Chapter 378

Mixed Models Tests for Two Proportions in a 3-Level Hierarchical Design (Level-2 Randomization)

Introduction

This procedure calculates power and sample size for a three-level hierarchical mixed-effects logistic regression model which is randomized at the **second** level. The goal of the study is to compare two group proportions. The study may be cross-sectional or longitudinal.

In a *cross-sectional* version of this design, students (first level units) are nested in classrooms (second level units) which are nested in schools (third level units). Each school is randomized into one of two intervention groups; e.g., treatment and control.

In a *longitudinal* version of this design, repeated measurements (first level units) are nested in patients (second level units) which are nested in clinics (third level units). Each clinic is randomized into one of two intervention groups; e.g., treatment and control.

Note that companion procedures analyze the other cases in which the randomization occurs at the first, or third, level.

Technical Details

Our formulation comes from Ahn, Heo, and Zhang (2015), chapter 6, section 6.7.2, pages 225-228. The hierarchical mixed model used for the analysis is

$$\log\left(\frac{p_{ijk}}{1 - p_{ijk}}\right) = \beta_0 + \delta X_{ijk} + u_i + u_{j(i)}$$

where

Y_{ijk}	is the binary response of the kth level-1 unit of the jth level-2 unit of the ith level-3 unit.
p_{ijk}	is an expected value defined by $p_{ijk} = E(Y_{ijk}|X_{ijk})$. Assume $\left[p_{ijk}|(X_{ijk} = 0)\right] = p_2$ and $\left[p_{ijk}|(X_{ijk} = 1)\right] = p_1$
β_0	is the fixed intercept.
δ	is the treatment effect of interest. It is the difference between the two group proportions.
X_{ijk}	is an indicator variable that is 1 if i^{th} unit is in group 1 and 0 if it is in group 2.
u_i	is the level-3 random intercept effect for the ith level-3 unit. It is distributed as $N(0, \sigma_3^2)$.
$u_{j(i)}$	is the level-2 random intercept effect for the $j(i)^{th}$ level-2 unit. It is distributed as $N(0, \sigma_2^2)$.
ρ_1	is the correlation among level-1 units which are in a particular level-2 unit. For fixed models like this, $\rho_1 = Corr(Y_{ijk}, Y_{ijk'}) = (\sigma_2^2 + \sigma_3^2)/\sigma^2$.
ρ_2	is the correlation among level-2 units which are in a particular level-3 unit. For fixed models like this, $\rho_2 = Corr(Y_{ijk}, Y_{ij'k'}) = (\sigma_3^2)/\sigma^2$.
C	is the number of level-3 units.
K_1	is the number of level-2 units assigned to group 1.
K_2	is the number of level-2 units assigned to group 2.
M	is the number of level-1 units per level-2 unit.

The test of significance of the X_{ijk} term in the logistic model is the test statistic of interest. It tests the difference of the two group proportions.

Assume that $\delta = p_1 - p_2$ is to be tested using a z-test (large sample). The statistical hypotheses are $H_0: \delta = 0$ vs. $H_a: \delta \neq 0$. The test statistic is the regression coefficient of the X_{ijk} term in a mixed model.

The power can be calculated using

$$Power = \frac{\Phi\{|p_1 - p_2|\sqrt{K_2 CM/f_2} - \Phi(1 - \alpha/2)\sqrt{(1 + 1/\lambda)\bar{p}(1 - \bar{p})}\}}{\sqrt{p_2(1 - p_2) + p_1(1 - p_1)/\lambda}}$$

where $\lambda = K_1/K_2$, $\bar{p} = (K_1 p_1 + K_2 p_2)/(K_1 + K_2)$, and $f_2 = 1 + (M - 1)\rho_1 - M\rho_2$.

Procedure Options

This section describes the options that are specific to this procedure. These are located on the Design tab. For more information about the options of other tabs, go to the Procedure Window chapter.

Design Tab

The Design tab contains most of the parameters and options that you will be concerned with.

