Chapter 362

Mixed Models Tests for Slope-Interaction in a 2×2 Factorial 3-Level Hierarchical Design with Fixed Slopes (Level-2 Randomization)

Introduction

This procedure calculates power and sample size for a three-level hierarchical mixed model which is randomized at the **second** level. The associated **longitudinal** study uses a 2-by-2 factorial design with two binary factors X and Z, each with two possible values (0 and 1). This results in four treatment arms. The goal of the study is to test whether the slopes of subjects across time are different from what would be expected if the effect of the two factors were additive. That is, one wants to test the three-way interaction between the two binary factors and time.

In most cases, this design is called a *repeated measures* design. The classic example is a study in which the level-2 units are subjects which are nested in level-3 units (e.g., classes, clinics, or hospitals). The level-1 units are time points at which measurements are taken. This factor is nested in the level-2 units.

This procedure is for longitudinal studies in which each subject is measured two or more times.

In this case of level-2 randomization, each level-2 unit (subject) is randomly assigned to one of the four treatments combinations. Hence, a level-3 unit will include subjects in all four treatment groups.

All subjects in a treatment group are assumed to have the same (*fixed*) slope.

Technical Details

Our formulation comes from Ahn, Heo, and Zhang (2015), chapter 6, section 6.6.2, pages 221-222. The hierarchical mixed model that is adopted is

$$Y_{ij} = \beta_0 + \delta_X X_{ijk} + \delta_Z Z_{ijk} + \delta_T T_{ijk} + \delta_{XZ} X_{ijk} Z_{ijk} + \delta_{XT} X_{ijk} T_{ijk} + \delta_{ZT} Z_{ijk} T_{ijk} + \boldsymbol{\delta_{XZT} X_{ijk} Z_{ijk} T_{ijk}} + u_i$$
$$+ u_{j(i)} + e_{ijk}$$

where

Y_{ijk} is the continuous response of the kth level-1 unit, within the jth level-2 unit, within the ith level-3 unit.

X_{ijk} is an indicator variable that is equal to "1" if the jth level-2 unit is assigned to receive intervention X and "0" otherwise. Thus, $X_{ijk} = Xj$ for all i and k.

Z_{ijk} is an indicator variable that is equal to "1" if the jth level-2 unit is assigned to receive intervention Z and "0" otherwise. Thus, $Z_{ijk} = Zj$ for all i and k.

β_0 is the fixed intercept.

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δ_X is the treatment effect of factor X.

δ_Z is the treatment effect of factor Z.

δ_{XZ} is the interaction effect of factors X and Z.

δ_{XT} is the interaction effect of factors X and Z.

δ_{ZT} is the interaction effect of factors X and Z.

δ_{XZT} is the 3-way interaction effect of X, Z, and time. **This is the coefficient of interest**.

u_i is the level-3 random intercept effect for the ith level-3 unit. It is distributed as $N(0, \sigma_3^2)$.

$u_{j(i)}$ is the level-2 random intercept effect for the $j(i)$th level-2 unit. It is distributed as $N(0, \sigma_2^2)$.

e_{ijk} is the level-1 random intercept effect that is distributed as $N(0, \sigma_e^2)$.

σ^2 is the variance of Y when slopes are fixed, where $\sigma^2 = \sigma_3^2 + \sigma_2^2 + \sigma_e^2$.

ρ_1 is the correlation among level-1 units which are in a particular level-2 unit.

$K_{0,0}$ is the number of level-2 units per level-3 unit for which X = 0 and Z = 0.

$K_{0,1}$ is the number of level-2 units per level-3 unit for which X = 0 and Z = 1.

$K_{1,0}$ is the number of level-2 units per level-3 unit for which X = 1 and Z = 0.

$K_{1,1}$ is the number of level-2 units per level-3 unit for which X = 1 and Z = 1.

C is the number of level-3 units.

M is the number of level-1 units per level-2 unit. It is the number of measurement times.

The test of significance of the product $X_{ijk}Z_{ijk}T_{ijk}$ is the interaction effect of X, Z, and Time. This is the test statistic of interest. It tests whether the subject-specific slopes behave the same across all treatment combinations.

