Chapter 553

# Levene Test of Variances (Simulation)

## Introduction

This procedure analyzes the power and significance level of Levene's homogeneity test. This test is used to test whether two or more population variances are equal. For each scenario that is set up, two simulations are run. One simulation estimates the significance level and the other estimates the power.

## Technical Details

*Computer simulation* allows us to estimate the power and significance level that is actually achieved by a test procedure in situations that are not mathematically tractable. Computer simulation was once limited to mainframe computers. But, in recent years, as computer speeds have increased, simulation studies can be completed on desktop and laptop computers in a reasonable period of time.

The steps to a simulation study are

1. Specify how the test is carried out. This includes indicating how the test statistic is calculated and how the significance level is specified.
2. Generate random samples from the distributions specified by the alternative hypothesis. Calculate the test statistics from the simulated data and determine if the null hypothesis is accepted or rejected. Tabulate the number of rejections and use this to calculate the test's power.
3. Generate random samples from the distributions specified by the null hypothesis. Calculate each test statistic from the simulated data and determine if the null hypothesis is accepted or rejected. Tabulate the number of rejections and use this to calculate the test's significance level.
4. Repeat steps 2 and 3 several thousand times, tabulating the number of times the simulated data leads to a rejection of the null hypothesis. The power is the proportion of simulated samples in step 2 that lead to rejection. The significance level is the proportion of simulated samples in step 3 that lead to rejection.

## Generating Random Distributions

Two methods are available in **PASS** to simulate random samples. The first method generates the random variates directly, one value at a time. The second method generates a large pool (over 10,000) of random values and then draws the random numbers from this pool. This second method can cut the running time of the simulation by 70%.

The second method begins by generating a large pool of random numbers from the specified distributions. Each of these pools is evaluated to determine if its mean is within a small relative tolerance (0.0001) of the target mean. If the actual mean is not within the tolerance of the target mean, individual members of the population are replaced with new random numbers if the new random number moves the mean towards its target. Only a few hundred such swaps are required to bring the actual mean to within tolerance of the target mean. This population is then sampled with replacement using the uniform distribution. We have found that this method works well as long as the size of the pool is at least the maximum of twice the number of simulated samples desired and 10,000.

## Levene's Test

Levene (1960) presents a test of homogeneity (equal variance). The test does not assume that all populations are normally distributed and is recommended when the normality assumption is not viable.

Suppose *g* groups each have a normal distribution with possibly different means and standard deviations $\sigma_1, \sigma_2, \ldots, \sigma_g$. Let $n_1, n_2, \ldots, n_g$ denote the number of subjects in each group, $Y_{ki}$ denote response values, and *N* denote the total sample size of all groups. The test assumes that the data are obtained by taking a simple random sample from each of the *g* populations.

The formula for the calculation of Levene's test is

$$W = \frac{(N-g)\sum_{k=1}^{g} n_k (Z_k - \bar{Z})^2}{(g-1)\left\{\sum_{k=1}^{g}\sum_{i=1}^{n_k}(Z_{ki} - \bar{Z}_k)^2\right\}}$$

where

$$Z_{ki} = |Y_{ki} - \bar{Y}_k|$$

$$\bar{Z}_k = \frac{1}{n_k}\sum_{i=1}^{n_k} Z_{ki}$$

$$\bar{Z} = \frac{1}{N}\sum_{k=1}^{g}\sum_{i=1}^{n_k} Z_{ki}$$

$$\bar{Y}_k = \frac{1}{n_k}\sum_{i=1}^{n_k} Y_{ki}$$

If the assumptions are met, the distribution of this test statistic follows the *F* distribution with degrees of freedom *g* - 1 and *N* - *g*.

# Example 1 – Power for a Range of Standard Deviations

A one-way design is being employed to compare the means of four groups using an *F* test. Past experimentation has shown that the data within each group can reasonably be assumed to be normally distributed. The researcher would like to use the Levene test to evaluate the assumption that the variances of all four groups are equal (homogeneity). Previous studies have shown that the standard deviation within a group is about 5. The researcher wants to investigate the power when the standard deviation of the second group is 6, 8, or 10. Treatment means of 10, 20, 10, and 10 are anticipated. The researcher wants to compute the power for group sample sizes of 10, 20, 30, 40, and 50. The group sample sizes are equal in a particular scenario. The value of alpha is 0.05.

