

Chapter 637

Meta-Analysis of Correlation Tests using a Random-Effects Model

Introduction

This procedure calculates power and number of studies for random-effect meta-analyses of parallel group studies that are summarized by the correlation coefficient of two variables of interest. The sample size formulas used by this procedure were published in Hedges and Pigott (2001) and Harrer *et al.* (2022).

Meta-Analysis

Meta-analysis refers to the systematic assessment of a set of two or more individual studies (experiments or clinical trials). The studies must be similar so that their results may be numerically compared. For example, in this procedure all studies must have as their outcome the correlation coefficient between two variables of interest.

By combining the results of several studies, their individual results may be summarized and assessed. The power of the combined test will be increased since it represents a larger sample.

Fixed-Effects versus Random-Effects Models

The *fixed-effects model*, which is assumed by another procedure, assumes that all studies have the same true effect size. That is, it assumes that all factors that might influence the effect size are the same so that the true effect size for all studies is the same. Therefore, only the *within study variability* needs to be considered since the effect size variation occurs because of random error.

The *random-effects model*, which is assumed by this procedure, assumes that the studies in the analysis are similar, but not to the extent that they all have the same true effect size. Instead, it is assumed that each study has its own true effect size and that the particular outcome of the study is a sample from a distribution with that mean. The random-effects model must not only account for the *within study variability*, but it must also account for the *between study variability*.

Technical Details

The Meta-Analysis of the Correlation Coefficient

The sample size formulas used by this procedure were published in Harrer *et al.* (2022).

A series of K studies are assumed to each have an effect size ρ which is the correlation coefficient between two variables. The results of these individual studies may be combined into one overall statistical test that draws a more precise conclusion.

Test Statistic

Suppose the i^{th} study yields a sample size of N as well as a *correlation coefficient* which is calculated as

$$r_i = \frac{S_{XY}^2}{S_X S_Y}$$

This sample correlation coefficient is used to estimate the mean correlation coefficient

$$\rho = \frac{\sigma_{XY}^2}{\sigma_X \sigma_Y}$$

The sample correlation coefficients do not directly follow the normal distribution, so the following normalizing transformation is used to achieve a normally-distributed test statistic.

$$T = \ln \left(\frac{1+r}{1-r} \right) \frac{1}{2}$$

This variable has the approximate within-study variance

$$V_W = \frac{1}{(N-3)}$$

When a *random-effect model* is assumed, the variance of T must augment a component that represents the between-study variation. Because the between-study variance-component is difficult to estimate during the planning stage of the study, it is estimated as a multiple of the within-study variance. That is, the between-study variance-component is given by

$$V_B = R V_W.$$

Here V_W represents the within-study variance and R is the heterogeneity ratio. This ratio allows us to specify the between-study variance, V_B , as a multiple of V_W . Hedges and Pigott (2001) recommend using 0.333 for a *small* value, 0.667 for a *medium* value, and 1.0 for a *large* value. Setting R to 0 results in the fixed-effect result.

Meta-Analysis of Correlation Tests using a Random-Effects Model

The test statistic may be represented by

$$Z = \frac{T - T_0}{\sqrt{(V_W + V_B)/K}} = \frac{T - T_0}{\sqrt{V_W(1 + R)/K}}$$

where T is a weighted average of the individual T_i and the within-study variation is estimated from the above formula with N set to the average sample size of all the studies in the meta-analysis.

We assume that Z follows the standard normal distribution and so it can be used to test the null hypothesis $H_0: \rho = \rho_0$ at significance level of α .

I^2 - Heterogeneity Index

Some authors set the amount of heterogeneity in the study using the heterogeneity index I^2 (also called the *degree of heterogeneity* or *inconsistency*) instead of R . This value is related to R using the relationship

$$R = \frac{I^2}{1 - I^2}$$

Hypotheses

Three sets of statistical hypotheses are commonly used. These are

$$H_0: \rho \leq \rho_0 \quad \text{vs.} \quad H_1: \rho > \rho_0$$

$$H_0: \rho \geq \rho_0 \quad \text{vs.} \quad H_1: \rho < \rho_0$$

$$H_0: \rho = \rho_0 \quad \text{vs.} \quad H_1: \rho \neq \rho_0$$

Usually, ρ_0 is set to zero.