Assume that δ_{XZT} is to be tested using a Wald test. The statistical hypotheses are $H_0: \delta_{XZT} = 0$ vs. $H_a: \delta_{XZT} \neq 0$.

The power is calculated using

$$Power = \Phi \left\{ \left| \frac{\delta_{XZT}}{\sigma} \right| \sqrt{ \frac{C K_{0,0} M \, \text{Var}(T)}{f \left(\frac{1}{K_{0,0}} + \frac{1}{K_{1,1}} + \frac{1}{K_{1,0}} + \frac{1}{K_{0,1}} \right)} } - \Phi^{-1}(1 - \alpha/2) \right\}$$

where $f = 1 - \rho_1$ and $\text{Var}(T) = \sum_{j=1}^{M} \left(T_j - \bar{T} \right)^2 / M$.

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Example 1 – Calculating Power

Suppose that a three-level hierarchical design is planned in which there will be two interventions. Each intervention will be whether one of two drugs is administered. This will result in four treatment groups. There will be five measurements per subject and the four treatments will be randomly applied to level-2 units.

The analysis will be a mixed model of continuous data using the model given earlier in this chapter. The subject-specific slopes will be assumed to be *fixed*. The goal of the study is to test the regression coefficient of the XZT interaction. The following parameter settings are to be used for the power analysis: $\delta_{XZT} = 3$; $\sigma = 9.8$; $\rho_1 = 0.1$; C = 5 or 10; M = 5; $\alpha = 0.05$; and K00 = K01 = K10 = K11 = 5 to 20 by 5. Find the power of each combination of parameter settings.

Setup

If the procedure window is not already open, use the PASS Home window to open it. The parameters for this example are listed below and are stored in the **Example 1** settings file. To load these settings to the procedure window, click **Open Example Settings File** in the Help Center or File menu.

Design Tab

Solve For ....**Power**
Alpha...**0.05**
C (Level 3 Units) **5 10**
K00 (Group 00 Count (X=0, Z=0)) **5 10 15 20**
K01 (Group 01 Count (X=0, Z=1)) **K00**
K10 (Group 10 Count (X=1, Z=0)) **K00**
K11 (Group 11 Count (X=1, Z=1)) **K00**
M (Level 1 Units Per Level-2 Unit) **5**
δ_{XZT} (Three-Way Interaction).........................**3**
σ (Standard Deviation)..................................**9.8**
ρ_1 (Correlation Among Level-1 Units)............**0.1**

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Output

Click the Calculate button to perform the calculations and generate the following output.

Numeric Reports

Numeric Results
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Solve For: Power
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Power	Total Sample Size N	Number of Level-3 Units C	Group 00 K00	Group 01 K01	Group 10 K10	Group 11 K11	Total K	Number of Level-1 Units per Level-2 Unit M	Three-Way Interaction δ_{XZT}	Standard Deviation σ	Correlation Among Level-1 Units ρ_1	Alpha
			Number of Level-2 Units per Level-3 Unit									
0.7228	500	5	5	5	5	5	20	5	3	9.8	0.1	0.05
0.9503	1000	5	10	10	10	10	40	5	3	9.8	0.1	0.05
0.9930	1500	5	15	15	15	15	60	5	3	9.8	0.1	0.05
0.9992	2000	5	20	20	20	20	80	5	3	9.8	0.1	0.05
0.9503	1000	10	5	5	5	5	20	5	3	9.8	0.1	0.05
0.9992	2000	10	10	10	10	10	40	5	3	9.8	0.1	0.05
1.0000	3000	10	15	15	15	15	60	5	3	9.8	0.1	0.05
1.0000	4000	10	20	20	20	20	80	5	3	9.8	0.1	0.05
_____

Power   The probability of rejecting a false null hypothesis when the alternative hypothesis is true.
N   The total number of Level-1 units in the study.
C   The number of Level-2 units (subjects) per Level-3 unit.
K00   The number of Level-2 units per Level-3 unit in Group (0,0) (the group in which X = 0 and Z = 0).
K01   The number of Level-2 units per Level-3 unit in Group (0,1) (the group in which X = 0 and Z = 1).
K10   The number of Level-2 units per Level-3 unit in Group (1,0) (the group in which X = 1 and Z = 0).
K11   The number of Level-2 units per Level-3 unit in Group (1,1) (the group in which X = 1 and Z = 1).
K   The total number of Level-2 units.
M   The number of Level-1 units per Level-2 unit (i.e., the number of time points).
δ_{XZT} The 3-way interaction among the subject-specific slopes (β_{11} - β_{10}) - (β_{01} - β_{00}) at which the power is calculated.
σ The standard deviation of the Yijk assuming a fixed-slope model.
ρ_1 The correlation among Level-1 units in a particular Level-2 unit.
Alpha   The probability of rejecting a true null hypothesis.

