Chapter 884

Latin Square Designs

Introduction

This module generates Latin Square and Graeco-Latin Square designs. Designs for three to ten treatments are available.

An introduction to experimental design is presented in Chapter 881 on Two-Level Designs and will not be repeated here.

Latin Square designs are similar to randomized block designs, except that instead of the removal of one blocking variable, these designs are carefully constructed to allow the removal of two blocking factors. They accomplish this while reducing the number of experimental units needed to conduct the experiment.

Following is an example of a four treatment Latin Square. The experimental layout is as follows:

		Columns		
Rows	**Col1**	**Col2**	**Col3**	**Col4**
Row 1	A	B	C	D
Row 2	B	C	D	A
Row 3	C	D	A	B
Row 4	D	A	B	C

In the above table, the four treatments are represented by the four letters: A, B, C, and D. The letters are arranged so that each letter occurs only once within each row and each column. Notice that a simple random design would require 4 x 4 x 4 = 64 experimental units. This Latin Square needs only 16 experimental units—a reduction of 75%!

The influence of a fourth factor may also be removed from the design by introducing a second set of letters, this time lower case. This design is known as the *Graeco-Latin Square*.

		Columns		
Rows	**Col1**	**Col2**	**Col3**	**Col4**
Row 1	Aa	Bb	Cc	Dd
Row 2	Bd	Ca	Db	Ac
Row 3	Cb	Dc	Ad	Ba
Row 4	Dc	Ad	Ba	Cb

Four factors at four levels each would normally require 256 experimental units, but this design only requires 16—a reduction in experimental units of almost 94%!

The Graeco-Latin Square is formed by combining two orthogonal Latin Squares. Graeco-Latin Squares are available for all numbers of treatments except six.

Latin Square Assumptions

It is important to understand the assumptions that are made when using the Latin Square design. The large reduction in the number of experimental units needed by this design occurs because it assumptions the magnitudes of the interaction terms are small enough that they may be ignored. That is, the Latin Square design is a main effects only design. Another way of saying this is that the treatments, the row factor, and the column factor affect the response independently of one another.

Assuming that there are no interactions is quite restrictive, so before you use this design you should be able to defend this assumption. In practice, the influence of the interactions is averaged into the experimental error of the analysis of variance table. We say that the experimental error is inflated. This results in a reduced F-ratio for testing the treatment factor, and a reduced F-ratio lessens the possibility of achieving statistical significance.

Randomization

Probability statements made during the analysis of the experimental data require strict attention to the randomization process. The randomization process is as follows:

1. Randomly select a design from the set of orthogonal designs available.
2. Randomly assign levels of the row factor to the rows.
3. Randomly assign levels of the column factor to the columns.
4. Randomly assign treatments to the treatment letters (or numbers as the case may be).

Orthogonal Sets

These designs were taken from Rao, Mitra, and Matthai (1966). We have included designs with up to ten treatments. The number of available squares depends on the number of treatments. The following table shows the number of orthogonal squares stored within this procedure.

Number of Treatments	Number of Orthogonal Designs
3	2
4	3
5	4
6	1
7	6
8	7
9	8
10	2

Graeco-Latin Squares are generated by combining two of the available orthogonal squares. Note that there are no six-level Graeco-Latin Squares.

Example 1 – Latin Square Design

This section presents an example of how to generate a Latin Square design using this program. **CAUTION: since the purpose of this routine is to generate data, you should begin with an empty output spreadsheet.**

In this example, we will show you how to generate a design with four treatments.

Setup

If the procedure window is not already open, use the PASS Home window to open it. The parameters for this example are listed below and are stored in the **Example 1** settings file. To load these settings to the procedure window, click **Open Example Settings File** in the Help Center or File menu.

Design Tab	
Row Values	**R1 R2 R3 R4**
Column Values	**C1 C2 C3 C4**
Treatment 1 Values	**A B C D**
Treatment 2 Values	**<Empty>**
Orthogonal Design Number I	**1**
Orthogonal Design Number II	**2**
Store Data on Spreadsheet	**Checked**
Store First Factor In	**1**

Output

Click the Calculate button to perform the calculations and generate the following output.

Four-Level Latin Square Design

Experimental Design

ID	Row	Column	Treatment 1
1	R1	C1	A
2	R1	C2	B
3	R1	C3	C
4	R1	C4	D
5	R2	C1	B
6	R2	C2	A
7	R2	C3	D
8	R2	C4	C
9	R3	C1	C
10	R3	C2	D
11	R3	C3	A
12	R3	C4	B
13	R4	C1	D
14	R4	C2	C
15	R4	C3	B
16	R4	C4	A

These values were also produced on the spreadsheet.

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