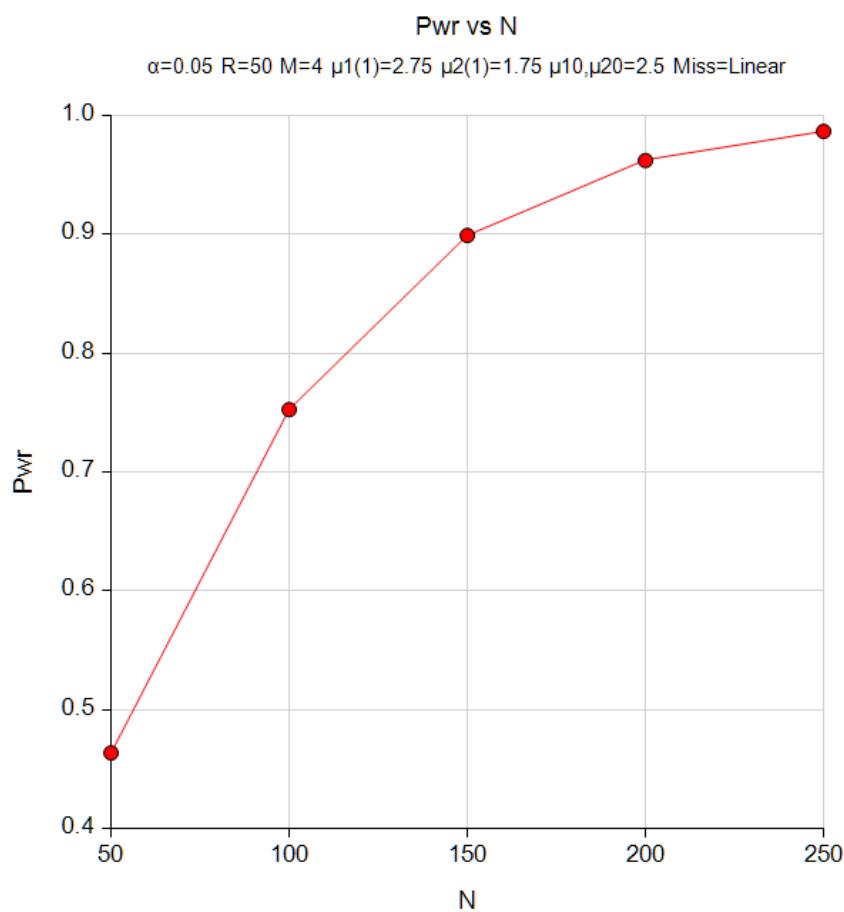
**Numeric Results for the Test of Slope-Difference in Count Data using GEE**

Solve For: Power  
 Measurement Times: Equally spaced  
 Correlation: Matrix stored on spreadsheet in columns C1-C4  
 Missing Pattern: Range of missing proportions  
 Observant Proportions: Assume independence  
 $\mu_k(t)$ : Event rate of group k (1 = treatment, 2 = control) at time proportion t

Power	Total Sample Size	Group 1 Allocation Percent	Number of Meas. Times	Event Rate								Alpha		
				at First Measurement			at Last Measurement			Base Corr.	First Row of Corr. Matrix	Missing Data Proportions	Measurement Times	
				$\mu_1(0)$	$\mu_2(0)$	$\mu_1(1)$	$\mu_2(1)$	Difference						
0.4636	50	50	4	2.5	2.5	2.75	1.75	1	N/A	$\rho_1(1)$	$Ms_1(1)$	T(1)	0.05	
0.7526	100	50	4	2.5	2.5	2.75	1.75	1	N/A	$\rho_1(1)$	$Ms_1(1)$	T(1)	0.05	
0.8991	150	50	4	2.5	2.5	2.75	1.75	1	N/A	$\rho_1(1)$	$Ms_1(1)$	T(1)	0.05	
0.9622	200	50	4	2.5	2.5	2.75	1.75	1	N/A	$\rho_1(1)$	$Ms_1(1)$	T(1)	0.05	
0.9867	250	50	4	2.5	2.5	2.75	1.75	1	N/A	$\rho_1(1)$	$Ms_1(1)$	T(1)	0.05	

