Chapter 605

Multivariate Analysis of Variance (MANOVA)

Introduction

This module calculates power for multivariate analysis of variance (MANOVA) designs having up to three factors. It computes power for three MANOVA test statistics: Wilks' lambda, Pillai-Bartlett trace, and Hotelling-Lawley trace.

MANOVA is an extension of common analysis of variance (ANOVA). In ANOVA, differences among various group means on a single-response variable are studied. In MANOVA, the number of response variables is increased to two or more. The hypothesis concerns a comparison of vectors of group means. The multivariate extension of the *F*-test is not completely direct. Instead, several test statistics are available. The actual distributions of these statistics are difficult to calculate, so we rely on approximations based on the *F*-distribution.

Assumptions

The following assumptions are made when using MANOVA to analyze a factorial experimental design.

1. The response variables are continuous.
2. The residuals follow the multivariate normal probability distribution with mean zero and constant variance-covariance matrix.
3. The subjects are independent.

Technical Details

General Linear Multivariate Model

This section provides the technical details of the MANOVA designs that can be analyzed by **PASS**. The approximate power calculations outlined in Muller, LaVange, Ramey, and Ramey (1992) are used. Using their notation, for *N* subjects, the usual general linear multivariate model is

$$\underset{(N\times p)}{\mathrm{Y}} = \underset{(N\times q\times p)}{XM} + \underset{(N\times p)}{\mathrm{R}}$$

where each row of the residual matrix *R* is distributed as a multivariate normal

$$row_k(R) \sim N_p(0, \Sigma)$$

Note that *p* is the number of response variables and *q* is the number of design variables, *Y* is the matrix of responses, *X* is the design matrix, *M* is the matrix of regression parameters (means), and *R* is the matrix of residuals.

Hypotheses about various sets of regression parameters are tested using

$$H_0\colon \underset{a\times p}{\Theta} = \Theta_0$$

$$\underset{a\times q\times p}{CM} = \Theta$$

where *C* is an orthonormal contrast matrix and Θ_0 is a matrix of hypothesized values, usually zeros. Note that *C* defines contrasts among the factor levels. Tests of the various main effects and interactions may be constructed with suitable choices of *C*. These tests are based on

$$\widehat{M} = (X'X)^{-}X'Y$$

$$\hat{\Theta} = C\widehat{M}$$

$$\underset{p\times p}{H} = \left(\hat{\Theta} - \Theta_0\right)'[C(X'X)^{-}C']^{-1}\left(\hat{\Theta} - \Theta_0\right)$$

$$\underset{p\times p}{E} = \hat{\Sigma}\cdot(N-r)$$

$$\underset{p\times p}{T} = H + E$$

where *r* is the rank of *X*.

Wilks' Lambda Approximate F Test

The hypothesis $H_0: \Theta = \Theta_0$ may be tested using Wilks' likelihood ratio statistic *W*. This statistic is computed using

$$W = |ET^{-1}|$$

An *F* approximation to the distribution of *W* is given by

$$F_{df_1,df_2} = \frac{\eta \,/\, df_1}{(1-\eta) \,/\, df_2}$$

where

$$\lambda = df_1 F_{df_1,df_2}$$

$$\eta = 1 - W^{1/g}$$

$$df1 = ap$$

$$df2 = g[(N-r) - (p-a+1)\,/\,2] - (ap-2)\,/\,2$$

$$g = \left(\frac{a^2p^2-4}{a^2+p^2-5}\right)^{\frac{1}{2}}$$

Pillai-Bartlett Trace Approximate F Test

The hypothesis $H_0: \Theta = \Theta_0$ may be tested using the Pillai-Bartlett Trace. This statistic is computed using

$$T_{PB} = \mathrm{tr}(HT^{-1})$$

A noncentral *F* approximation to the distribution of T_{PB} is given by

$$F_{df_1,df_2} = \frac{\eta \,/\, df_1}{(1-\eta) \,/\, df_2}$$

where

$$\lambda = df_1 F_{df_1,df_2}$$

$$\eta = \frac{T_{PB}}{s}$$

$$s = \min(a, p)$$

$$df1 = ap$$

$$df2 = s[(N-r) - p + s]$$

Hotelling-Lawley Trace Approximate F Test

The hypothesis $H_0\colon \Theta = \Theta_0$ may be tested using the Hotelling-Lawley Trace. This statistic is computed using