Solve For

Solve For

This option specifies the parameter to be solved for from the other parameters. The parameters that may be selected are *Effect Size*, *Power*, *C*, *K1*, and *M*.

Under most situations, you will select either *Power* to calculate power or *K*1 to calculate the number of level-2 unit2. Occasionally, you may want to fix *K*1 and *K*2 and calculate *C* or *M*.

Note that the value selected here always appears as the vertical axis on the charts.

The program is set up to calculate power directly. To find appropriate values of the other parameters, a binary search is made using an iterative procedure until an appropriate value is found. This search considers only integer values of *C*, *K1*, or *M*.

Power and Alpha

Power

This option specifies one or more values for power. Power is the probability of rejecting a false null hypothesis, and is equal to one minus Beta. Beta is the probability of a type-II error, which occurs when a false null hypothesis is not rejected.

Values must be between zero and one. Historically, the value of 0.80 (Beta = 0.20) was used for power. Now, 0.90 (Beta = 0.10) is also commonly used.

A single value may be entered or a range of values such as *0.8 to 0.95 by 0.05* may be entered.

If your only interest is in determining the appropriate sample size for a confidence interval, set power to 0.5.

Alpha

This option specifies one or more values for the probability of a type-I error. A type-I error occurs when a true null hypothesis is rejected.

Values must be between zero and one. Usually, the value of 0.05 is used for alpha and this has become a standard. This means that about one test in twenty will falsely reject the null hypothesis. You should pick a value for alpha that represents the risk of a type-I error you are willing to take in your experimental situation.

You may enter a range of values such as *0.01 0.05 0.10* or *0.01 to 0.10 by 0.01*.

Sample Size

C (Level 3 Units)

This is the number of level 3 units in group 1 (treatment group). The sample size for this group is equal to Ceiling(C1 * K * M).

Examples of level 3 units are schools, clinics, hospitals, or cities.

A cross-sectional study might include schools as the level 3 units, classrooms as the level 2 units, and students are the level 1 units.

A longitudinal study might include clinics as the level 3 units, patients as the level 2 units, and repeated measurements are the level 1 units.

This value must be a positive integer.

You can enter a list of values such as "10 20 30". A separate analysis will be run for each element in the list.

K1 (Level 2 Units Assigned to Group 1)

Enter the average number of level 2 units per level 3 unit that are assigned to group 1 (treatment group). The sample size for this group is equal to Ceiling(C * K1 * M).

Examples of level 2 units are classrooms or patients. A *cross-sectional* study might include schools as the level 3 units, classrooms as the level 2 units, and students are the level 1 units. A *longitudinal* study might include clinics as the level 3 units, patients as the level 2 units, and repeated measurements are the level 1 units.

This value must be a positive number. Decimal values such as 4.5 can be entered.

You can enter a list of values such as "10 20 30". A separate analysis will be run for each element in the list.

K2 (Level 2 Units Assigned to Group 2)

This is the number of level 2 units per level 3 in group 2 (control group). The sample size for this group is equal to Ceiling(C * K2 * M).

Examples of l2 units are classrooms or patients. A cross-sectional study might include schools as the level 3 units, classrooms as the level 2 units, and students are the level 1 units. A longitudinal study might include clinics as the level 3 units, patients as the level 2 units, and repeated measurements are the level 1 units.

This value must be a positive number. Decimal values such as 5.7 can be entered.

Using Multiples of K1

If you simply want a multiple of the value for group 1 (K1), you would enter the multiple followed by "K1", with no blanks. If you want to use K1 directly, you do not have to enter the leading "1". For example, all of the following are valid entries: K1, 2K1, 0.5K1, and 1.5K1.

You can use a list of values such as "10 20 30" or "K1 2K1 3K1".

M (Level 1 Units Per Level 2 Unit)

This is the average number of level 1 units in each level 2 unit. Examples of level 1 units are repeated measurements or students.

