Chapter 385

Mixed Models Tests for Two Proportions in a 2-Level Hierarchical Design (Level-1 Randomization)

Introduction

This procedure calculates power and sample size for a two-level hierarchical mixed-effects logistic regression in which clusters (groups, classes, hospitals, etc.) of subjects are measured one time (cross-sectional) on a binary variable. The goal of the study is to compare the two group proportions.

In this design, the subjects are the level one units, and the clusters are the level two units. Each subject in a particular cluster (level two unit) is randomized individually to one of two possible interventions. Note that a companion procedure power analyzes the other case in which the randomization occurs for the level two units (the clusters).

Technical Details

Our formulation comes from Ahn, Heo, and Zhang (2015), chapter 5, section 5.7.2, pages 179-181. The hierarchical mixed-effects logistic regression model that is adopted is

$$\log\left(\frac{p_{ij}}{1-p_{ij}}\right) = \beta_0 + \delta X_{ij} + u_i$$

where

Y_{ij}	is the binary response of the j^{th} subject in the i^{th} cluster.
p_{ij}	is $\mathrm{E}(Y_{ij}\|X_{ij})$. Assume $[p_{ij}\|(X_{ij}=0)] = p_2$ and $[p_{ij}\|(X_{ij}=1)] = p_1$
β_0	is the fixed intercept.
δ	is the treatment effect of interest.
X_{ij}	is an indicator variable that is set to 1 if subject *j* in cluster *i* receives intervention 1 and 0 otherwise.
u_i	is a random effect (subject-specific intercept) term for the i^{th} cluster that is distributed as $N(0,\sigma_u^2)$.
σ_u^2	is variance of the level two (cluster) random effects.
ρ	is the intraclass correlation (ICC), where $\rho = Corr(Y_{ij}, Y_{ij'}) = (\sigma_u^2/(\sigma_u^2+\pi^2/3))$.

The test of significance of the δ coefficient in the logistic regression analysis is the test statistic of interest.

The power can be calculated using

$$Power = \Phi\left\{\frac{|p_1 - p_2|\sqrt{KM_2/[1-\rho]} - \Phi^{-1}(1-\alpha/2)\sqrt{\left(1+\frac{1}{\lambda}\right)\bar{p}(1-\bar{p})}}{\sqrt{p_2(1-p_2) + p_1(1-p_1)/\lambda}}\right\}$$

where

K	is the number of clusters in the study.
$M1$	is the average number of subjects per cluster in group 1.
$M2$	is the average number of subjects per cluster in group 2.
λ	is M_1/M_2.
$\bar{p}$	is $\left(\frac{p_1+\lambda p_2}{1+\lambda}\right)$.

This power function is used in a binary search algorithm to determine p_1, K_1, or M.

Example 1 – Calculating Power

Suppose that a two-level hierarchical design is planned in which there will be only one measurement per subject and treatments will be applied to subjects (level-one units). The analysis will be a mixed-effect logistic regression. The following parameter settings are to be used for the power analysis: $P1$ = 0.6; $P2$ = 0.5; ρ = 0.05; $M1$= 15 or 25; $M2$ = $M1$; α = 0.05; and K = 10, 20, 30, 40.

Setup

If the procedure window is not already open, use the PASS Home window to open it. The parameters for this example are listed below and are stored in the **Example 1** settings file. To load these settings to the procedure window, click **Open Example Settings File** in the Help Center or File menu.

Design Tab

Solve For .... **Power**
Alpha.. **0.05**
K (Number of Clusters) **10 20 30 40**
M1 (Group 1 Subjects Per Cluster) **15 25**
M2 (Group 2 Subjects Per Cluster) **M1**
P1 Input Type .... **Proportions**
P1 (Group 1 Proportion|H1) **0.6**
P2 (Group 2 Proportion)................................. **0.5**
ρ (Intracluster Correlation, ICC) **0.05**

Output

Click the Calculate button to perform the calculations and generate the following output.

