

## Chapter 636

# Meta-Analysis of Correlation Tests using a Fixed-Effects Model

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## Introduction

This procedure calculates power and number of studies for fixed-effect meta-analyses of parallel group studies that are summarized by the correlation coefficient of two variables of interest. The sample size formulas used by this procedure were published in Hedges and Pigott (2001) and Harrer *et al.* (2022).

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## Meta-Analysis

Meta-analysis refers to the systematic assessment of a set of two or more individual studies (experiments or clinical trials). The studies must be similar so that their results may be numerically compared. For example, in this procedure all studies must have as their outcome the correlation coefficient between two variables of interest.

By combining the results of several studies, their individual results may be summarized and assessed. The power of the combined test will be increased since it represents a larger sample.

## Fixed-Effects versus Random-Effects Models

The *fixed-effects model*, which is assumed in this procedure, assumes that all studies have the same true effect size. That is, it assumes that all factors that might influence the effect size are the same so that the true effect size for all studies is the same. Therefore, only the *within study variability* needs to be considered since the effect size variation occurs because of random error.

The *random-effects model*, which is assumed in another procedure, assumes that the studies in the analysis are similar, but not to the extent that they all have the same true effect size. Instead, it is assumed that each study has its own true effect size and that the particular outcome of the study is a sample from a distribution with that mean. The random-effects model must not only account for the *within study variability*, but it must also account for the *between study variability*.

## Technical Details

### The Meta-Analysis of the Correlation Coefficient

The sample size formulas used by this procedure were published in Harrer *et al.* (2022).

A series of  $K$  studies are assumed to each have an effect size  $\rho$  which is the correlation coefficient between two variables. The results of these individual studies may be combined into one overall statistical test that draws a more precise conclusion.

### Test Statistic

Suppose the  $i^{\text{th}}$  study yields a sample size of  $N$  as well as a *correlation coefficient* which is calculated as

$$r_i = \frac{S_{XY}^2}{S_X S_Y}$$

This sample correlation coefficient is used to estimate a population correlation coefficient

$$\rho = \frac{\sigma_{XY}^2}{\sigma_X \sigma_Y}$$

The sample correlation coefficients do not directly follow the normal distribution, so the following normalizing transformation is used to achieve a normally-distributed test statistic.

$$T = \ln \left( \frac{1+r}{1-r} \right) \frac{1}{2}$$

This variable has the approximate within-study standard deviation

$$\sigma_w = \frac{1}{\sqrt{K(N-3)}}$$

If a *fixed-effect model* is assumed, the variance of  $T$  ignores the possibility of between-study variation, so the following test statistic may be constructed

$$Z = \frac{T - T_0}{S_w}$$

where  $T$  is a weighted average of the individual  $T_i$  and  $S_w$  is estimated from the above formula with  $N$  set to the average sample size of all the studies in the meta-analysis.

We assume that  $Z$  follows the standard normal distribution and so it can be used to test the null hypothesis  $H_0: \rho = \rho_0$  at significance level of  $\alpha$ .

## Hypotheses

Three sets of statistical hypotheses are commonly used. These are

$$H_0: \rho \leq \rho_0 \quad \text{vs.} \quad H_1: \rho > \rho_0$$

$$H_0: \rho \geq \rho_0 \quad \text{vs.} \quad H_1: \rho < \rho_0$$

$$H_0: \rho = \rho_0 \quad \text{vs.} \quad H_1: \rho \neq \rho_0$$

Usually,  $\rho_0$  is set to zero.

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## Power Calculation

The power of a one-tailed test that  $\rho = \rho_0$  is given by

$$\text{Power} = 1 - \Phi(c_\alpha - \lambda)$$

where  $\rho$  is set by the alternative hypothesis,  $c_\alpha$  is the  $1 - \alpha$  quantile of the standard normal distribution, and

$$\lambda = \frac{\rho - \rho_0}{\sqrt{V}}$$

$$V = \sigma_W^2$$

The power of a two-tailed test that  $\rho = \rho_0$  is given by

$$\text{Power} = 1 - \Phi(c_{\alpha/2} - \lambda) + \Phi(-c_{\alpha/2} - \lambda)$$

These formulas can be used in search algorithms to determine the necessary number of studies required to achieve a desired level of power.

## Example 1 – Calculating Number of Studies

Researchers wish to determine a reasonable number of studies for a meta-analysis with the following characteristics: outcome is a correlation coefficient, two-sided significance level of 0.05, power of 0.8, average sample size of all studies is 10,  $\rho_1 = 0.2, 0.3, 0.4$ , and  $\rho_0 = 0.0$ .