## Setup

If the procedure window is not already open, use the PASS Home window to open it. The parameters for this example are listed below and are stored in the **Example 1** settings file. To load these settings to the procedure window, click **Open Example Settings File** in the Help Center or File menu.

| Design Tab | |
|---|---|
| Solve For | **Power** |
| Alpha | **0.05** |
| Simulations | **5000** |
| Random Seed | **5503023** (for Reproducibility) |
| Number of Groups | **4** |
| Group Allocation | **Equal (n = n1 = n2 = ⋯)** |
| n (Group Size) | **10 20 30 40 50** |
| Group 1 Power Distribution | **Normal(10 S1)** |
| Group 2 Power Distribution | **Normal(20 S2)** |
| Group 3 Power Distribution | **Normal(10 S1)** |
| Group 4 Power Distribution | **Normal(10 S1)** |
| Parameter 2 Name | **S1** |
| Parameter 2 Value(s) | **5** |
| Parameter 3 Name | **S2** |
| Parameter 3 Value(s) | **7 8 9** |
| Specify Alpha Distributions | **All equal to the group 1 distribution of the power simulation** |

# Output

Click the Calculate button to perform the calculations and generate the following output.

## Simulation Summary and Numeric Results

**Simulation Summary**

| | Simulation Distribution | |
|---|---|---|
| **Group** | **Power (H1)** | **Alpha (H0)** |
| 1 | Normal(10 S1) | Normal(10 S1) |
| 2 | Normal(20 S2) | Normal(10 S1) |
| 3 | Normal(10 S1) | Normal(10 S1) |
| 4 | Normal(10 S1) | Normal(10 S1) |

| | |
|---|---|
| Number of Groups | 4 |
| Random Number Pool Size | 10000 |
| Number of Simulations | 5000 |
| Random Seed | 5503023 (User-Entered) |
| Run Time | 15.97 seconds |

**Numeric Results**

Solve For: Power

| | | Sample Size | | Alpha | | | | | |
|---|---|---|---|---|---|---|---|---|---|
| **Scenario** | **Power** | **Average Group n** | **Total N** | **Target** | **Actual** | **Standard Deviation of σ's \| H1** | **Average of σ's \| H1** | **S1** | **S2** |
| 1 | 0.167 | 10 | 40 | 0.05 | 0.068 | 0.9 | 5.5 | 5 | 7 |
| 2 | 0.289 | 20 | 80 | 0.05 | 0.063 | 0.9 | 5.5 | 5 | 7 |
| 3 | 0.391 | 30 | 120 | 0.05 | 0.048 | 0.8 | 5.5 | 5 | 7 |
| 4 | 0.543 | 40 | 160 | 0.05 | 0.055 | 0.9 | 5.5 | 5 | 7 |
| 5 | 0.625 | 50 | 200 | 0.05 | 0.052 | 0.9 | 5.5 | 5 | 7 |
| 6 | 0.268 | 10 | 40 | 0.05 | 0.070 | 1.3 | 5.7 | 5 | 8 |
| 7 | 0.466 | 20 | 80 | 0.05 | 0.060 | 1.3 | 5.8 | 5 | 8 |
| 8 | 0.674 | 30 | 120 | 0.05 | 0.057 | 1.3 | 5.8 | 5 | 8 |
| 9 | 0.842 | 40 | 160 | 0.05 | 0.054 | 1.3 | 5.8 | 5 | 8 |
| 10 | 0.898 | 50 | 200 | 0.05 | 0.052 | 1.3 | 5.8 | 5 | 8 |
| 11 | 0.403 | 10 | 40 | 0.05 | 0.071 | 1.7 | 6.0 | 5 | 9 |
| 12 | 0.710 | 20 | 80 | 0.05 | 0.056 | 1.7 | 6.0 | 5 | 9 |
| 13 | 0.868 | 30 | 120 | 0.05 | 0.054 | 1.7 | 6.0 | 5 | 9 |
| 14 | 0.957 | 40 | 160 | 0.05 | 0.049 | 1.8 | 6.0 | 5 | 9 |
| 15 | 0.981 | 50 | 200 | 0.05 | 0.055 | 1.7 | 6.0 | 5 | 9 |