A *cross-sectional* study might include schools as the level 3 units, classrooms as the level 2 units, and students are the level 1 units. A *longitudinal* study might include clinics as the level 3 units, patients as the level 2 units, and repeated measurements are the level 1 units.

This value must be a positive number that is at least 1.

It can be a decimal (fractional) number such as '2.7'. The resulting total number of level 1 units will be rounded up to the next largest integer.

You can use a list of values such as "5 10 15". A separate analysis will be run for each element in the list.

Effect Size

P1 Input Type

Indicate what type of values to enter to specify the effect size: P1 - P2. Regardless of the entry type chosen, the test statistics used in the power and sample size calculations are the same. The value of P1 is calculated from the value entered. This option is simply given for convenience in specifying the effect size.

The choices are

- **Proportions**

  Enter P1 (Group 1 Proportion|H1) and P2 (Group 2 Proportion).

- **Differences**

  Enter D1 (Difference|H1 = P1-P2) and P2 (Group 2 Proportion).

- **Ratios**

  Enter R1 (Ratio|H1 = P1/P2) and P2 (Group 2 Proportion).

- **Odds Ratios**

  Enter OR1 (Odds Ratio|H1 = Odds1/Odds2) and P2 (Group 2 Proportion).

P1 (Group 1 Proportion |H1)

Enter a value for the proportion in group 1 (the experimental or treatment group) under the alternative hypothesis, H1. The power calculations assume that this is the actual value of the proportion.

You can enter a single value such as 0.1 or a series of values such as 0.1 0.2 0.3 or 0.1 to 0.9 by 0.1.

Note that values must be between zero and one and cannot be equal to P2.

D1 (Difference|H1 = P1 – P2)

This option specifies the difference between the two proportions under the alternative hypothesis, H1. This difference is used with P2 to calculate the value of P1 using the formula: P1=Diff+P2.

The power calculations assume that P1 is the actual value of the proportion in group 1 (experimental or treatment group).

You can enter a single value such as 0.05 or a series of values such as 0.03 0.05 0.10 or 0.01 to 0.05 by 0.01.

Differences must be between -1 and 1. They cannot take on the values -1, 0, or 1.

R1 (Ratio|H1 = P1/P2)

This option specifies the ratio between the two proportions P1 and P2. This ratio is used with P2 to calculate the value of P1 using the formula: P1=(Ratio) x (P2).

The power calculations assume that P1 is the actual value of the proportion in group 1 (experimental or treatment group).

You can enter a single value such as 0.5 or a series of values such as 0.5 0.6 0.7 0.8 or 1.25 to 2.0 by 0.25.

Ratios must be greater than zero. They cannot take on the value of one.

OR1 (Ratio|H1 = Odds1/Odds2)

This option specifies the ratio of P1 and P2, where P1 is the proportion in group 1 under the alternative hypothesis. This ratio is used with P2 to calculate the value of P1 using the formula: $P1 = R1 \times P2$. The power calculations assume that P1 is the actual value of the proportion in group 1 (experimental or treatment group).

You may enter a range of values such as *0.5 0.6 0.7 0.8* or *1.25 to 2.0 by 0.25*. Ratios must be greater than zero. They cannot take on the value of one.

P2 (Group 2 Proportion)

Enter a value for the proportion in group 2 (the control, baseline, standard, or reference group).

Values must be between 0 and 1.

You can enter a single value such as 0.1 or a series of values such as 0.1 0.2 0.3 or 0.1 to 0.5 by 0.1.

Effect Size – Within Level Correlations

ρ1 (Correlation Among Level 1 Units)

This is the correlation of each pair of level 1 units in a particular level 2 unit.

Possible values are from 0 to just below 1. For fixed slope models, $\rho1 \geq \rho2$. Typical values are between 0.0001 and 0.5.

You may enter a single value or a list of values.

ρ2 (Correlation Among Level 2 Units)

This is the correlation of each pair of level 2 units in a particular level 3 unit.

Possible values are from 0 to just below 1. For fixed slope models, $\rho1 \geq \rho2$. Typical values are between 0.0001 and 0.5.