Power Calculation

The power of a one-tailed test that $\rho = \rho_0$ is given by

$$Power = 1 - \Phi(c_\alpha - \lambda)$$

where ρ is set by the alternative hypothesis, c_α is the $1 - \alpha$ quantile of the standard normal distribution, and

$$\lambda = \frac{\rho - \rho_0}{\sqrt{V}}$$

$$V = V_W(1 + R)$$

The power of a two-tailed test that $\rho = \rho_0$ is given by

$$Power = 1 - \Phi(c_{\alpha/2} - \lambda) + \Phi(-c_{\alpha/2} - \lambda)$$

These formulas can be used in search algorithms to determine the necessary number of studies required to achieve a desired level of power.

Example 1 – Calculating Sample Size

Researchers wish to determine a reasonable number of studies for a meta-analysis with the following characteristics: outcome is a correlation coefficient, two-sided significance level of 0.05, power of 0.8, average sample size of all studies is 10, $\rho_1 = 0.2$, and $\rho_0 = 0.0$. They want to compare the results of setting $R = 0.333, 0.667, \text{ or } 1.0$.

Setup

If the procedure window is not already open, use the PASS Home window to open it. The parameters for this example are listed below and are stored in the **Example 1** settings file. To load these settings to the procedure window, click **Open Example Settings File** in the Help Center or File menu.

Design Tab	
Solve For	Number of Studies
Alternative Hypothesis	Two-Sided (H1: $\rho \neq \rho_0$)
Power.....	0.8
Alpha.....	0.05
N (Average Sample Size Per Study).....	10
ρ_0 (Correlation H0)	0.0
ρ_1 (Correlation H1)	0.2
Calculate Between-Study Variance From	R (Between-Study Var / Within-Study Var)
R (Between-Study Var / Within-Study Var)	0.333 0.667 1

Output

Click the Calculate button to perform the calculations and generate the following output.

Numeric Reports

Numeric Results

Solve For: **Number of Studies**
 Hypotheses: $H_0: \rho = \rho_0$ vs. $H_1: \rho \neq \rho_0$
 Between-Study Variance Input: R (Between-Study / Within-Study Variances)

Power		Average Sample Size Per Study N	Number of		Correlation		Between-Study Variance Calculation		Alpha
Target	Actual		Studies K	Subjects KN	ρH_0 ρ_0	ρH_1 ρ_1	R	I^2	
0.8	0.80674	10	37	370	0	0.2	0.333	0.24981	0.05
0.8	0.80446	10	46	460	0	0.2	0.667	0.40012	0.05
0.8	0.80313	10	55	550	0	0.2	1.000	0.50000	0.05

- Target Power The power desired. Power is the probability of rejecting a false null hypothesis.
- Actual Power The power actually achieved. Power is the probability of rejecting a false null hypothesis about the correlation. This is the power calculated at the actual correlation.
- N The average sample size per study calculated on all studies included in the meta-analysis.
- K The number of studies included in the meta-analysis.
- KN The number of subjects included in the meta-analysis. $KN = K \times N$.
- ρ_0 The value of the correlation coefficient assumed by the null hypothesis, H_0 .
- ρ_1 The value of the correlation coefficient assumed by the alternative hypothesis, H_1 . This is the correlation at which the power is calculated.
- R The value of the heterogeneity ratio used to specify the between study variance. The formula is (Between Study Variance) = $R \times$ (Within Study Variance). The following values are often used: Small = 0.333, Medium = 0.667, Large = 1.000.
- I^2 The value of the heterogeneity index used to specify the Between Study variance. The formula is (Between Study Variance) = $I^2 / (1 - I^2) \times$ (Within Study Variance). The following values are often used: Small = 0.25, Medium = 0.5, Large = 0.75.
- Alpha The probability of rejecting a true null hypothesis.