| | |
|---|---|
| Scenario | Identifies this line of the report. It will be used cross-reference this line in other reports. |
| σ | The standard deviation within a group. There is a (possibly different) σ for each group. |
| H0 | The null hypothesis that all σ's are equal. |
| H1 | The alternative hypothesis that at least one σ is different from the others. |
| Power | The probability of rejecting H0 when it is false. This is the actual value calculated by the power simulation. |
| n | The average group sample size. The average of the individual group sample sizes. |
| N | The total sample size found by summing all group sample sizes. |
| Target Alpha | The planned probability of rejecting a true H0. This is the planned Type-1 error rate. |
| Actual Alpha | The alpha achieved by the test as calculated by the alpha simulation. Note that the alpha simulation is separate from the power simulation. |
| SD of σ's \| H1 | The standard deviation of the group σ's used in the power simulation. This measures the magnitude of the difference among the σ's. |
| Average of σ's \| H1 | The mean of the group σ's used in the power simulation. This measures the magnitude of the σ's. |
| M1, S1, S2, etc. | The value(s) of the user-specified distribution parameters. |

**Summary Statements**

A one-way design with 4 groups will be used to test whether there is a difference among the variances of the 4 groups (H0: variance homogeneity versus H1: variance heterogeneity). The comparison will be made using a Levene test, with a target Type I error rate (α) of 0.05. Based on 5000 simulations of the null distributions: Normal(10 S1); Normal(10 S1); Normal(10 S1); and Normal(10 S1), and of the alternative distributions: Normal(10 S1); Normal(20 S2); Normal(10 S1); and Normal(10 S1) (where S1 = 5, and S2 = 7), with group sample sizes of 10, 10, 10, and 10 (for a total of 40 subjects), the power is 0.167. (Additional details: The standard deviation and average of the within-group standard deviations, assuming the alternative hypothesis distributions, are 0.9 and 5.5, respectively. The actual Type I error rate (α), based on the null hypothesis distribution simulations, is 0.068.)

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**References**

Conover, W.J. 1999. Practical Nonparametric Statistics. 3rd Edition. John Wiley & Sons. New York.
Devroye, Luc. 1986. Non-Uniform Random Variate Generation. Springer-Verlag. New York.
Gibbons, J.D. and Chakraborti, S. 2011. Nonparametric Statistical Inference, 5th Edition. CRC Press.
Milliken, G.A and Johnson, D.E. 1984. Analysis of Messy Data: Volume 1. Van Nostrand Reinhold. New York.
Winer, B.J., Brown, D.R, and Michels, K.M. 1991. Statistical Principles in Experimental Design. 3rd Edition. McGraw-Hill. New York.

These reports show the output for this run. We will annotate the Numeric Results report.

## Power

This is the probability of rejecting a false null hypothesis. This value is estimated by the power simulation. The Power and Alpha Confidence Interval report displayed next will provide estimates of the precision of these power values.

## Average Group Size n

This is the average of the group sample sizes.

## Total Sample Size N

This is the total sample size of the study.

## Target Alpha

The target value of alpha: the probability of rejecting a true null hypothesis. This is often called the significance level.

## Actual Alpha

This is the value of alpha estimated by the alpha simulation. It should be compared with the Target Alpha. The Power and Alpha Confidence Interval report displayed next will provide estimates of the precision of these Actual Alpha values.

## Standard Deviation of σ's | H1

This is the standard deviation of the hypothesized within-group standard deviations of the power (H1) simulation distributions. Under the H0, this value is zero. So, this value represents the magnitude of the difference among the standard deviations that is being tested.

### Average of σ's | H1

This is the mean of the group standard deviations calculated from the power simulation distributions.

### S1

These are the values entered for S1, the group standard deviation in the power simulation.

### S2

These are the values entered for S2, the group two standard deviation of the power simulation.