Item	Values
$\rho_1(1)$	1, 0.7, 0.49, 0.343
$Ms_1(1)$	0, 0.1, 0.2, 0.3
T(1)	0, 0.33, 0.67, 1

## GEE Tests for the Slope of Two Groups in a Repeated Measures Design (Count Outcome)

**Plots****Autocorrelation Matrix for Report Row 1**

Time	T(0)	T(0.33)	T(0.67)	T(1)
T(0)	1.000	0.70	0.49	0.343
T(0.33)	0.700	1.00	0.70	0.490
T(0.67)	0.490	0.70	1.00	0.700
T(1)	0.343	0.49	0.70	1.000

**Autocorrelation Matrix for Report Row 2**

Time	T(0)	T(0.33)	T(0.67)	T(1)
T(0)	1.000	0.70	0.49	0.343
T(0.33)	0.700	1.00	0.70	0.490
T(0.67)	0.490	0.70	1.00	0.700
T(1)	0.343	0.49	0.70	1.000

(More Reports Follow)

The standard reports and plots are displayed.

## Example 8 – Entering an Observant Probabilities Matrix

This example will show how an observant probabilities matrix can be loaded directly.

In this example we set  $\mu_1(0) = 2.5$ ,  $\mu_2(0) = 2.5$ ,  $\mu_1(1) = 2.75$ ,  $\mu_2(1) = 1.75$ . There are to be 4 equally spaced time measurements. The significance level is 0.05, the sample size ranges from 50 to 250, and R is 50%. The correlation pattern is Linear Exponential Decay with a base correlation of 0.4, Base Time Proportion of 0.10, and Emax of 4. The significance level is 0.05, the sample size ranges from 50 to 250, and R is 50%. The missing input type will be set to Matrix of Pairwise Missing.

**Table of Observant Probabilities**

Row	C1	C2	C3	C4
1	1.00	0.90	0.80	0.70
2	0.90	0.90	0.72	0.63
3	0.80	0.72	0.80	0.56
4	0.70	0.63	0.56	0.70

This table gives the pairwise observant probabilities. That is, each entry gives the probability of obtaining a response for both the row and column time points. For example, 0.63 is the probability of observing both the second response and the fourth response.

### Setup

If the procedure window is not already open, use the PASS Home window to open it. The parameters for this example are listed below and are stored in the **Example 8** settings file. To load these settings to the procedure window, click **Open Example Settings File** in the Help Center or File menu.

#### Design Tab

Solve For .....	<b>Power</b>
Alternative Hypothesis .....	<b>Two-Sided</b>
Alpha.....	<b>0.05</b>
N (Subjects) .....	<b>50 to 250 by 50</b>
R (Group 1 Allocation %) .....	<b>50</b>
Measurement Time Input Type .....	<b>Equally Spaced Measurement Times</b>
M (Number of Measurements) .....	<b>4</b>
$\mu_1(0)$ and $\mu_2(0)$ Input Type .....	<b><math>\mu_1(0) = \mu_2(0)</math></b>
$\mu_1(0)$ and $\mu_2(0)$ .....	<b>2.5</b>
$\mu_1(1)$ Input Type .....	<b><math>\mu_1(1)</math></b>
$\mu_1(1)$ .....	<b>2.75</b>
$\mu_2(1)$ .....	<b>1.75</b>
Pattern of p's Across Time.....	<b>Linear Exponential Decay</b>
$\rho$ (Base Correlation).....	<b>0.4</b>
Base Time Proportion .....	<b>0.1</b>
Emax (Max Decay Exponent) .....	<b>4</b>
Missing Input Type.....	<b>Pairwise Observed Proportions on Spreadsheet</b>
Columns of Pairwise Observed.....	<b>C1-C4</b>

## GEE Tests for the Slope of Two Groups in a Repeated Measures Design (Count Outcome)

**Input Spreadsheet Data**

Row	C1	C2	C3	C4
1	1.0	0.90	0.80	0.70
2	0.9	0.90	0.72	0.63
3	0.8	0.72	0.80	0.56
4	0.7	0.63	0.56	0.70

**Output**

Click the Calculate button to perform the calculations and generate the following output.

