

Chapter 629

Multi-Arm Equivalence Tests for Survival Curves using Cox's Proportional Hazards Model in a Cluster-Randomized Design

Introduction

This procedure calculates power and sample size for testing the equivalence of multiple treatment hazard rates versus a common control hazard rate using Cox's proportional hazards regression when the data are obtained in a cluster-randomized design. Because survival times are not normally distributed and because some survival times are censored, Cox proportional-hazards regression is often used to analyze the data. The formulation for testing the significance of a Cox regression coefficient is identical to the standard logrank test. Thus, the power and sample size formulas for one analysis also work for the other.

The procedure is documented in Chow, Shao, Wang, and Lokhnygina (2018) and Machin, Campbell, Tan, and Tan (2018) which are based on the work of Schoenfeld (1981, 1983).

A *cluster (group) randomized design* is one in which whole units, or clusters, of subjects are randomized to the groups rather than the individual subjects in those clusters. However, the conclusions of the study concern individual subjects rather than the clusters. Examples of clusters are families, school classes, neighborhoods, hospitals, and doctor's practices.

Cluster-randomized designs are often adopted when there is a high risk of contamination if cluster members were randomized individually. For example, it may be difficult for doctors to use two treatment methods in their practice. The price of randomizing by clusters is a loss of efficiency--the number of subjects needed to obtain a certain level of precision in a cluster-randomized trial is usually much larger than the number needed when the subjects are randomized individually. Hence, standard methods of sample size estimation cannot be used.

The Bonferroni adjustment of the type I error rate may be optionally made because several comparisons are being tested using the same data. Making a multiplicity adjustment is usually recommended, but not always. In fact, Saville (1990) advocates not applying it and Machin, Campbell, Tan, and Tan (2018) include omitting it as a possibility.

Technical Details

Cox's Proportional Hazards Regression

Cox's proportional hazards regression is widely used for survival data. The regression model is

$$h(t|z) = h(t|0) \exp(bz)$$

where

b is the regression coefficient which is equal to $\log[h(t|1)/h(t|0)] = \log(HR)$

z is a binary indicator variable of treatment group

t is elapsed time

$h(t|z)$ is the hazard rate at time t , given covariate z

HR is the hazard ratio $h(t|1)/h(t|0)$

HR_0 and $1 / HR_0$ are the upper and lower equivalence limits of the hazard ratio

Equivalence Hypothesis

The equivalence of two hazard rates is established by concluding that their ratio is within a clinically insignificant margin from one. The statistical hypotheses that yield this conclusion when the null hypothesis is rejected is

$$H_0: |\log(HR)| \geq \log(HR_0) \quad \text{vs.} \quad H_1: |\log(HR)| < \log(HR_0), \quad \text{assuming } HR_0 > 1$$

Test Statistic

It can be shown that the test of b , the regression coefficient representing the group, based on the partial likelihood method of Cox (1972) coincides with the common logrank test statistic. When testing equivalence, you can use b from a Cox regression or calculate the modified logrank statistic as follows

Logrank Test

In this case, the test statistic uses two one-sided logrank tests. Define

$$L_U = \frac{\sum_{k=1}^K \left(I_k - \frac{Y_{1i}HR_0}{Y_{1i}HR_0 + Y_{2i}} \right)}{\left[\sum_{k=1}^K \left(\frac{Y_{1i}Y_{2i}HR_0}{(Y_{1i}HR_0 + Y_{2i})^2} \right) \right]^{-\frac{1}{2}}}$$

$$L_L = \frac{\sum_{k=1}^K \left(I_k - \frac{Y_{1i}(1/HR_0)}{Y_{1i}(1/HR_0) + Y_{2i}} \right)}{\left[\sum_{k=1}^K \left(\frac{Y_{1i}Y_{2i}(1/HR_0)}{(Y_{1i}(1/HR_0) + Y_{2i})^2} \right) \right]^{-\frac{1}{2}}}$$

where K is the number of deaths, Y_{ij} is the number of subjects at risk just prior to the j^{th} observed event in the i^{th} group, and I_k is a binary variable indicating whether the k^{th} event is from group 1 or not.

The null hypothesis is rejected if $L_U < z_\alpha$ and $L_L > z_{1-\alpha}$.