### Setup

If the procedure window is not already open, use the PASS Home window to open it. The parameters for this example are listed below and are stored in the **Example 1** settings file. To load these settings to the procedure window, click **Open Example Settings File** in the Help Center or File menu.

#### Design Tab

Solve For .....	<b>Number of Studies</b>
Alternative Hypothesis .....	<b>Two-Sided (H1: <math>\rho \neq \rho_0</math>)</b>
Power.....	<b>0.8</b>
Alpha.....	<b>0.05</b>
N (Average Sample Size Per Study).....	<b>10</b>
$\rho_0$ (Correlation H0) .....	<b>0.0</b>
$\rho_1$ (Correlation H1) .....	<b>0.2 0.3 0.4 0.5</b>

## Output

Click the Calculate button to perform the calculations and generate the following output.

### Numeric Reports

#### Numeric Results

Solve For: [Number of Studies](#)  
 Hypotheses:  $H_0: \rho = \rho_0$  vs.  $H_1: \rho \neq \rho_0$

Power		Average Sample Size Per Study N	Number of		Correlation		Alpha
Target	Actual		Studies K	Subjects KN	$\rho H_0$ $\rho_0$	$\rho H_1$ $\rho_1$	
0.8	0.81011	10	28	280	0	0.2	0.05
0.8	0.80971	10	12	120	0	0.3	0.05
0.8	0.84269	10	7	70	0	0.4	0.05
0.8	0.82810	10	4	40	0	0.5	0.05

Target Power	The power desired. Power is the probability of rejecting a false null hypothesis.
Actual Power	The power actually achieved. Power is the probability of rejecting a false null hypothesis about the correlation. This is the power calculated at the actual correlation.
N	The average sample size per study calculated on all studies included in the meta-analysis.
K	The number of studies included in the meta-analysis.
KN	The number of subjects included in the meta-analysis. $KN = K \times N$ .
$\rho_0$	The value of the correlation coefficient assumed by the null hypothesis, $H_0$ .
$\rho_1$	The value of the correlation coefficient assumed by the alternative hypothesis, $H_1$ . This is the correlation at which the power is calculated.
Alpha	The probability of rejecting a true null hypothesis.

#### Summary Statements

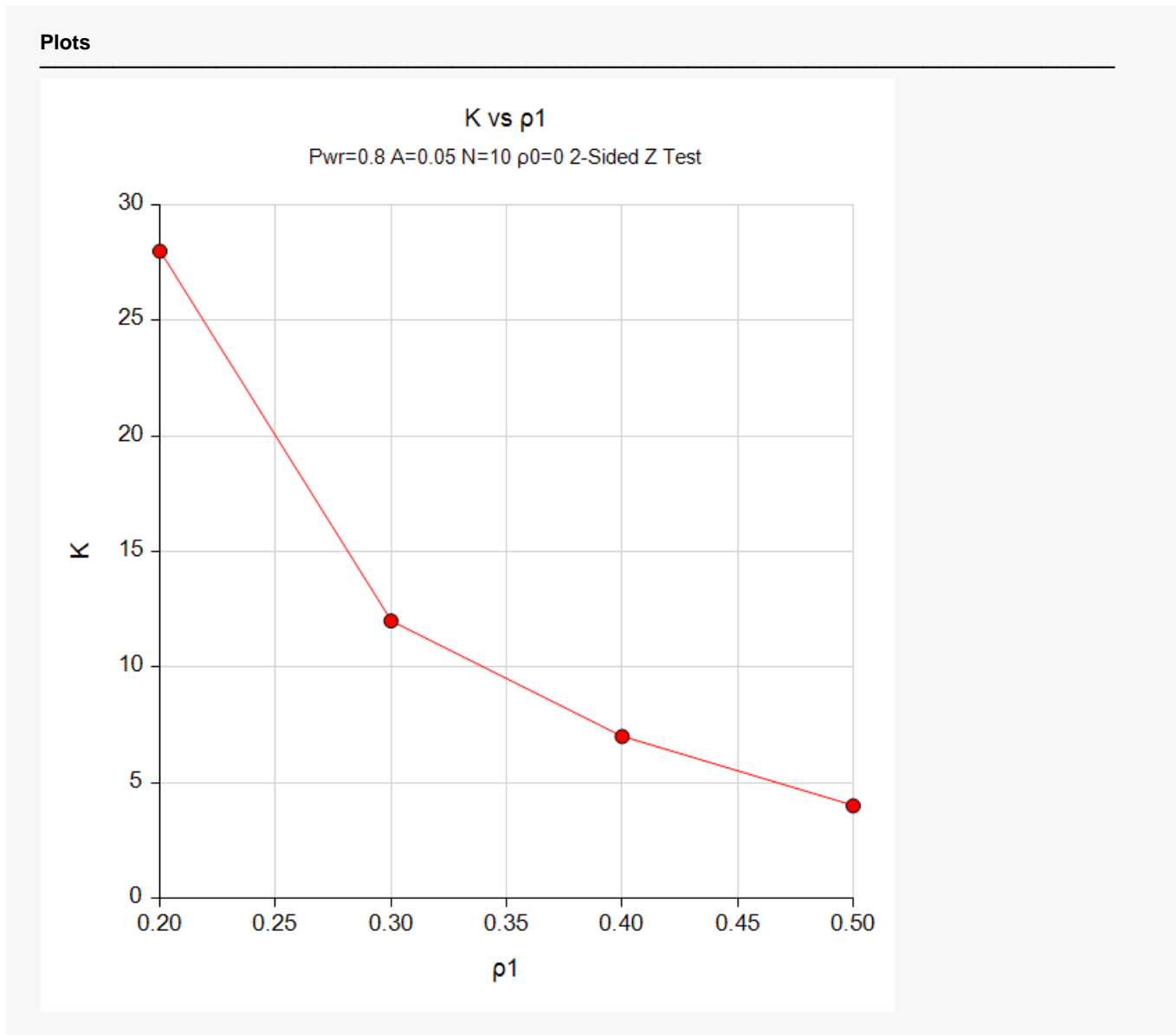
A collection of correlation studies will be used to test whether the correlation is different from 0 ( $H_0: \rho = 0$  versus  $H_1: \rho \neq 0$ ). The meta-analysis comparison will be made using a two-sided fixed-effects-model Z-test, with a Type I error rate ( $\alpha$ ) of 0.05. With an average study sample size of 10, to detect a correlation of 0.2 with 80% power, the number of needed studies will be 28.