You may enter a single value or a list of values.

Example 1 – Calculating Power

Suppose that a three-level hierarchical design is planned in which there will be students (level-1) which are nested in classrooms (level-2) which are nested in schools (level-3). This analysis will calculate the power for testing the significance of the difference in proportions of two interventions. There will be one measurement per student and treatments will be applied to schools (level-3 units).

The analysis will use a mixed logistic regression model. The following parameter settings are to be used for the power analysis: $P1 = 0.6$; $P2 = 0.5$; $\rho1 = 0.02$; $\rho2 = 0.01$; $C = 6$; $M = 5, 10$; $\alpha = 0.05$; and $K1 = K2 = 5$ to 20 by 5.

Setup

This section presents the values of each of the parameters needed to run this example. First, from the PASS Home window, load the **Mixed Models Tests for Two Proportions in a 3-Level Hierarchical Design (Level-2 Randomization)** procedure window. You may then make the appropriate entries as listed below, or open **Example 1** by going to the **File** menu and choosing **Open Example Template**.

Option   **Value**

Design Tab
Solve For .... **Power**
Alpha.. **0.05**
C (Level 3 Units) **6**
K1 (Level 2 Unit Assigned to Group 1) ... **5 10 15 20**
K2 (Level 2 Unit Assigned to Group 2) ... **K1**
M (Level 1 Units Per Level 2 Unit).......... **5 10**
P1 Input Type .... **Proportions**
P1 (Group 1 Proportion|H1).................... **0.6**
P2 (Group 2 Proportion) **0.5**
ρ1 (Correlation Among Level 1 Units) **0.02**
ρ2 (Correlation Among Level 2 Units) **0.01**

Annotated Output

Click the Calculate button to perform the calculations and generate the following output.

Numeric Results

Numeric Results for Comparing Two Proportions
H0: P1 = P2. H1: P1 ≠ P2

Power	Total N	Lvl 3 Units C	Grp 1 Lvl 2 Units K1	Grp 2 Lvl 2 Units K2	Lvl 1 Units M	Grp 1 Prop P1	Grp 2 Prop P2	Prop Diff P1-P2	Lvl 1 Corr ρ1	Lvl 2 Corr ρ2	Alpha
0.4029	300	6	5	5	5	0.6000	0.5000	0.1000	0.020	0.010	0.050
0.6595	600	6	5	5	10	0.6000	0.5000	0.1000	0.020	0.010	0.050
0.6802	600	6	10	10	5	0.6000	0.5000	0.1000	0.020	0.010	0.050
0.9188	1200	6	10	10	10	0.6000	0.5000	0.1000	0.020	0.010	0.050
0.8452	900	6	15	15	5	0.6000	0.5000	0.1000	0.020	0.010	0.050
0.9844	1800	6	15	15	10	0.6000	0.5000	0.1000	0.020	0.010	0.050
0.9303	1200	6	20	20	5	0.6000	0.5000	0.1000	0.020	0.010	0.050
0.9974	2400	6	20	20	10	0.6000	0.5000	0.1000	0.020	0.010	0.050

References

Ahn, C., Heo, M., and Zhang, S. 2015. Sample Size Calculations for Clustered and Longitudinal Outcomes in Clinical Research. CRC Press. New York.

Report Definitions

Power is the probability of rejecting a false null hypothesis. It should be close to one.

N is the total number of level-1 units.

C is the number of level-3 units.

K1, K2 are the average number of level-2 units per level-3 unit assigned to groups 1 and 2, respectively.

M is the average number of level-1 units per level-2 unit.

P1 is the proportion for group 1 (treatment group) assuming the alternative hypothesis.

P2 is the proportion for group 2 (control group). This is the proportion in the standard, reference, baseline, or control group.

Prop Diff = P1 - P2 is the difference in the group proportions assumed by the alternative hypothesis.

$\rho1$ is the correlation among level-1 units in a particular level-2 unit.

