

Chapter 388

Mixed Models Tests for Two Means in a 2-Level Hierarchical Design (Level-2 Randomization)

Introduction

This procedure calculates power and sample size for a two-level hierarchical mixed model in which clusters (groups, classes, hospitals, etc.) of subjects are measured one time (cross-sectional) on a continuous variable. The goal of the study is to compare the two group means.

In this design, the subjects are the level one units, and the clusters are the level two units. All subjects in a particular cluster (level two unit) receive one of two possible interventions. This intervention is selected at random. Note that a companion procedure power analyzes the other case in which the randomization occurs for the level one units (the subjects).

Technical Details

Our formulation comes from Ahn, Heo, and Zhang (2015), chapter 5, section 5.3.1, pages 151-154. The hierarchical mixed model that is adopted is

$$Y_{ij} = \beta_0 + \delta X_{ij} + u_i + e_{ij}$$

where

- Y_{ij} is the continuous response of the j^{th} subject in the i^{th} cluster.
- β_0 is the fixed intercept.
- δ is the treatment effect of interest. It is the difference between the two treatment means.
- X_{ij} is an indicator variable that is = 1 if cluster i is assigned to group 1 and 0 otherwise.
- u_i is a random effect (subject-specific intercept) term for the i^{th} cluster that is distributed as $N(0, \sigma_u^2)$.
- e_{ij} is a random effect for the j^{th} subject in the i^{th} cluster that is distributed as $N(0, \sigma_e^2)$.
- σ_u^2 is variance of the level two (cluster) random effects.
- σ_e^2 is variance of the level one (subject) random effects.
- σ^2 is the variance of Y , where $\sigma^2 = \sigma_u^2 + \sigma_e^2$.
- ρ is the intraclass correlation (ICC), where $\rho = \text{Corr}(Y_{ij}, Y_{ij'}) = (\sigma_u^2 / \sigma^2) = \sigma_u^2 / (\sigma_u^2 + \sigma_e^2)$.

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The test of significance of the X_{ij} term in the mixed model analysis is the test statistic of interest. It tests the difference of the two treatment means. Since these are asymptotic results, the specific type of mixed model is not stated.

Assume that $\delta = \mu_1 - \mu_2$ is to be tested using a z-test (large sample). The statistical hypotheses are $H_0: \delta = 0$ vs. $H_a: \delta \neq 0$. When $K_1 = K_2$, the test statistic, given by

$$z = \frac{(\bar{Y}_1 - \bar{Y}_2)\sqrt{K_1 M}}{\sigma\sqrt{2(1 + (M - 1)\rho)}}$$

where

$$\bar{Y}_g = \frac{1}{K_1 M} \sum_{i=1}^{K_1} \sum_{j=1}^M Y_{ij}, g = 1, 2$$

has an approximate normal distribution.

When $K_1 = K_2$, the power can be calculated using

$$Power = \Phi \left\{ \frac{\delta}{\sigma} \sqrt{K_1 M / [2(1 + (M - 1)\rho)]} - \Phi^{-1}(1 - \alpha/2) \right\}$$

Example 1 – Calculating Power

Suppose that a two-level hierarchical design is planned in which there will be only one measurement per subject and treatments will be applied to clusters (level-two units). The analysis will be a mixed model of continuous data. The following parameter settings are to be used for the power analysis: $\delta = 0.5$; $\sigma = 1$; $\rho = 0.01$; $M = 5$ or 10 ; $ICC = 0.01$; $\alpha = 0.05$; and $K1$ and $K2 = 5$ to 20 by 5 .

Setup

If the procedure window is not already open, use the PASS Home window to open it. The parameters for this example are listed below and are stored in the **Example 1** settings file. To load these settings to the procedure window, click **Open Example Settings File** in the Help Center or File menu.

Design Tab

| | |
|---|-------------------|
| Solve For | Power |
| Alpha..... | 0.05 |
| K1 (Number of Group 1 Clusters) | 5 10 15 20 |
| K2 (Number of Group 2 Clusters) | K1 |
| M (Number of Subjects Per Cluster) | 5 10 |
| δ (Mean Difference = $\mu_1 - \mu_2$) | 0.5 |
| σ (Standard Deviation)..... | 1 |
| ρ (Intracluster Correlation, ICC) | 0.01 |

Output

Click the Calculate button to perform the calculations and generate the following output.