## Power and Alpha Confidence Intervals Report

**Power and Alpha Confidence Intervals from Simulations**

| Scenario | Total Sample Size N | Power | 95% Confidence Interval Limits for Power | | Alpha | | 95% Confidence Interval Limits for Alpha | | S1 | S2 |
|---|---|---|---|---|---|---|---|---|---|---|
| | | | Lower | Upper | Target | Actual | Lower | Upper | | |
| 1 | 40 | 0.167 | 0.157 | 0.178 | 0.05 | 0.068 | 0.061 | 0.075 | 5 | 7 |
| 2 | 80 | 0.289 | 0.276 | 0.302 | 0.05 | 0.063 | 0.056 | 0.070 | 5 | 7 |
| 3 | 120 | 0.391 | 0.378 | 0.405 | 0.05 | 0.048 | 0.042 | 0.054 | 5 | 7 |
| 4 | 160 | 0.543 | 0.529 | 0.557 | 0.05 | 0.055 | 0.048 | 0.061 | 5 | 7 |
| 5 | 200 | 0.625 | 0.611 | 0.638 | 0.05 | 0.052 | 0.046 | 0.058 | 5 | 7 |
| 6 | 40 | 0.268 | 0.256 | 0.281 | 0.05 | 0.070 | 0.063 | 0.077 | 5 | 8 |
| 7 | 80 | 0.466 | 0.452 | 0.479 | 0.05 | 0.060 | 0.054 | 0.067 | 5 | 8 |
| 8 | 120 | 0.674 | 0.661 | 0.687 | 0.05 | 0.057 | 0.050 | 0.063 | 5 | 8 |
| 9 | 160 | 0.842 | 0.832 | 0.852 | 0.05 | 0.054 | 0.048 | 0.060 | 5 | 8 |
| 10 | 200 | 0.898 | 0.889 | 0.906 | 0.05 | 0.052 | 0.046 | 0.058 | 5 | 8 |
| 11 | 40 | 0.403 | 0.390 | 0.417 | 0.05 | 0.071 | 0.064 | 0.078 | 5 | 9 |
| 12 | 80 | 0.710 | 0.697 | 0.722 | 0.05 | 0.056 | 0.049 | 0.062 | 5 | 9 |
| 13 | 120 | 0.868 | 0.859 | 0.878 | 0.05 | 0.054 | 0.048 | 0.060 | 5 | 9 |
| 14 | 160 | 0.957 | 0.952 | 0.963 | 0.05 | 0.049 | 0.043 | 0.055 | 5 | 9 |
| 15 | 200 | 0.981 | 0.977 | 0.985 | 0.05 | 0.055 | 0.049 | 0.062 | 5 | 9 |

| | |
|---|---|
| N | The total sample size found by summing all group sample sizes. |
| Power | The probability of rejecting H0 when it is false. This is the actual value calculated by the power simulation. |
| Lower and Upper Limits of a 95% C.I. for Power | The limits of an exact, 95% confidence interval for power based on the binomial distribution. They are calculated from the power simulation. |
| Target Alpha | The desired probability of rejecting a true null hypothesis at which the tests were run. |
| Actual Alpha | The alpha achieved by the test as calculated by the alpha simulation. |
| Lower and Upper Limits of a 95% C.I. for Alpha | The limits of an exact, 95% confidence interval for alpha based on the binomial distribution. They are calculated from the alpha simulation. |
| M1, S1, S2, etc. | The value(s) of the user-specified distribution parameters. |

### Total Sample Size N

This is the total sample size of the study.

### 95% Confidence Interval Limits for Power

These are the limits of an exact, 95% confidence interval for power using the power simulation. The confidence interval is based on the binomial distribution. The width of this confidence interval is directly related to the number of simulations that were used.

### 95% Confidence Interval Limits for Alpha

These are the limits of an exact, 95% confidence interval for alpha using the alpha simulation. The confidence interval is based on the binomial distribution. The width of this confidence interval is directly related to the number of simulations that were used. Since the target alpha is 0.05, 0.05 should be within these limits.

## Detailed Results Reports

**Details for Scenario 1 of the Numeric Results**

| Group | ni | Percent of N | σ \| H0 | σ \| H1 | μ \| H0 | μ \| H1 |
|---|---|---|---|---|---|---|
| 1 | 10 | 25 | 5.0 | 5.0 | 10 | 10.0 |
| 2 | 10 | 25 | 5.0 | 7.0 | 10 | 20.0 |
| 3 | 10 | 25 | 5.1 | 4.9 | 10 | 10.0 |
| 4 | 10 | 25 | 5.0 | 5.0 | 10 | 10.0 |

| Group | N | | SD of σ's \| H0 | SD of σ's \| H1 | SD of μ's \| H0 | SD of μ's \| H1 | Average of σ's \| H0 | Average of σ's \| H1 |
|---|---|---|---|---|---|---|---|---|
| All | 40 | 100 | 0.0 | 0.9 | 0 | 4.3 | 5 | 5.5 |

(More Reports Follow)

These reports show the details of each scenario.