$\rho2$ is the correlation among level-2 units in a particular level-3 unit.

Alpha is the probability of rejecting a true null hypothesis, that is, rejecting when the means are actually equal.

Summary Statements

The total sample size of 300, which were obtained from 6 level-3 units with 5 level-2 units in group one and 5 level-2 units in group two and an average of 5 level-1 units per level-2 unit, achieve 40% power to detect a difference between the group proportions of at least 0.1000. The proportion in group 1 is assumed to be 0.6000 under the alternative hypothesis. The proportion in group 2 is 0.5000. The correlation of level-1 units within a level-2 unit is 0.020. The correlation of level-2 units within a level-3 unit is 0.010. A test based on a mixed-effects logistic regression is anticipated at a significance level of 0.050.

This report shows the power for each of the scenarios.

Plots Section
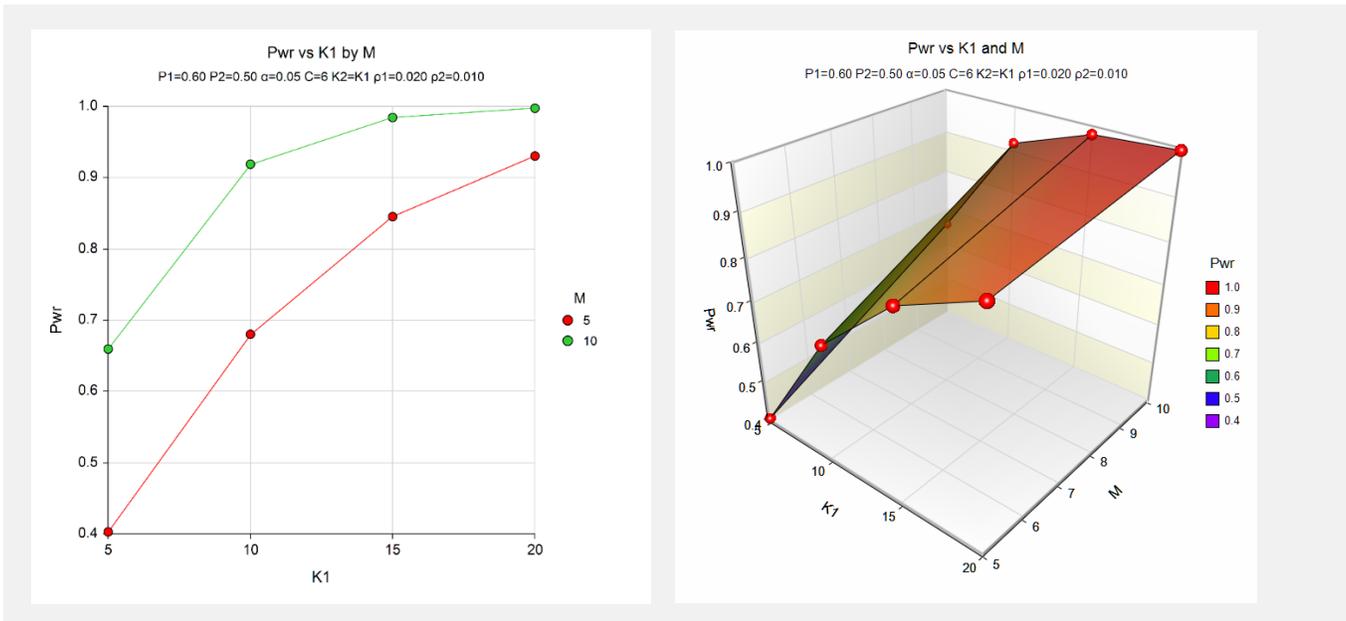
These plots show the power versus the level-2 count for the two values of M.

Example 2 – Calculating Sample Size (Level-2 Units)

Continuing with the last example, suppose the researchers want to determine the number of level 2 needed to achieve 90% power for the two values of M.