Summary Statements
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A 2×2 factorial (X = 0,1 and Z = 0,1) 3-level hierarchical design will have subjects (level-2 units) in clusters (level-3 units, e.g., classes, clinics, or hospitals), with random assignment of subjects to each of the 4 treatment arms (Groups 00, 01, 10, and 11) (level-2 randomization), and with repeated measurements (level-1 units) on each subject (over time). This design will be used to test whether the outcome trends (slopes) are different for each of the treatment combinations (or, equivalently, whether the group combination mean differences change across time). This hypothesis will be evaluated by testing the three-way interaction term (X × Z × time) of the linear mixed-effects model, assuming fixed slopes, with a Type I error rate (α) of 0.05. This interaction is formed from the four group slopes across time using the following formula: δ_{XZT} = (β_{11} - β_{10}) - (β_{01} - β_{00}). The standard deviation of Yijk, assuming a fixed-slope model, is assumed to be 9.8 (this standard deviation is the square-root of the variance of Yijk, where the variance is the sum of the error term variance, the level-2 random intercept variance, and the level-3 random intercept variance). The intraclass correlation coefficient of level-1 units (repeated measurements on a subject) is assumed to be 0.1. To detect a three-way interaction among the subject-specific slopes (δ_{XZT}) of at least 3, with 5 level-3 units (clusters), and within each level-3 unit, level-2 (subject-level) sample sizes of 5 in Group 00 (X = 0, Z = 0), 5 in Group 01 (X = 0, Z = 1), 5 in Group 10 (X = 1, Z = 0), and 5 in Group 11 (X = 1, Z = 1), with 5 level-1 units (repeated measurements) obtained from each level-2 unit (subject) (for a grand total of 500 level-1 measurements), the power is 0.7228.
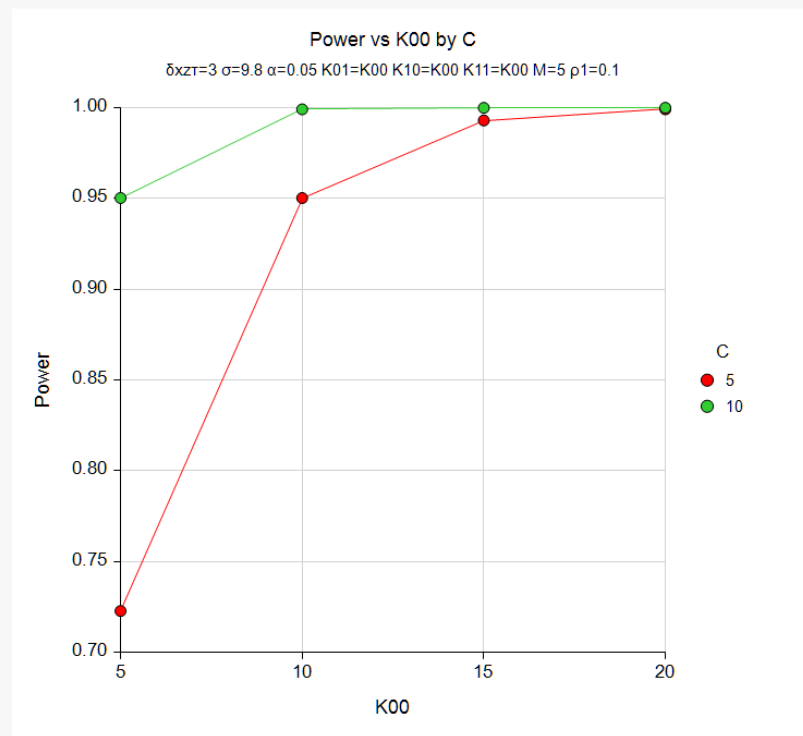_____

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References
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Ahn, C., Heo, M., and Zhang, S. 2015. Sample Size Calculations for Clustered and Longitudinal Outcomes in
    Clinical Research. CRC Press. New York.
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This report shows the power for each of the scenarios.