Numeric Reports

Numeric Results for a Two-Level Hierarchical Mixed Model

Solve For: **Power**

Groups: 1 = Treatment, 2 = Control

Difference: $\delta = \mu_1 - \mu_2$

| Power | Number of Subjects | | | Number of Clusters | | | Number of Subjects per Cluster M | Mean Difference δ | Standard Deviation σ | Intraclass Correlation ρ | Alpha |
|--------|--------------------|---------------|------------|--------------------|---------------|------------|-------------------------------------|-----------------------------|--------------------------------|----------------------------------|-------|
| | Group 1 N1 | Group 2 N2 | Total N | Group 1 K1 | Group 2 K2 | Total K | | | | | |
| 0.4104 | 25 | 25 | 50 | 5 | 5 | 10 | 5 | 0.5 | 1 | 0.01 | 0.05 |
| 0.6681 | 50 | 50 | 100 | 5 | 5 | 10 | 10 | 0.5 | 1 | 0.01 | 0.05 |
| 0.6885 | 50 | 50 | 100 | 10 | 10 | 20 | 5 | 0.5 | 1 | 0.01 | 0.05 |
| 0.9231 | 100 | 100 | 200 | 10 | 10 | 20 | 10 | 0.5 | 1 | 0.01 | 0.05 |
| 0.8514 | 75 | 75 | 150 | 15 | 15 | 30 | 5 | 0.5 | 1 | 0.01 | 0.05 |
| 0.9856 | 150 | 150 | 300 | 15 | 15 | 30 | 10 | 0.5 | 1 | 0.01 | 0.05 |
| 0.9341 | 100 | 100 | 200 | 20 | 20 | 40 | 5 | 0.5 | 1 | 0.01 | 0.05 |
| 0.9977 | 200 | 200 | 400 | 20 | 20 | 40 | 10 | 0.5 | 1 | 0.01 | 0.05 |

Power The probability of rejecting a false null hypothesis when the alternative hypothesis is true.

N1, N2, and N The number of subjects in groups 1, 2, and both, respectively.

K1, K2, and K The number of clusters in groups 1, 2, and both, respectively.

M The average number of items (subjects) per cluster.

δ The mean difference in the response at which the power is calculated. $\delta = \mu_1 - \mu_2$.

σ The standard deviation of the subject responses.

ρ The intraclass correlation (ICC).

Alpha The probability of rejecting a true null hypothesis.

Summary Statements

A 2-group 2-level hierarchical design will have level-1 units (e.g., students, subjects, or patients) in level-2 units (e.g., classes, clinics, hospitals, or clusters) with random assignment of level-2 units to each of the 2 groups (level-2 randomization). This design will be used to test the difference of two means ($\mu_1 - \mu_2$), using the appropriate term of the hierarchical mixed-effects model, with a Type I error rate (α) of 0.05. The standard deviation of level-1 units (subjects) is assumed to be 1. The correlation of level-1 units within a level-2 unit (intraclass correlation) is assumed to be 0.01. To detect a mean difference ($\mu_1 - \mu_2$) of 0.5, with 5 level-2 units (clusters) in Group 1 and 5 level-2 units (clusters) in Group 2, and with 5 level-1 units (subjects) in each level-2 unit (for a grand total of 50 level-1 units), the power is 0.4104.