Setup

This section presents the values of each of the parameters needed to run this example. First, from the PASS Home window, load the **Mixed Models Tests for Two Proportions in a 3-Level Hierarchical Design (Level-2 Randomization)** procedure window. You may then make the appropriate entries as listed below, or open **Example 2** by going to the **File** menu and choosing **Open Example Template**.

Option	**Value**
Design Tab	
Solve For	**K1 (Number Level 2 Units Per Level 3 Unit)**
Power..	**0.90**
Alpha...	**0.05**
C (Level 3 Units)	**6**
K2 (Level 2 Unit Assigned to Group 2) ...	**K1**
M (Level 1 Units Per Level 2 Unit)..........	**5 10**
P1 Input Type	**Proportions**
P1 (Group 1 Proportion|H1)....................	**0.6**
P2 (Group 2 Proportion)	**0.5**
ρ1 (Correlation Among Level 1 Units)	**0.02**
ρ2 (Correlation Among Level 2 Units)	**0.01**

Output

Click the Calculate button to perform the calculations and generate the following output.

Numeric Results

Numeric Results for Comparing Two Proportions
H0: P1 = P2. H1: P1 ≠ P2

Power	Total N	Lvl 3 Units C	Grp 1 Lvl 2 Units K1	Grp 2 Lvl 2 Units K2	Lvl 1 Units M	Grp 1 Prop P1	Grp 2 Prop P2	Prop Diff P1-P2	Lvl 1 Corr ρ1	Lvl 2 Corr ρ2	Alpha
0.9034	1080	6	18	18	5	0.6000	0.5000	0.1000	0.020	0.010	0.050
0.9188	1200	6	10	10	10	0.6000	0.5000	0.1000	0.020	0.010	0.050

This report shows the power for each of the scenarios.

Example 3 – Validation using Ahn, Heo, and Zhang (2015)

Ahn, Heo, and Zhang (2015) page 228 provide a table in which several scenarios are reported. We will validate this procedure by the first row of the table. The following parameter settings were for the analysis: power = 0.80; $P1 = 0.5$; $P2 = 0.4$; $\rho1 = 0.1$; $\rho2 = 0.05$; $C = 24$; $M = 5$; and $\alpha = 0.05$. These settings resulted in a value of $K1$ and $K2$ (their $N_2^{(0)}$) of 4 and an attained power of 0.829.

Setup

This section presents the values of each of the parameters needed to run this example. First, from the PASS Home window, load the **Mixed Models Tests for Two Proportions in a 3-Level Hierarchical Design (Level-2 Randomization)** procedure window. You may then make the appropriate entries as listed below, or open **Example 3** by going to the **File** menu and choosing **Open Example Template**.

Option   **Value**

Design Tab

Solve For .... **K1 (Number Level 2 Units Per Level 3 Unit)**

Power.. **0.80**

Alpha.. **0.05**

C (Level 3 Units) **24**

K (Level 2 Units Per Level 3 Unit) **4**

M (Level 1 Units Per Level 2 Unit).......... **5**

P1 Input Type .... **Proportions**

P1 (Group 1 Proportion|H1).................... **0.5**

P2 (Group 2 Proportion) **0.4**

ρ1 (Correlation Among Level 1 Units) **0.1**

ρ2 (Correlation Among Level 2 Units) **0.05**

Output

Click the Calculate button to perform the calculations and generate the following output.

Numeric Results

Numeric Results for Comparing Two Proportions
H0: P1 = P2. H1: P1 ≠ P2

Power	Total N	Lvl 3 Units C	Grp 1 Lvl 2 Units K1	Grp 2 Lvl 2 Units K2	Lvl 1 Units M	Grp 1 Prop P1	Grp 2 Prop P2	Prop Diff P1-P2	Lvl 1 Corr ρ1	Lvl 2 Corr ρ2	Alpha
0.8286	960	24	4	4	5	0.5000	0.4000	0.1000	0.100	0.050	0.050

PASS calculates the same values of *K1* and power: 4 and 0.8286.