Plots Section

Plots
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Power vs K00 by C

$\delta x z \tau = 3$ $\sigma = 9.8$ $\alpha = 0.05$ K01=K00 K10=K00 K11=K00 M=5 $\rho 1 = 0.1$

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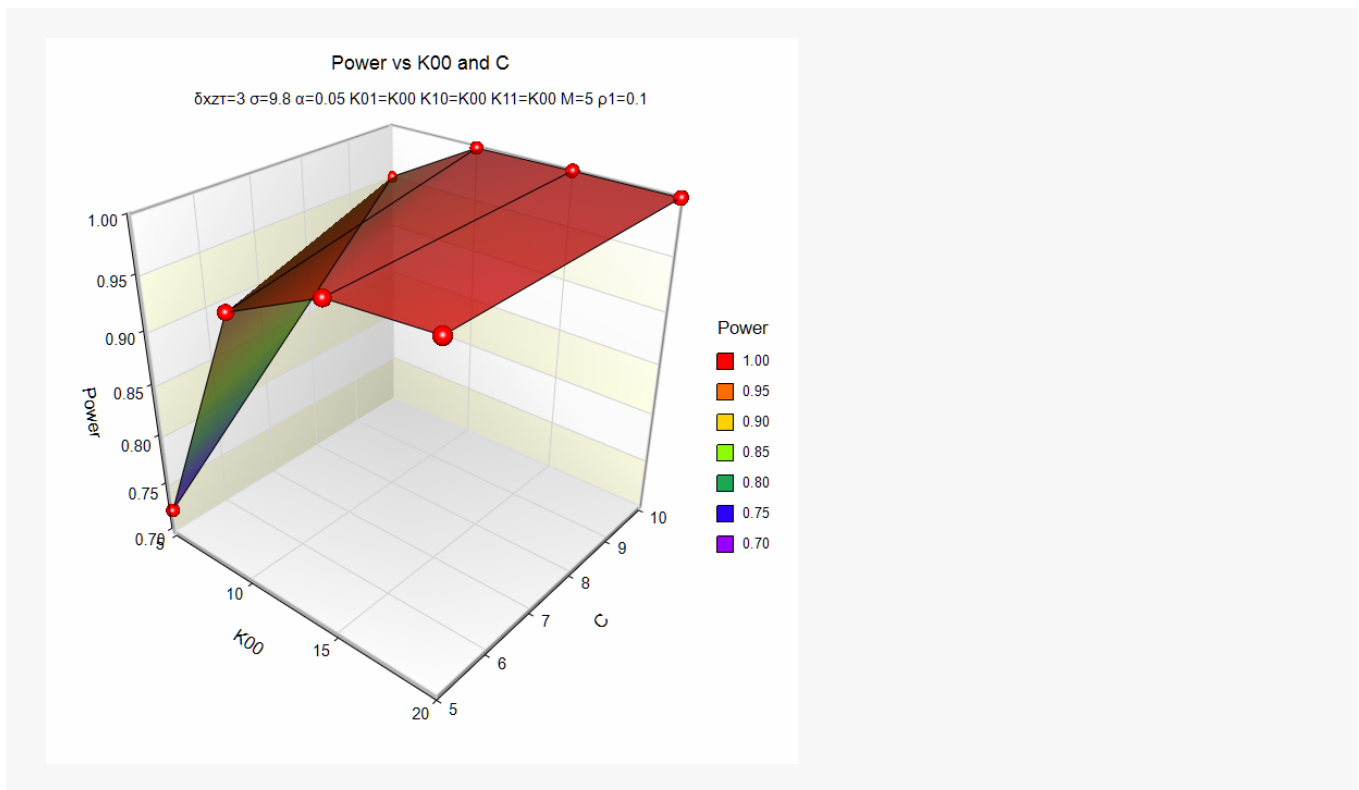
These plots show the power for the various parameter settings.