The distribution of L_U is approximately normal with mean $(\log(HR_i) - \log(HR_0))\sqrt{P_C P_i d_i N}$ and unit variance and the distribution of L_L is approximately normal with mean $(\log(HR_i) - \log(1/HR_0))\sqrt{P_C P_i d_i N}$ and unit variance, where

P_C is the proportion of N that is in the control group

P_i is the proportion of N that is in the i^{th} treatment group

N is the total sample size

N_C is the sample size from the control group, $N_C = N(P_C)$

N_i is the sample size from the i^{th} treatment group, $N_i = N(P_i)$

P_{ev_C} is probability of the event of interest in the control group

P_{ev_i} is probability of the event of interest in the i^{th} treatment group

d_i is the overall probability of an event, $d_i = P_{ev_C}P_C + P_{ev_i}P_i$

HR_i is the observed hazard ratio for the i^{th} treatment group vs. the control group

HR_0 is the upper equivalence boundary (limit) of the hazard ratio

Cluster-Randomized Designs

Denote an observation by Y_{ijk} where $i = c, 1, 2, \dots, G$ gives the group, $j = 1, 2, \dots, K_i$ gives the cluster within group i , and $k = 1, 2, \dots, m_{ij}$ denotes an individual in cluster j of group i . In this chapter, we will assume that group c is the control group and groups $1, \dots, G$ are the treatment groups.

Let ρ denote the intraclass correlation coefficient (ICC) among individuals from the same cluster. This correlation is the correlation of censor indicator variable. Let COV denote the coefficient of variation of the cluster sizes. Machin *et al.* (2018) page 101 shows that the number of events, e , that are needed to obtain a given power of $1 - \beta$ and a significance level of α to detect a hazard ratio of $HR_i (h_i / h_c)$ is given by

$$e = \frac{(1 + r)^2}{r} \left[\frac{(z_{1-\alpha} + z_{1-\beta})^2}{(\log HR_i - \log HR_0)^2} \right]$$

where $r = N_i / N_c$ and $z_x = \Phi(x)$ is the standard normal distribution function.

The number of subjects in a regular design can be determined using

$$N = N_c + N_i = \frac{e}{F}$$

where

$$F = \frac{(Pev_c + r(Pev_i))}{1 + r}$$

Design Effect

When using a cluster-randomized design Machin *et al.* (2018) page 202 show that the above formula is modified using a quantity that is known as the *design effect* (DE). The version of DE that is used in **PASS** is given as formula 12.7 on page 197 of Machin *et al.* (2018) which is

$$DE = 1 + \{[CV^2 + 1]\bar{M} - 1\}\rho$$

where \bar{M} is the average cluster size of all clusters given by

$$\bar{M} = \frac{K_c M_c + K_i M_i}{K_c + K_i}$$

CV is the coefficient of variation of cluster sizes of all clusters in the study and ρ is the ICC as defined above.

The resulting sample size formula in terms of cluster counts and size is

$$N = K_c M_c + K_i M_i = DE \left(\frac{e}{F} \right)$$

Power Calculations

The power of this test is given by

$$\Phi\left(\left(\log(HR_0) - \log(HR_i)\right)\sqrt{P_c P_i d_i N} - z_{1-\alpha}\right) + \Phi\left(\left(\log(HR_0) + \log(HR_i)\right)\sqrt{P_c P_i d_i N} - z_{1-\alpha}\right) - 1$$

where HR_i is the actual value of the hazard ratio under the alternative hypothesis and N is defined as above.

Multiplicity Adjustment

Because G tests between the treatment groups and the control group are run when analyzing the results of this study, many statisticians recommend that a Bonferroni adjustment be applied. This adjustment is easy to apply: the value of alpha that is used in the test is found by dividing the original alpha by the number of tests. For example, if the original alpha is set at 0.05 and the number of treatment (not including the control) groups is five, the individual tests should be conducted using an alpha of 0.01.

The main criticism of this procedure is that if there are many tests, the value of alpha becomes very small. To mitigate against this complaint, some statisticians recommend separating the treatment groups into those that are of primary interest and those that are of secondary interest. The Bonferroni adjustment is made by the using the number of primary treatments rather than the total number of treatments.