#### References

- Borenstein, M., Hedges, L.V., Higgins, J.P.T., and Rothstein, H.R. 2021. Introduction to Meta-Analysis, Second Edition. John Wiley and Sons.
- Cooper, J., Hedges, L.V., Valentine, J.C. 2019. The Handbook of Research Synthesis and Meta-Analysis, Third Edition. Russell Sage Foundation.
- Griffin, J.W. 2021. 'Calculating statistical power for meta-analysis using metapower', The Quantitative Methods for Psychology, Vol. 17, No. 1, pages 24-39.
- Harrer, M., Cuijpers, P., Furukawa, T.A., and Ebert, D.D. 2022. Doing Meta-Analysis with R: A Hands-On Guide. CRC Press.
- Hedges, L.V. and Pigott, T.D. 2001. 'The Power of Statistical Tests in Meta-Analysis', Psychological Methods, Volume 6, No. 3, pages 203-217.
- Valentine, J.C., Pigott, T.D., and Rothstein, H.R. 2010. 'How Many Studies Do You Need? A Primer on Statistical Power for Meta-Analysis', Journal of Education and Behavioral Statistics, Volume 35, No. 2, pages 215-247.

This report shows the sample sizes for the indicated scenarios.

## Plots Section



This plot represents the required number of studies for various values of the correlation coefficient.

## Example 2 – Validation using Hedges and Pigott (2001)

Hedges and Pigott (2001) page 208 present an example of solving for power for a test of a correlation coefficient if the meta-analysis uses two-sided test with significance level of 0.05, number of studies = 10, average sample size of 10,  $\rho_1 = 0.1$ , and  $\rho_0 = 0.0$ . They find the power to be 0.32.

### Setup

If the procedure window is not already open, use the PASS Home window to open it. The parameters for this example are listed below and are stored in the **Example 2** settings file. To load these settings to the procedure window, click **Open Example Settings File** in the Help Center or File menu.

Design Tab

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Solve For ..... **Power**  
 Alternative Hypothesis ..... **Two-Sided (H1:  $\rho \neq \rho_0$ )**  
 Alpha..... **0.05**  
 K (Number of Studies) ..... **10**  
 N (Average Sample Size Per Study)..... **25**  
 $\rho_0$  (Correlation|H0) ..... **0.0**  
 $\rho_1$  (Correlation|H1) ..... **0.1**

### Output

Click the Calculate button to perform the calculations and generate the following output.

**Numeric Results**

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Solve For: **Power**  
 Hypotheses:  $H_0: \rho = \rho_0$  vs.  $H_1: \rho \neq \rho_0$

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	Average Sample Size Per Study N	Number of		Correlation		Alpha
		Studies K	Subjects KN	$\rho H_0$ $\rho_0$	$\rho H_1$ $\rho_1$	
<b>Power</b>	25	10	250	0	0.1	0.05

PASS also calculates a power of 0.32 which validates the procedure.