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Mixed Models Tests for Slope-Interaction in a 2×2 Factorial 3-Level Hierarchical Design with Fixed Slopes (Level-2 Rand.)

Example 2 – Calculating Sample Size (K00)

Continuing with the last example, suppose the researchers want to determine the value of K00 needed to achieve 90% power for both values of M.

Setup

If the procedure window is not already open, use the PASS Home window to open it. The parameters for this example are listed below and are stored in the **Example 2** settings file. To load these settings to the procedure window, click **Open Example Settings File** in the Help Center or File menu.

Design Tab

Solve For ....**K00 (Group 00 Count (X=0, Z=0))**
Power...**0.90**
Alpha..**0.05**
C (Level 3 Units) **5 10**
K01 (Group 01 Count (X=0, Z=1)) **K00**
K10 (Group 10 Count (X=1, Z=0)) **K00**
K11 (Group 11 Count (X=1, Z=1)) **K00**
M (Level 1 Units Per Level-2 Unit) **5**
δxzτ (Three-Way Interaction).........................**3**
σ (Standard Deviation)..................................**9.8**
ρ1 (Correlation Among Level-1 Units)............**0.1**

Output

Click the Calculate button to perform the calculations and generate the following output.

Numeric Results

Solve For: K00 (Group 00 Count (X=0, Z=0))

Power	Total Sample Size N	Number of Level-3 Units C	Number of Level-2 Units per Level-3 Unit					Number of Level-1 Units per Level-2 Unit M	Three-Way Interaction δxzτ	Standard Deviation σ	Correlation Among Level-1 Units ρ1	Alpha
			Group 00 K00	Group 01 K01	Group 10 K10	Group 11 K11	Total K					
0.9282	900	5	9	9	9	9	36	5	3	9.8	0.1	0.05
0.9503	1000	10	5	5	5	5	20	5	3	9.8	0.1	0.05

This report shows the required K00 count for each of the scenarios.

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Example 3 – Validation using Ahn, Heo, and Zhang (2015)

Ahn, Heo, and Zhang (2015) page 222 provide a table in which several scenarios are reported. We will validate this procedure by duplicating the results of the <u>first row</u>. Note that in using this table in this case, we reverse the second and third levels. Hence, instead of searching for C00, we will search for K00. Also, K in the table becomes C in our example.

The following parameter settings are used for the power analysis: Power = 0.80; δ_{XZT} = 0.3; σ = 4; ρ1 = 0.1; *M* = 5; C = 8; and *α* = 0.05. The values of K00, K01, K10, K11 are found to be 63 and the resulting power is 0.801.

Setup

If the procedure window is not already open, use the PASS Home window to open it. The parameters for this example are listed below and are stored in the **Example 3** settings file. To load these settings to the procedure window, click **Open Example Settings File** in the Help Center or File menu.

Design Tab

Solve For	**K00 (Group 00 Count (X=0, Z=0))**
Power...	**0.80**
Alpha...	**0.05**
C (Level 3 Units)	**8**
K01 (Group 01 Count (X=0, Z=1))	**K00**
K10 (Group 10 Count (X=1, Z=0))	**K00**
K11 (Group 11 Count (X=1, Z=1))	**K00**
M (Level 1 Units Per Level-2 Unit)	**5**
δxzτ (Three-Way Interaction).........................	**0.3**
σ (Standard Deviation).................................	**4**
ρ1 (Correlation Among Level-1 Units)............	**0.1**

Output

Click the Calculate button to perform the calculations and generate the following output.

Numeric Results

Solve For: K00 (Group 00 Count (X=0, Z=0))

Power	Total Sample Size N	Number of Level-3 Units C	Number of Level-2 Units per Level-3 Unit					Number of Level-1 Units per Level-2 Unit M	Three-Way Interaction δxzτ	Standard Deviation σ	Correlation Among Level-1 Units ρ1	Alpha
			Group 00 K00	Group 01 K01	Group 10 K10	Group 11 K11	Total K					
0.8013	10080	8	63	63	63	63	252	5	0.3	4	0.1	0.05

PASS also calculates K00 to be 63 and the power to be 0.8013.