There are some who advocate ignoring the adjustment entirely in the case of randomized clinical trials. See for example Saville (1990) and the discussion in chapter 14 of Machin, Campbell, Tan, and Tan (2018).

Size of the Control Group

Because the control group is used over and over, some advocate increasing the number of clusters in this group. The standard adjustment is to include \sqrt{G} clusters in the control group for each cluster in one of the treatment groups. See Machin, Campbell, Tan, and Tan (2018, pages 231-232). Note that often, the treatment groups all have the same sample size.

Example 1 – Finding the Sample Size

A company is planning an equivalence trial using a four-arm, cluster-randomized design in which $HR_0 = 1.25$; $HR_1 = HR_2 = HR_3 = 1$, $\rho = 0.05$, $Pev_1 = Pev_2 = Pev_3 = 0.75$, $Pev_C = 0.75$, $M_i = 10, 20, \text{ or } 30$, $COV = 0.65$, $alpha = 0.05$, and number of clusters is to be calculated. The target power is 0.9.

The control group cluster allocation will be set to $\sqrt{G} = \sqrt{3} = 1.732$ since the control group is used for three comparisons in this design.

Setup

If the procedure window is not already open, use the PASS Home window to open it. The parameters for this example are listed below and are stored in the **Example 1** settings file. To load these settings to the procedure window, click **Open Example Settings File** in the Help Center or File menu.

Design Tab	
Solve For	Sample Size
Power of Each Test	0.90
Overall Alpha	0.05
Bonferroni Adjustment	Standard Bonferroni
Group Allocation	Enter Group Allocation Pattern, solve for group numbers of clusters
M (Average Cluster Size).....	10 20 30
COV of Cluster Sizes.....	0.65
Pev (Default Probability of an Event)	0.75
HR0 (Upper Equivalence Hazard Ratio)	1.25
Control Probability of an Event	Pev
Control Average Cluster Size.....	M
Control Cluster Allocation	1.732
Set A Number of Groups.....	3
Set A Hazard Ratio	1
Set A Probability of an Event	Pev
Set A Average Cluster Size	M
Set A Cluster Allocation	1
Set B Number of Groups.....	0
Set C Number of Groups	0
Set D Number of Groups	0
More.....	Unchecked
ρ (Intracluster Correlation)	0.05

Output

Click the Calculate button to perform the calculations and generate the following output.

Numeric Reports

Numeric Results

Solve For: Sample Size
 Group Allocation: Enter Group Allocation Pattern, solve for group numbers of clusters
 Hypotheses: H0: $HR \leq 1 / HR0$ or $HR \geq HR0$ vs. H1: $1 / HR0 < HR < HR0$
 Number of Groups: 4
 Bonferroni Adjustment: Standard Bonferroni (Divisor = 3)

Comparison	Power	Number of			Cluster Size			Hazard Ratio			Alpha			
		Sample Size Ni	Events Ei	Clusters Ki	Cluster Allocation	Average Mi	COV	Probability of an Event Pevi	Upper Equivalence HR0	Actual HRi	ICC ρ	Design Effect DE	Overall	Bonferroni-Adjusted
Control vs A1	0.90029	1730	2155.5	173	1.732	10	0.65	0.75			0.05	1.66125	0.05	0.01667
Control vs A2	0.90029	1000	1245.9	100	1.000	10	0.65	0.75	1.25	1	0.05	1.66125	0.05	0.01667
Control vs A3	0.90029	1000	1245.9	100	1.000	10	0.65	0.75	1.25	1	0.05	1.66125	0.05	0.01667
Total		4730	5893.3	473										
Control vs A1	0.90396	2500	4448.4	125	1.732	20	0.65	0.75			0.05	2.37250	0.05	0.01667
Control vs A2	0.90396	1440	2562.3	72	1.000	20	0.65	0.75	1.25	1	0.05	2.37250	0.05	0.01667
Control vs A3	0.90396	1440	2562.3	72	1.000	20	0.65	0.75	1.25	1	0.05	2.37250	0.05	0.01667
Total		6820	12135.3	341										
Control vs A1	0.90072	3210	7424.1	107	1.732	30	0.65	0.75			0.05	3.08375	0.05	0.01667
Control vs A2	0.90072	1860	4301.8	62	1.000	30	0.65	0.75	1.25	1	0.05	3.08375	0.05	0.01667
Control vs A3	0.90072	1860	4301.8	62	1.000	30	0.65	0.75	1.25	1	0.05	3.08375	0.05	0.01667
Total		8790	20329.6	293										