### Scenario (in Title)

This is the row number of the Numeric Results report about which this report gives the details.

### Group

This is the number of the group shown on this line.

### ni

This is the sample size of each group. This column is especially useful when the sample sizes are unequal.

### Percent of N

This is the percentage of the total sample that is allocated to each group.

### σ | H0 and σ | H1

These are the standard deviations that were obtained by the alpha and power simulations, respectively. Note that they often are not exactly equal to what was specified because of the error introduced by simulation.

### μ | H0 and μ | H1

These are the means that were used in the alpha and power simulations, respectively.

### SD of σ's | H0, H1

These are the standard deviations of the within-group standard deviations that were obtained by the alpha and power simulations, respectively. Under H0, this value should be near zero. The H0 value lets you determine if your alpha simulation was correctly specified. The H1 value represents the magnitude of the effect size (when divided by an appropriate measure of the standard deviation).

### SD of μ's | H0, H1

These are the standard deviations of the within-group means that were obtained by the alpha and power simulations, respectively. These values are essentially ignored by the test, but they are provided for completeness.

### Average of σ's | H0, H1

These are the average of the individual group standard deviations that were obtained by the alpha and power simulations, respectively. They give an overall value for the variation in the design.

## Dropout-Inflated Sample Size Report

**Dropout-Inflated Sample Size**

| Average Group Sample Size n | Group | Dropout Rate | Sample Size ni | Dropout-Inflated Enrollment Sample Size ni' | Expected Number of Dropouts Di |
|---|---|---|---|---|---|
| 10 | 1 - 4 | 20% | 10 | 13 | 3 |
| | Total | | 40 | 52 | 12 |
| 20 | 1 - 4 | 20% | 20 | 25 | 5 |
| | Total | | 80 | 100 | 20 |
| 30 | 1 - 4 | 20% | 30 | 38 | 8 |
| | Total | | 120 | 152 | 32 |
| 40 | 1 - 4 | 20% | 40 | 50 | 10 |
| | Total | | 160 | 200 | 40 |
| 50 | 1 - 4 | 20% | 50 | 63 | 13 |
| | Total | | 200 | 252 | 52 |

| | |
|---|---|
| n | The average group sample size. |
| Group | Lists the group numbers. |
| Dropout Rate | The percentage of subjects (or items) that are expected to be lost at random during the course of the study and for whom no response data will be collected (i.e., will be treated as "missing"). Abbreviated as DR. |
| ni | The evaluable sample size for each group at which power is computed (as entered by the user). If ni subjects are evaluated out of the ni' subjects that are enrolled in the study, the design will achieve the stated power. |
| ni' | The number of subjects that should be enrolled in each group in order to obtain ni evaluable subjects, based on the assumed dropout rate. ni' is calculated by inflating ni using the formula ni' = ni / (1 - DR), with ni' always rounded up. (See Julious, S.A. (2010) pages 52-53, or Chow, S.C., Shao, J., Wang, H., and Lokhnygina, Y. (2018) pages 32-33.) |
| Di | The expected number of dropouts in each group. Di = ni' - ni. |