Summary Statements

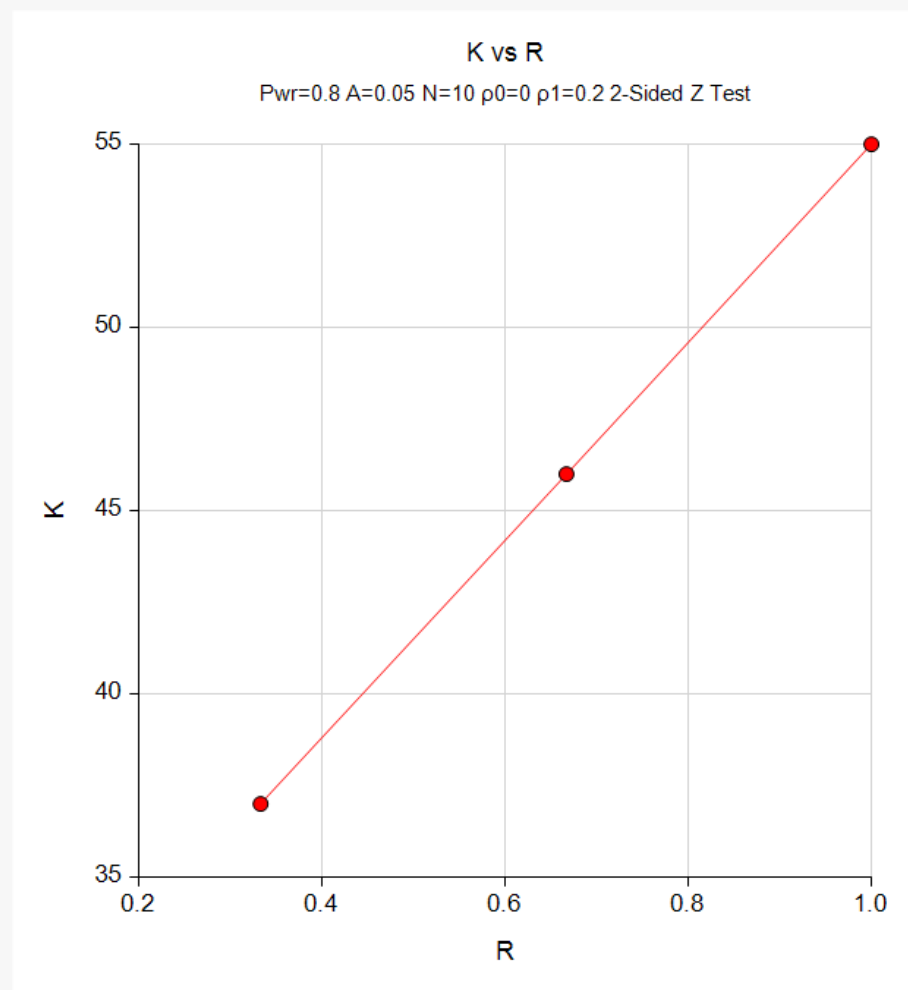
A collection of correlation studies will be used to test whether the correlation is different from 0 ($H_0: \rho = 0$ versus $H_1: \rho \neq 0$). The meta-analysis comparison will be made using a two-sided random-effects-model Z-test, with a Type I error rate (α) of 0.05. The heterogeneity ratio used to create the between-study variance is assumed to be 0.333. With an average study sample size of 10, to detect a correlation of 0.2 with 80% power, the number of needed studies will be 37.