- Comparison: The group that is involved in the comparison between the treatment and control displayed on this report line. The comparison is made using the hazard ratio.
- Power: The probability of rejecting a false null hypothesis for this comparison. This power is of the comparison shown on this line only.
- Ni: The number of items in the ith group. The total sample size is shown as the last row of the column.
- Ei: The number of events in the ith group required to achieve the power indicated. $Ei = Pevi \times Ni$.
- Ki: The number of clusters in the ith group. The total number of clusters is reported in the last row of the column.
- Allocation: The cluster allocation ratio of the ith group. The value on each row represents the relative number of clusters assigned to the group.
- Mi: The average number of items per cluster (or average cluster size) in the ith group.
- COV: The coefficient of variation of the cluster sizes within the group.
- Pevi: The average probability that a subject the ith group will have an event during the study. Pevi also represents the proportion of individuals in the ith group that are expected to have an event during the study. This probability includes the impact of various kinds of censoring.
- HR0: The upper equivalence limit of the hazard ratio. The lower equivalence limit of the hazard ratio is $1 / HR0$.
- HRi: The hazard ratio of the ith treatment group. $HR = hi / hc$.
- ρ : The intracluster correlation (ICC). The correlation between subjects within a cluster.
- DE: The design effect. This value is used to increase the sample size because of the cluster randomization that is used in the design.
- Overall Alpha: The probability of rejecting at least one of the comparisons in this experiment when each null hypothesis is true.
- Bonferroni Alpha: The adjusted significance level at which each individual comparison is made.

Multi-Arm Equivalence Tests for Survival Curves using Cox's Prop. Hazards Model in a Cluster-Randomized Design