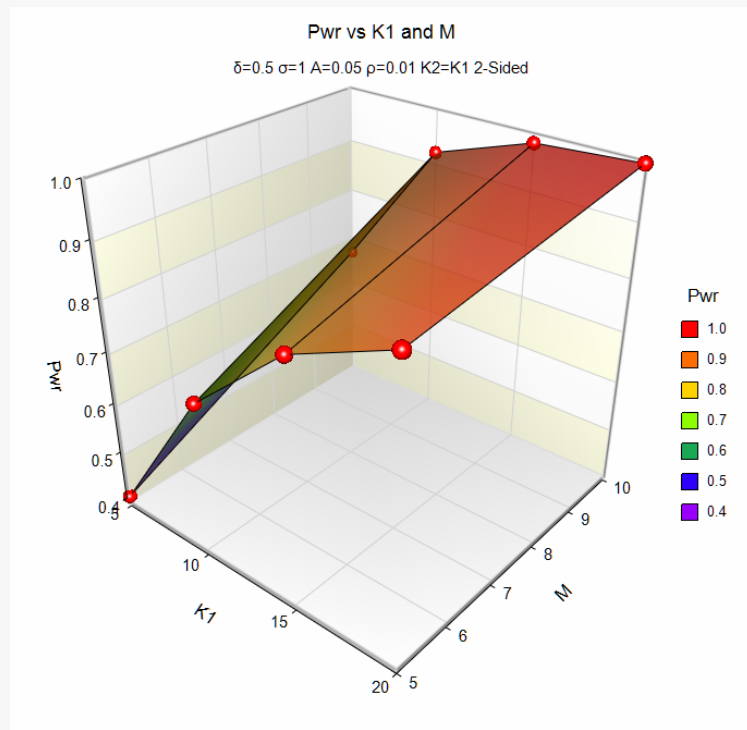
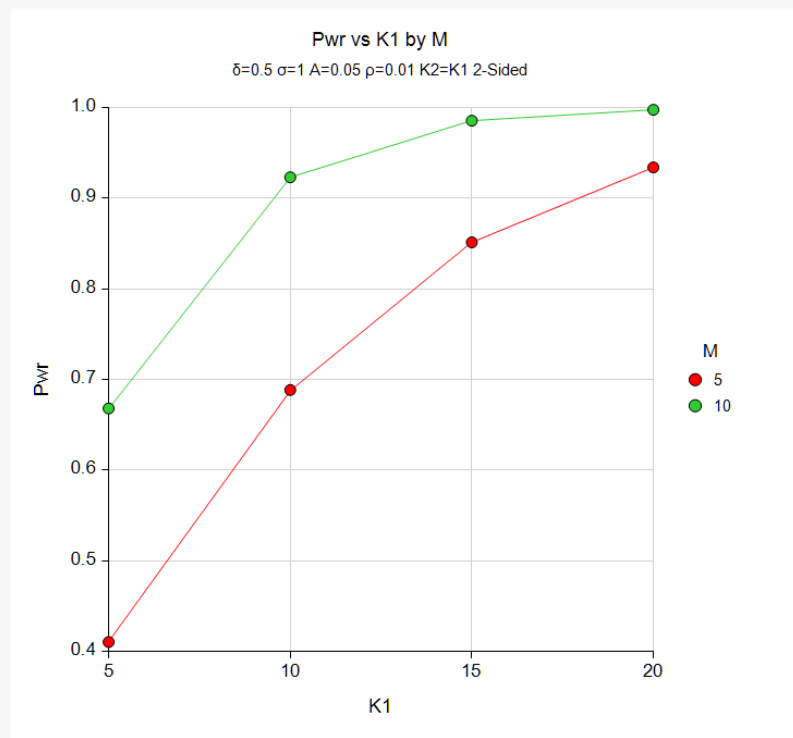
References

Ahn, C., Heo, M., and Zhang, S. 2015. Sample Size Calculations for Clustered and Longitudinal Outcomes in Clinical Research. CRC Press. New York.

This report shows the power for each of the scenarios.

Plots Section

Plots



These plots show the power versus the cluster count for the two cluster size values.

Example 2 – Calculating Sample Size (Number of Clusters)

Continuing with the last example, suppose the researchers want to determine the number of clusters needed to achieve 90% power for both values of M.

Setup

If the procedure window is not already open, use the PASS Home window to open it. The parameters for this example are listed below and are stored in the **Example 2** settings file. To load these settings to the procedure window, click **Open Example Settings File** in the Help Center or File menu.

Design Tab

Solve For **K1 (Number of Clusters)**
 Power..... **0.90**
 Alpha..... **0.05**
 K2 (Number of Group 2 Clusters) **K1**
 M (Number of Subjects Per Cluster) **5 10**
 δ (Mean Difference = $\mu_1 - \mu_2$) **0.5**
 σ (Standard Deviation)..... **1**
 ρ (Intraclass Correlation, ICC)..... **0.01**

Output

Click the Calculate button to perform the calculations and generate the following output.

Numeric Results for a Two-Level Hierarchical Mixed Model

Solve For: **K1 (Number of Clusters)**
 Groups: 1 = Treatment, 2 = Control
 Difference: $\delta = \mu_1 - \mu_2$

| Power | Number of Subjects | | | Number of Clusters | | | Number of Subjects per Cluster M | Mean Difference δ | Standard Deviation σ | Intraclass Correlation ρ | Alpha |
|--------|--------------------|---------------|------------|--------------------|---------------|------------|-------------------------------------|-----------------------------|--------------------------------|----------------------------------|-------|
| | Group 1 N1 | Group 2 N2 | Total N | Group 1 K1 | Group 2 K2 | Total K | | | | | |
| 0.9081 | 90 | 90 | 180 | 18 | 18 | 36 | 5 | 0.5 | 1 | 0.01 | 0.05 |
| 0.9231 | 100 | 100 | 200 | 10 | 10 | 20 | 10 | 0.5 | 1 | 0.01 | 0.05 |

This report shows the power for each of the scenarios.