Numeric Reports

Numeric Results

Solve For: Power
Groups: 1 = Treatment, 2 = Control
Hypotheses: H0: P1 = P2 vs. H1: P1 ≠ P2

			Number of Subjects per Cluster		Proportion				
Power	Number of Subjects N	Number of Clusters K	Group 1 M1	Group 2 M2	Group 1 P1	Group 2 P2	Difference P1 - P2	Intracluster Correlation ρ	Alpha
0.4306	300	10	15	15	0.6	0.5	0.1	0.05	0.05
0.6359	500	10	25	25	0.6	0.5	0.1	0.05	0.05
0.7152	600	20	15	15	0.6	0.5	0.1	0.05	0.05
0.9045	1000	20	25	25	0.6	0.5	0.1	0.05	0.05
0.8727	900	30	15	15	0.6	0.5	0.1	0.05	0.05
0.9795	1500	30	25	25	0.6	0.5	0.1	0.05	0.05
0.9474	1200	40	15	15	0.6	0.5	0.1	0.05	0.05
0.9962	2000	40	25	25	0.6	0.5	0.1	0.05	0.05

Power The probability of rejecting a false null hypothesis when the alternative hypothesis is true.
N The total number of subjects in the study.
K The number of clusters.
M1 The average number of items (subjects) per cluster in group one, which is designated as the treatment group.
M2 The average number of items (subjects) per cluster in group two, which is designated as the control group.
P1 The proportion for group 1 (treatment) assumed by the alternative hypothesis.
P2 The proportion for group 2 (control, standard, reference, baseline).
P1 - P2 The difference in the group proportions assumed by the alternative hypothesis.
ρ The intracluster correlation.
Alpha The probability of rejecting a true null hypothesis.

Summary Statements

A 2-group 2-level hierarchical design will have level-1 units (e.g., students, subjects, or patients) in level-2 units (e.g., classes, clinics, hospitals, or clusters) with random assignment of level-1 units to each of the 2 groups (level-1 randomization). This design will be used to test the difference between two proportions, using the appropriate term of the hierarchical mixed-effects logistic regression model, with a Type I error rate (α) of 0.05. The correlation of level-1 units within a level-2 unit (intracluster correlation) is assumed to be 0.05. To detect a proportion difference (P1 - P2) of 0.1 (with P1 = 0.6 and P2 = 0.5), with 10 level-2 units, and within each level-2 unit, 15 level-1 units in Group 1 and 15 level-1 units in Group 2 (for a grand total of 300 level-1 units), the power is 0.4306.