Summary Statements

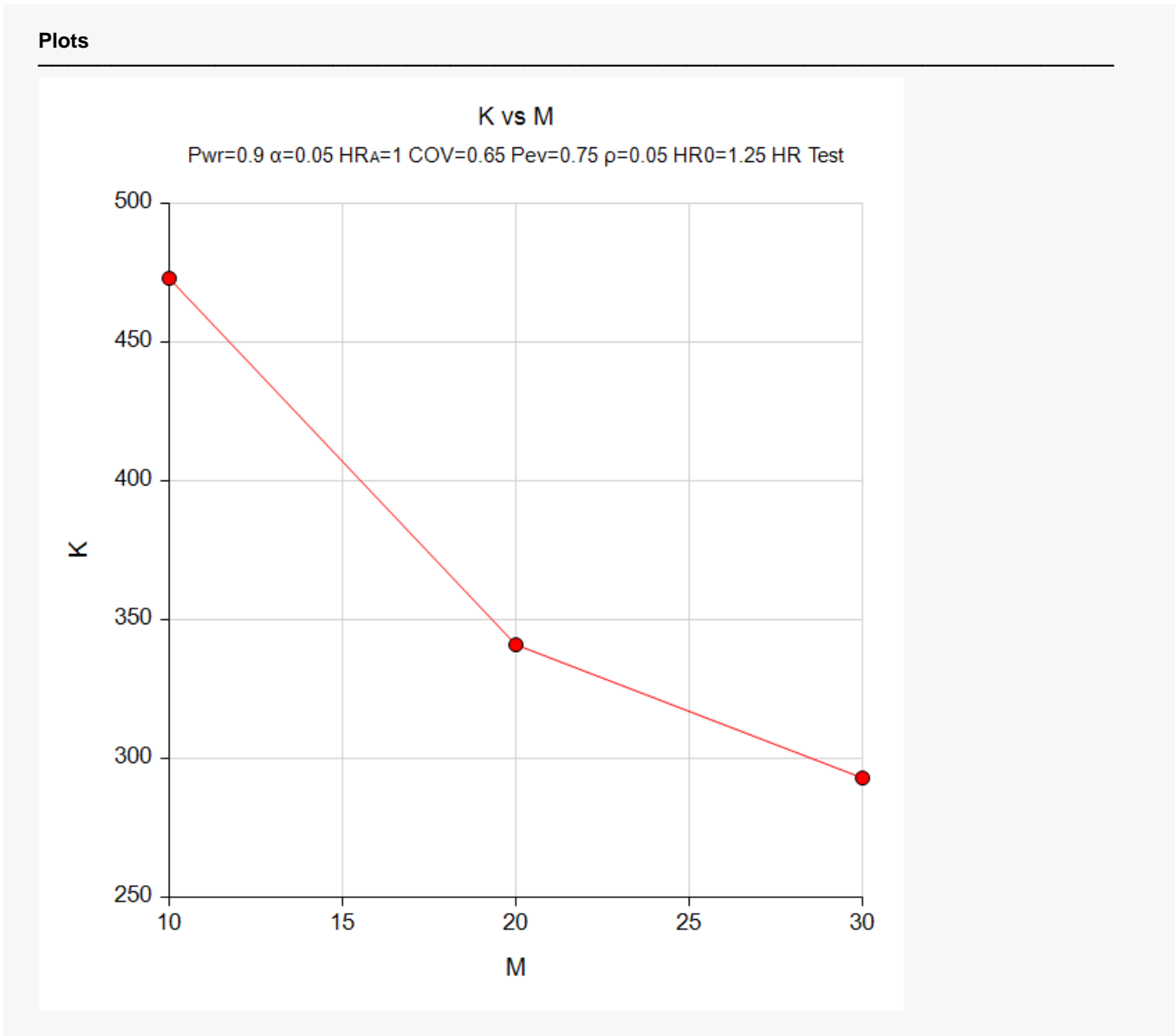
A parallel, 4-group cluster-randomized design (with one control group and 3 treatment groups) will be used to test whether the hazard rate for each treatment group is equivalent to the control group hazard rate, with lower and upper equivalence limits for the hazard ratio of 0.8 and 1.25 ($H_0: HR_i \leq 0.8$ or $HR_i \geq 1.25$ versus $H_1: 0.8 < HR_i < 1.25$, $HR_i = h_i / h_c$). The hypotheses will be evaluated using 3 two-one-sided, two-sample, Bonferroni-adjusted (divisor = 3) Cox regression coefficient tests, with an overall (experiment-wise) Type I error rate (α) of 0.05. The coefficient of variation of the cluster sizes in all clusters is assumed to be 0.65. The average probability of an event for a subject in the control group is assumed to be 0.75, and the event probabilities for the treatment groups are assumed to be 0.75, 0.75, and 0.75. The calculations are based on the assumption that the hazard ratio is constant throughout the study. The intracluster correlation is assumed to be 0.05. The average cluster size (number of subjects or items per cluster) for the control group is assumed to be 10, and the average cluster size for each of the treatment groups is assumed to be 10, 10, and 10. To detect the treatment to control hazard ratios 1, 1, and 1 with at least 90% power for each test, the control group cluster count needed will be 173 and the number of needed clusters for the treatment groups will be 100, 100, and 100 (totaling 473 clusters overall).

References

- Ahn, C., Heo, M., and Zhang, S. 2015. *Sample Size Calculations for Clustered and Longitudinal Outcomes in Clinical Research*. CRC Press. New York.
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- Campbell, M.J. and Walters, S.J. 2014. *How to Design, Analyse and Report Cluster Randomised Trials in Medicine and Health Related Research*. Wiley. New York.
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- Machin, D., Campbell, M.J., Tan, S.B, and Tan, S.H. 2018. *Sample Sizes for Clinical, Laboratory, and Epidemiology Studies*, 4th Edition. Wiley Blackwell.
- Schoenfeld, David A. 1983. 'Sample Size Formula for the Proportional-Hazards Regression Model', *Biometrics*, Volume 39, Pages 499-503.
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This report shows the numeric results of this sample size study. Notice that the results are shown in blocks of five rows at a time. Each block represents an individual treatment.

Plots Section



This plot gives a visual presentation to the results in the Numeric Report. We can quickly see the impact on the total cluster count, K, of increasing the average cluster size, M.

Example 2 – Validation using a Previously Validated Procedure

We could not find a validation result in the statistical literature, so we will use a previously validated **PASS** procedure (**Equivalence Tests for Two Survival Curves using Cox's Proportional Hazards Model in a Cluster-Randomized Design**) to produce the results for the following example.