Example 3 – Calculating Sample Size (Number of Subjects per Cluster)

Continuing with the last example, suppose the researchers want to determine the number of subjects needed to achieve 90% power for all values of K1.

Note that this calculation is sensitive to the value of ICC in that if ICC is too large, no value of M is possible.

Setup

If the procedure window is not already open, use the PASS Home window to open it. The parameters for this example are listed below and are stored in the **Example 3** settings file. To load these settings to the procedure window, click **Open Example Settings File** in the Help Center or File menu.

Design Tab

Solve For **M (Number of Subjects Per Cluster)**
 Power..... **0.90**
 Alpha..... **0.05**
 K1 (Number of Group 1 Clusters) **5 10 15 20**
 K2 (Number of Group 2 Clusters) **K1**
 δ (Mean Difference = $\mu_1 - \mu_2$) **0.5**
 σ (Standard Deviation)..... **1**
 ρ (Intracluster Correlation, ICC)..... **0.01**

Output

Click the Calculate button to perform the calculations and generate the following output.

Numeric Results for a Two-Level Hierarchical Mixed Model

Solve For: **M (Number of Subjects Per Cluster)**
 Groups: 1 = Treatment, 2 = Control
 Difference: $\delta = \mu_1 - \mu_2$

| Power | Number of Subjects | | | Number of Clusters | | | Number of Subjects per Cluster M | Mean Difference δ | Standard Deviation σ | Intracluster Correlation ρ | Alpha |
|--------|--------------------|---------------|------------|--------------------|---------------|------------|-------------------------------------|-----------------------------|--------------------------------|------------------------------------|-------|
| | Group 1 N1 | Group 2 N2 | Total N | Group 1 K1 | Group 2 K2 | Total K | | | | | |
| 0.9110 | 105 | 105 | 210 | 5 | 5 | 10 | 21 | 0.5 | 1 | 0.01 | 0.05 |
| 0.9231 | 100 | 100 | 200 | 10 | 10 | 20 | 10 | 0.5 | 1 | 0.01 | 0.05 |
| 0.9055 | 90 | 90 | 180 | 15 | 15 | 30 | 6 | 0.5 | 1 | 0.01 | 0.05 |
| 0.9341 | 100 | 100 | 200 | 20 | 20 | 40 | 5 | 0.5 | 1 | 0.01 | 0.05 |

This report shows the values of M needed for each scenario.

Example 4 – Validation using Ahn, Heo, and Zhang (2015)

Ahn, Heo, and Zhang (2015) page 154 provide a table in which several scenarios are reported. We will validate this procedure by duplicating two of the table entries. The following parameter settings are used for the power analysis: Power = 0.80; $\delta = 0.4$; $\sigma = 1$; $\rho = 0.01$; $M = 10$ or 20 ; $ICC = 0.1$; and $\alpha = 0.05$. The reported values of $K1$ and $K2$ are 19 and 15.

Setup

If the procedure window is not already open, use the PASS Home window to open it. The parameters for this example are listed below and are stored in the **Example 4** settings file. To load these settings to the procedure window, click **Open Example Settings File** in the Help Center or File menu.

Design Tab

Solve For **K1 (Number of Clusters)**
 Power..... **0.80**
 Alpha..... **0.05**
 K2 (Number of Group 2 Clusters) **K1**
 M (Number of Subjects Per Cluster) **10 20**
 δ (Mean Difference = $\mu_1 - \mu_2$) **0.4**
 σ (Standard Deviation)..... **1**
 ρ (Intracluster Correlation, ICC) **0.1**

Output

Click the Calculate button to perform the calculations and generate the following output.

Numeric Results for a Two-Level Hierarchical Mixed Model

Solve For: **K1 (Number of Clusters)**
 Groups: 1 = Treatment, 2 = Control
 Difference: $\delta = \mu_1 - \mu_2$

| Power | Number of Subjects | | | Number of Clusters | | | Number of Subjects per Cluster M | Mean Difference δ | Standard Deviation σ | Intracluster Correlation ρ | Alpha |
|--------|--------------------|---------------|------------|--------------------|---------------|------------|-------------------------------------|-----------------------------|--------------------------------|------------------------------------|-------|
| | Group 1 N1 | Group 2 N2 | Total N | Group 1 K1 | Group 2 K2 | Total K | | | | | |
| 0.8074 | 190 | 190 | 380 | 19 | 19 | 38 | 10 | 0.4 | 1 | 0.1 | 0.05 |
| 0.8204 | 300 | 300 | 600 | 15 | 15 | 30 | 20 | 0.4 | 1 | 0.1 | 0.05 |

PASS calculates the same values of $K1$: 19 and 15.