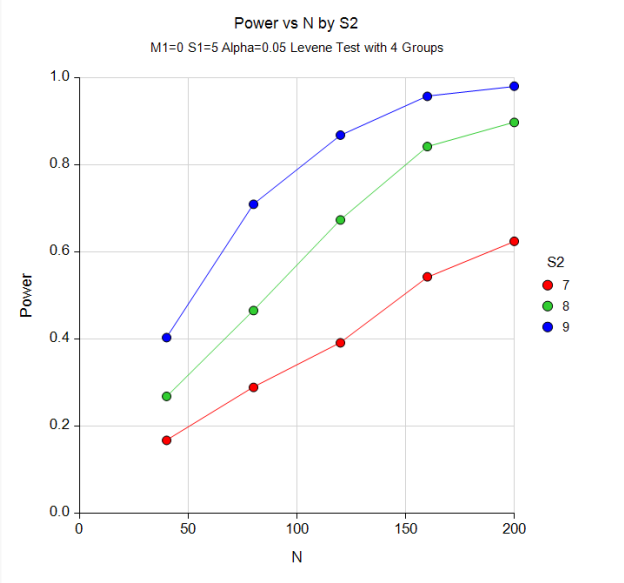
**Dropout Summary Statements**

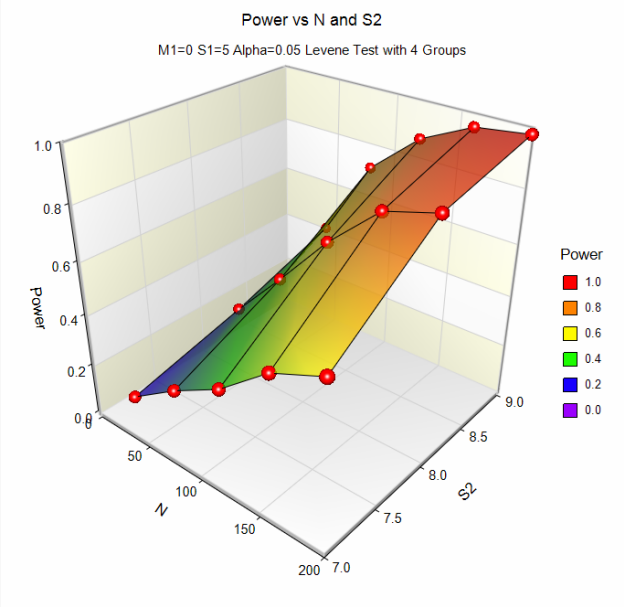
Anticipating a 20% dropout rate, group sizes of 13, 13, 13, and 13 subjects should be enrolled to obtain final group sample sizes of 10, 10, 10, and 10 subjects.

This report shows the sample sizes adjusted for dropout. In this example, dropout is assumed to be 20%. You can change the dropout rate on the Reports tab.

## Plots Section

**Plots**




These plots give a visual presentation to the results in the Numeric Report. We can quickly see the impact on the power of increasing the standard deviation and the sample size.

# Example 2 – Validation

To validate this procedure, we will compare its results to an example run in the *Tests for Two Variances* procedure. For this example, alpha was set to 0.05, the Scale was Standard Deviation, V1 was 1, and V2 was 2. The desired power was 0.8. The resulting sample size as N1 = N2 = 19.

For reproducibility, we'll use a random seed of 5692178.

## Setup

If the procedure window is not already open, use the PASS Home window to open it. The parameters for this example are listed below and are stored in the **Example 2** settings file. To load these settings to the procedure window, click **Open Example Settings File** in the Help Center or File menu.

| Design Tab | |
|---|---|
| Solve For | **Sample Size** |
| Power | **0.80** |
| Alpha | **0.05** |
| Simulations | **5000** |
| Random Seed | **5692178** (for Reproducibility) |
| Number of Groups | **2** |
| Group Allocation | **Equal (n1 = n2 = ⋯)** |
| Group 1 Power Distribution | **Normal(1 S1)** |
| Group 2 Power Distribution | **Normal(2 S2)** |
| Parameter 2 Name | **S1** |
| Parameter 2 Value(s) | **1** |
| Parameter 3 Name | **S2** |
| Parameter 3 Value(s) | **2** |
| Specify Alpha Distributions | **All equal to the group 1 distribution of the power simulation** |

## Output

Click the Calculate button to perform the calculations and generate the following output.

**Simulation Summary**

| | Simulation Distribution | |
|---|---|---|
| **Group** | **Power (H1)** | **Alpha (H0)** |
| 1 | Normal(1 S1) | Normal(1 S1) |
| 2 | Normal(2 S2) | Normal(1 S1) |

| | |
|---|---|
| Number of Groups | 2 |
| Random Number Pool Size | 10000 |
| Number of Simulations | 5000 |
| Random Seed | 5692178 (User-Entered) |
| Run Time | 3.29 seconds |

Levene Test of Variances (Simulation)

**Numeric Results**

Solve For: Sample Size

| | Power | | Sample Size | | Alpha | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|
| Scenario | Target | Actual | Average Group n | Total N | Target | Actual | Standard Deviation of σ's \| H1 | Average of σ's \| H1 | S1 | S2 |
| 1 | 0.8 | 0.801 | 22 | 44 | 0.05 | 0.056 | 0.5 | 1.5 | 1 | 2 |

Note that **PASS** calculates the sample size as 44 (22 per group). This is close to the analytic answer of 38 (19 per group) for the regular variance ratio test.