**Numeric Results for the Test of Slope-Difference in Count Data using GEE**

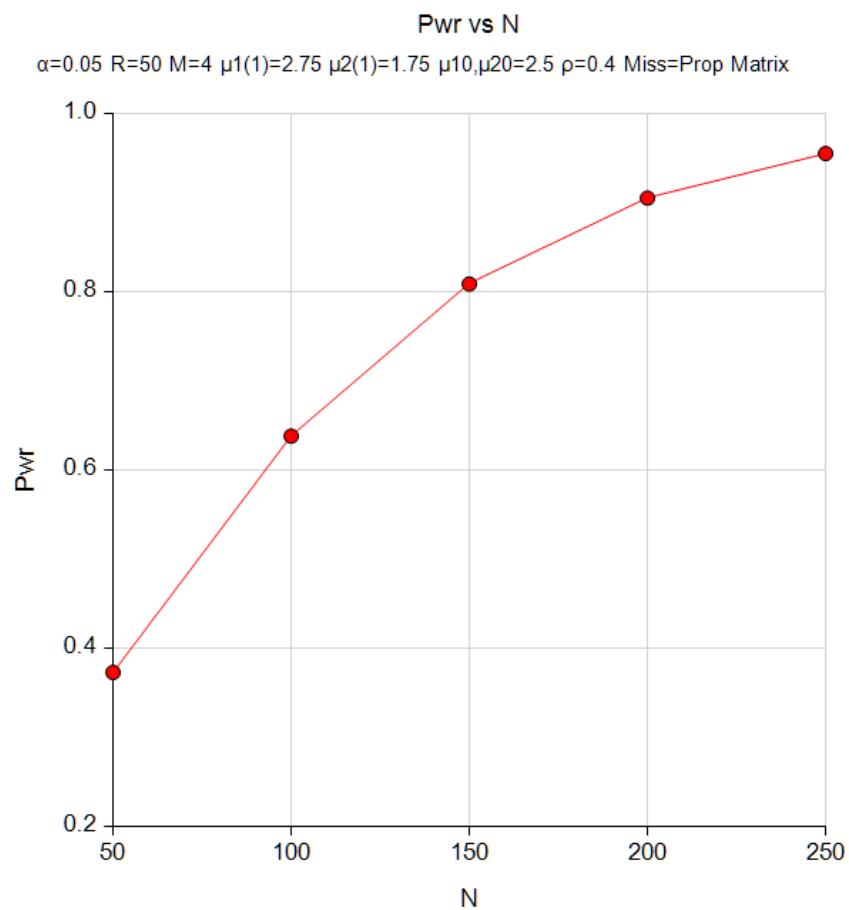
Solve For: Power  
 Measurement Times: Equally spaced  
 Correlation: Linear exponential decay, with Emax = 4 and Base Time Prop = 0.1  
 Missing Pattern: N/A. Matrix of observant probabilities entered in columns C1-C4.  
 $\mu_k(t)$ : Event rate of group k (1 = treatment, 2 = control) at time proportion t

Power	Total Sample Size N	Group 1 Allocation Percent R	Number of Meas. Times M	Event Rate								First Row of Corr. Matrix	Missing Data Proportions	Measurement Times	Alpha				
				at First Measurement			at Last Measurement			Base Corr. $\rho$									
				$\mu_1(0)$	$\mu_2(0)$	$\mu_1(1)$	$\mu_2(1)$	Difference											
0.3730	50	50	4	2.5	2.5	2.75	1.75	1	0.4	$\rho_1(1)$	N/A	T(1)	0.05						
0.6382	100	50	4	2.5	2.5	2.75	1.75	1	0.4	$\rho_1(1)$	N/A	T(1)	0.05						
0.8088	150	50	4	2.5	2.5	2.75	1.75	1	0.4	$\rho_1(1)$	N/A	T(1)	0.05						
0.9052	200	50	4	2.5	2.5	2.75	1.75	1	0.4	$\rho_1(1)$	N/A	T(1)	0.05						
0.9553	250	50	4	2.5	2.5	2.75	1.75	1	0.4	$\rho_1(1)$	N/A	T(1)	0.05						

**Item Values**

$\rho_1(1)$	1, 0.196, 0.071, 0.026
T(1)	0, 0.33, 0.67, 1

## GEE Tests for the Slope of Two Groups in a Repeated Measures Design (Count Outcome)

**Plots**

The standard reports are displayed.