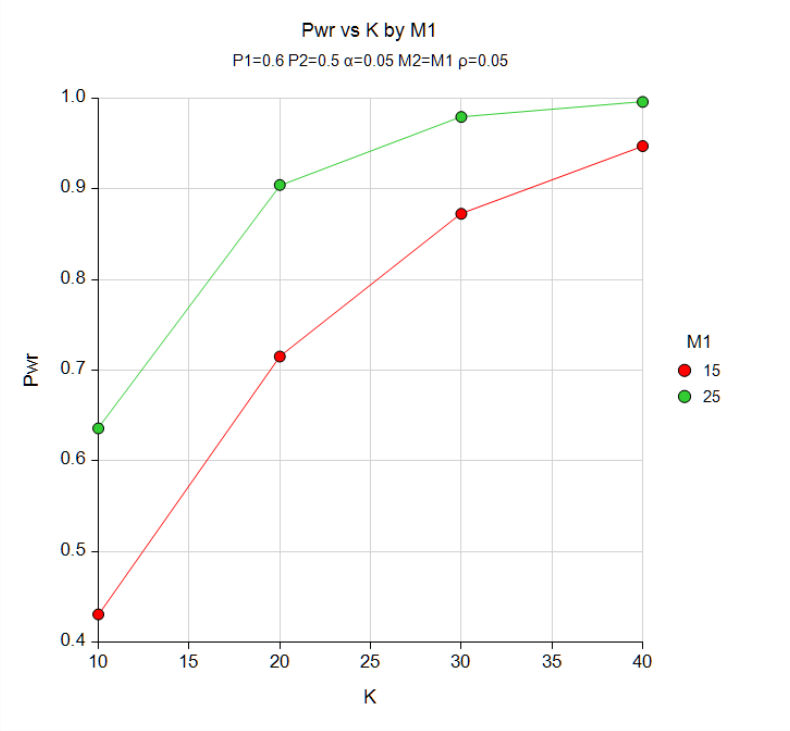
References

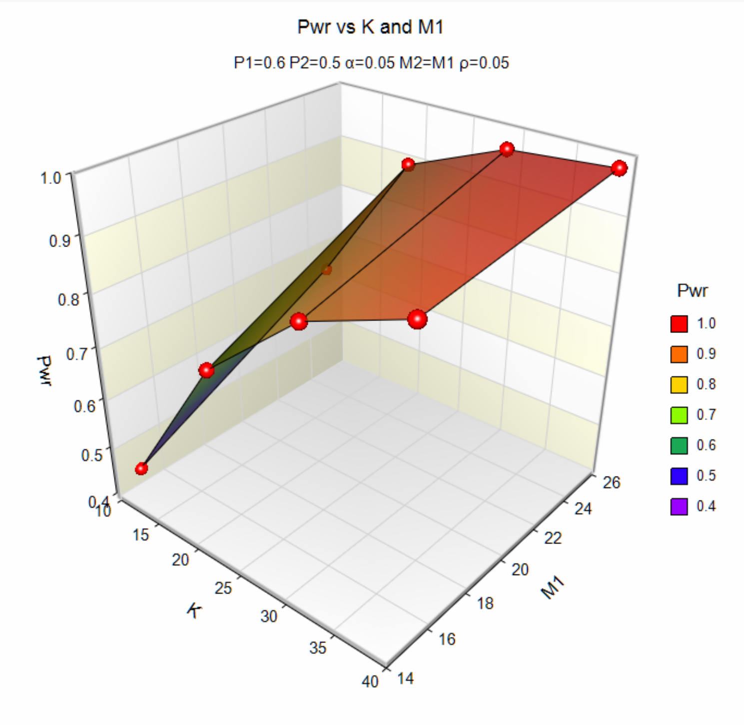
Ahn, C., Heo, M., and Zhang, S. 2015. Sample Size Calculations for Clustered and Longitudinal Outcomes in Clinical Research. CRC Press. New York.

This report shows the power for each of the scenarios.

Plots Section

Plots





These plots show the power versus the cluster count for the two cluster size values.

Example 2 – Calculating Sample Size (Number of Clusters)

Continuing with the last example, suppose the researchers want to determine the number of clusters needed to achieve 90% power for both values of M1 and M2.

Setup

If the procedure window is not already open, use the PASS Home window to open it. The parameters for this example are listed below and are stored in the **Example 2** settings file. To load these settings to the procedure window, click **Open Example Settings File** in the Help Center or File menu.

Design Tab

Solve For	**K (Number of Clusters)**
Power	**0.90**
Alpha	**0.05**
M1 (Group 1 Subjects Per Cluster)	**15 25**
M2 (Group 2 Subjects Per Cluster)	**M1**
P1 Input Type	**Proportions**
P1 (Group 1 Proportion|H1)	**0.6**
P2 (Group 2 Proportion)	**0.5**
ρ (Intracluster Correlation, ICC)	**0.05**

Output

Click the Calculate button to perform the calculations and generate the following output.

Numeric Results

Solve For: K (Number of Clusters)
Groups: 1 = Treatment, 2 = Control
Hypotheses: H0: P1 = P2 vs. H1: P1 ≠ P2

			Number of Subjects per Cluster		Proportion				
Power	Number of Subjects N	Number of Clusters K	Group 1 M1	Group 2 M2	Group 1 P1	Group 2 P2	Difference P1 - P2	Intracluster Correlation ρ	Alpha
0.9016	990	33	15	15	0.6	0.5	0.1	0.05	0.05
0.9045	1000	20	25	25	0.6	0.5	0.1	0.05	0.05

This report shows the necessary value of K for each scenario.