Suppose that a two-arm, cluster-randomized study is to be conducted in which $HR_1 = HR_2 = 1, HR_0 = 1.25, \rho = 0.05, Pev_1 = Pev_2 = 0.7, Pev_c = 0.7, M_i = 2, COV = 0.65, alpha = 0.0125$, and number of clusters is 400 in each group. The resulting power is 0.89321.

The **Equivalence Tests for Two Survival Curves using Cox's Proportional Hazards Model in a Cluster-Randomized Design** procedure is set up as follows.

Design Tab

Solve For	Power
Alpha.....	0.025
K1 (Number of Clusters)	400
M1 (Average Cluster Size).....	2
K2 (Number of Clusters)	K1
M2 (Average Cluster Size).....	M1
COV of Cluster Sizes	0.65
Pev1 (Probability of a Control Event).....	0.7
Pev2 (Probability of a Treatment Event)	Pev1
HR0 (Upper Equivalence Hazard Ratio)	1.25
HR1 (Actual Hazard Ratio)	1
ρ (Intraclass Correlation)	0.05

This set of options generates the following report.

Numeric Results

Solve For: **Power**
 Groups: 1 = Control, 2 = Treatment
 Hypotheses: $H_0: HR \leq 1 / HR_0$ or $HR \geq HR_0$ vs. $H_1: 1 / HR_0 < HR < HR_0$

Group	Power	Sample Size Ni	Number of Events Ei	Number of Clusters Ki	Cluster Size		Probability of an Event Pevi	Hazard Ratio		Actual HR1	Intraclass Correlation ρ	Alpha
					Average Mi	COV		Equivalence				
								Lower 1/HR0	Upper HR0			
1: Control		800	611.7	400	2	0.65	0.7				0.05	
2: Treatment	0.89321	800	611.7	400	2	0.65	0.7	0.8	1.25	1	0.05	0.025
Total		1600	1223.3	800								

The power is computed to be 0.89321.

Setup

If the procedure window is not already open, use the PASS Home window to open it. The parameters for this example are listed below and are stored in the **Example 2** settings file. To load these settings to the procedure window, click **Open Example Settings File** in the Help Center or File menu.

Design Tab

Solve For **Power**
 Overall Alpha **0.05**
 Bonferroni Adjustment **Standard Bonferroni**
 Group Allocation **Equal (Kc = K1 = K2 = ...)**
 Ki (Group Number of Clusters) **400**
 M (Average Cluster Size)..... **2**
 COV of Cluster Sizes..... **0.65**
 Pev (Default Probability of an Event)..... **0.7**
 HR0 (Upper Equivalence Hazard Ratio) **1.25**
 Control Probability of an Event **Pev**
 Control Average Cluster Size..... **M**
 Set A Number of Groups..... **2**
 Set A Hazard Ratio **1**
 Set A Probability of an Event **Pev**
 Set A Average Cluster Size **M**
 Set B Number of Groups..... **0**
 Set C Number of Groups **0**
 Set D Number of Groups **0**
 More..... **Unchecked**
 ρ (Intracluster Correlation)..... **0.05**

Output

Click the Calculate button to perform the calculations and generate the following output.

Numeric Results

Solve For: [Power](#)
 Hypotheses: H0: $HR \leq 1 / HR_0$ or $HR \geq HR_0$ vs. H1: $1 / HR_0 < HR < HR_0$
 Number of Groups: 3
 Bonferroni Adjustment: Standard Bonferroni (Divisor = 2)

Comparison	Power	Sample Size Ni	Number of			Cluster Size		Probability of an Event Pevi	Hazard Ratio			Alpha		
			Events Ei	Clusters Ki	Average Mi	COV	Upper Equivalence HR0		Actual HRi	ICC ρ	Design Effect DE	Overall	Bonferroni-Adjusted	
Control		800	611.7	400	2	0.65	0.7				0.05	1.09225		
vs A1	0.89321	800	611.7	400	2	0.65	0.7	1.25	1	0.05	1.09225	0.05	0.025	
vs A2	0.89321	800	611.7	400	2	0.65	0.7	1.25	1	0.05	1.09225	0.05	0.025	
Total		2400	1835.0	1200										

As you can see, the power is 0.89321 for both treatment groups which matches the power found in the validation run above. The procedure is validated.