Meta-Analysis of Correlation Tests using a Random-Effects Model

References

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- Borenstein, M., Hedges, L.V., Higgins, J.P.T., and Rothstein, H.R. 2021. Introduction to Meta-Analysis, Second Edition. John Wiley and Sons.
- Cooper, J., Hedges, L.V., Valentine, J.C. 2019. The Handbook of Research Synthesis and Meta-Analysis, Third Edition. Russell Sage Foundation.
- Griffin, J.W. 2021. 'Calculating statistical power for meta-analysis using metapower', The Quantitative Methods for Psychology, Vol. 17, No. 1, pages 24-39.
- Harrer, M., Cuijpers, P., Furukawa, T.A., and Ebert, D.D. 2022. Doing Meta-Analysis with R: A Hands-On Guide. CRC Press.
- Hedges, L.V. and Pigott, T.D. 2001. 'The Power of Statistical Tests in Meta-Analysis', Psychological Methods, Volume 6, No. 3, pages 203-217.
- Valentine, J.C., Pigott, T.D., and Rothstein, H.R. 2010. 'How Many Studies Do You Need? A Primer on Statistical Power for Meta-Analysis', Journal of Education and Behavioral Statistics, Volume 35, No. 2, pages 215-247.
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This report shows the sample sizes for the indicated scenarios.

Plots Section**Plots**

This plot represents the required number of studies for various values of the heterogeneity ratio, R .

Example 2 – Validation using Hand Calculations

We could not find a validation example for this procedure in the literature so we will validate the procedure using hand calculations.

Suppose a meta-analysis is contemplated with the following characteristics: outcome is a correlation coefficient, two-sided significance level of 0.05, power of 0.8, average sample size of all studies is 20, $\rho_1 = 0.2$, and $\rho_0 = 0.0$. They want to compare the results of setting $R = 1.0$ when the number of studies is 15.

Step 1: Compute the normalized effect size.

$$T = \ln\left(\frac{1+r}{1-r}\right)\frac{1}{2} = \ln\left(\frac{1+0.2}{1-0.2}\right)\frac{1}{2} = 0.2027326$$

Step 2: Compute the within-study variance component.

$$V_W = \frac{1}{N-3} = \frac{1}{17} = 0.0588235$$

Step 3: Compute the between-study variance component.

$$V_B = R(V_W) = 0.0588235$$

Step 4: Compute the standard error of the effect size.

$$SE_T = \sqrt{\frac{V_W + V_B}{K}} = \sqrt{\frac{0.0588235 + 0.0588235}{15}} = 0.0885615$$

Step 5: Compute λ .

$$\lambda = \frac{T - T_0}{SE_T} = \frac{0.2027326 - 0}{0.0885615} = 2.2891725$$

Step 6: Compute $c_{\alpha/2}$.

$$c_{\alpha/2} = \Phi(1 - 0.025) = 1.95996398$$

Step 7: Compute the power.

$$\begin{aligned} \text{Power} &= 1 - \Phi(c_{\alpha/2} - \lambda) + \Phi(-c_{\alpha/2} - \lambda) \\ &= 1 - \Phi(1.95996398 - 2.2891725) + \Phi(-1.95996398 - 2.2891725) \\ &= 1 - 0.3709990 + 0.0000107298 \\ &= 0.6290117 \end{aligned}$$

Setup

If the procedure window is not already open, use the PASS Home window to open it. The parameters for this example are listed below and are stored in the **Example 2** settings file. To load these settings to the procedure window, click **Open Example Settings File** in the Help Center or File menu.

Design Tab	
Solve For	Power
Alternative Hypothesis	Two-Sided (H1: $\rho \neq \rho_0$)
Alpha.....	0.05
K (Number of Studies)	15
N (Average Sample Size Per Study).....	20
ρ_0 (Correlation H0)	0.0
ρ_1 (Correlation H1)	0.2
Calculate Between-Study Variance From	R (Between-Study Var / Within-Study Var)
R (Between-Study Var / Within-Study Var)	1

Output

Click the Calculate button to perform the calculations and generate the following output.

Numeric Results								
Solve For:	Power							
Hypotheses:	H0: $\rho = \rho_0$ vs. H1: $\rho \neq \rho_0$							
Between-Study Variance Input:	R (Between-Study / Within-Study Variances)							
Power	Average Sample Size Per Study	Number of		Correlation		Between-Study Variance Calculation		Alpha
	N	Studies K	Subjects KN	ρH_0 ρ_0	ρH_1 ρ_1	R	I ²	
0.62901	20	15	300	0	0.2	1	0.5	0.05

PASS also calculates a power that rounds to 0.62901 which validates the procedure.