Example 3 – Calculating Sample Size (Number of Subjects per Cluster)

Continuing with the last example, suppose the researchers want to determine the number of subjects per cluster needed to achieve 90% power for all values of K. They want to consider what will happen if the M2 is twice as large as M1.

Setup

If the procedure window is not already open, use the PASS Home window to open it. The parameters for this example are listed below and are stored in the **Example 3** settings file. To load these settings to the procedure window, click **Open Example Settings File** in the Help Center or File menu.

Design Tab

Solve For .... **M1 (Group 1 Subjects Per Cluster)**
Power.. **0.90**
Alpha... **0.05**
K (Number of Clusters) **10 20 30 40**
M2 (Group 2 Subjects Per Cluster)................ **2M1**
P1 Input Type .... **Proportions**
P1 (Group 1 Proportion|H1) **0.6**
P2 (Group 2 Proportion)................................ **0.5**
ρ (Intracluster Correlation, ICC)..................... **0.05**

Output

Click the Calculate button to perform the calculations and generate the following output.

Numeric Results

Solve For: M1 (Group 1 Subjects Per Cluster)
Groups: 1 = Treatment, 2 = Control
Hypotheses: H0: P1 = P2 vs. H1: P1 ≠ P2

			Number of Subjects per Cluster		Proportion				
Power	Number of Subjects N	Number of Clusters K	Group 1 M1	Group 2 M2	Group 1 P1	Group 2 P2	Difference P1 - P2	Intracluster Correlation ρ	Alpha
0.9013	1110	10	37	74	0.6	0.5	0.1	0.05	0.05
0.9086	1140	20	19	38	0.6	0.5	0.1	0.05	0.05
0.9155	1170	30	13	26	0.6	0.5	0.1	0.05	0.05
0.9219	1200	40	10	20	0.6	0.5	0.1	0.05	0.05

This report shows the necessary values of M1 and M2 for each scenario.

Example 4 – Validation using Ahn, Heo, and Zhang (2015)

Ahn, Heo, and Zhang (2015) page 181 provide a table in which several scenarios are reported. We will validate this procedure by duplicating the first row.

The following parameter settings were used: Power = 0.80; *P1* = 0.6; *P2 = 0.4*; *ρ* = 0.1; *M*1 = *M*2 = 10; *ICC* = 0.1; and *α* = 0.05. The value of *K* is 10. The realized power value is 0.851.

Setup

If the procedure window is not already open, use the PASS Home window to open it. The parameters for this example are listed below and are stored in the **Example 4** settings file. To load these settings to the procedure window, click **Open Example Settings File** in the Help Center or File menu.

Design Tab

Solve For .... **Power**
Alpha... **0.05**
K (Number of Clusters) **10**
M1 (Group 1 Subjects Per Cluster)............... **10**
M2 (Group 2 Subjects Per Cluster)............... **M1**
P1 Input Type .... **Proportions**
P1 (Group 1 Proportion|H1) **0.6**
P2 (Group 2 Proportion)................................ **0.4**
ρ (Intracluster Correlation, ICC) **0.1**

Output

Click the Calculate button to perform the calculations and generate the following output.

Numeric Results

Solve For: Power
Groups: 1 = Treatment, 2 = Control
Hypotheses: H0: P1 = P2 vs. H1: P1 ≠ P2

			Number of Subjects per Cluster		Proportion				
Power	Number of Subjects N	Number of Clusters K	Group 1 M1	Group 2 M2	Group 1 P1	Group 2 P2	Difference P1 - P2	Intracluster Correlation ρ	Alpha
0.8514	200	10	10	10	0.6	0.4	0.2	0.1	0.05

PASS calculates the same power: 0.8514.

(We noticed that if you search for the smallest value of K, the power condition is met with K = 9.)