$$T_{HL} = \mathrm{tr}(HE^{-1})$$

An *F* approximation to the distribution of T_{HL} is given by

$$F_{df_1,df_2} = \frac{\eta \,/\, df_1}{(1-\eta) \,/\, df_2}$$

where

$$\lambda = df_1 F_{df_1,df_2}$$

$$\eta = \frac{\frac{T_{HL}}{s}}{1 + \frac{T_{HL}}{s}}$$

$$s = \min(a, p)$$

$$df1 = ap$$

$$df2 = s[(N-r) - p - 1] + 2$$

M (Means) Matrix

In the general linear multivariate model presented above, *M* represents a matrix of regression coefficients. Although other structures and interpretations of *M* are possible, in this module we assume that the elements of *M* are the cell means. The rows of *M* represent the factor categories and the columns of *M* represent the response variables. (Note that this is just the opposite of the orientation used when entering *M* into the spreadsheet.)

The *q* rows of *M* represent the *q* groups into which the subjects can be classified. For example, if a design includes three factors with 2, 3, and 4 categories, the matrix *M* would have 2 *x* 3 *x* 4 = 24 rows. That is, *q* = 24.

Consider now an example in which *q* = 3 and *p* = 4. That is, there are three groups into which subjects can be placed. Each subject has four made. The matrix *M* would appear as follows.

$$M = \begin{bmatrix} \mu_{11} & \mu_{12} & \mu_{13} & \mu_{14} \\ \mu_{21} & \mu_{22} & \mu_{23} & \mu_{24} \\ \mu_{31} & \mu_{32} & \mu_{33} & \mu_{34} \end{bmatrix}$$

For example, the element μ_{12} is the mean of the second response of subjects in the first group. To calculate the power of this design, you would need to specify appropriate values of all twelve means.

C Matrix – Contrasts

The *C* matrix is comprised of contrasts that are applied to the rows of *M*. You do not have to specify these contrasts. They are generated for you. You should understand that a different *C* matrix is generated for each term in the model.

Generating the C Matrix when there are Multiple Between Factors

Generating the *C* matrix when there is more than one factor is more difficult. We use the method of O'Brien and Kaiser (1985) which we briefly summarize here.

Step 1. Write a complete set of contrasts suitable for testing each factor separately. For example, if you have three factors with 2, 3, and 4 categories, you might use

$$\ddot{C}_{B1} = \begin{bmatrix} \frac{-1}{\sqrt{2}} & \frac{1}{\sqrt{2}} \end{bmatrix},\ \ddot{C}_{B2} = \begin{bmatrix} \frac{-2}{\sqrt{6}} & \frac{1}{\sqrt{6}} & \frac{1}{\sqrt{6}} \\ 0 & \frac{-1}{\sqrt{2}} & \frac{1}{\sqrt{2}} \end{bmatrix},\ \text{and}\ \ddot{C}_{B3} = \begin{bmatrix} \frac{-3}{\sqrt{12}} & \frac{1}{\sqrt{12}} & \frac{1}{\sqrt{12}} & \frac{1}{\sqrt{12}} \\ 0 & \frac{-2}{\sqrt{6}} & \frac{1}{\sqrt{6}} & \frac{1}{\sqrt{6}} \\ 0 & 0 & \frac{-1}{\sqrt{2}} & \frac{1}{\sqrt{2}} \end{bmatrix}.$$

Step 2. Define appropriate J_k matrices corresponding to each factor. These matrices comprised of one row and *k* columns whose equal element is chosen so that the sum of its elements squared is one. In this example, we use

$$J_2 = \begin{bmatrix} \frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} \end{bmatrix},\ J_3 = \begin{bmatrix} \frac{1}{\sqrt{3}} & \frac{1}{\sqrt{3}} & \frac{1}{\sqrt{3}} \end{bmatrix},\ J_4 = \begin{bmatrix} \frac{1}{\sqrt{4}} & \frac{1}{\sqrt{4}} & \frac{1}{\sqrt{4}} & \frac{1}{\sqrt{4}} \end{bmatrix}$$

Step 3. Create the appropriate contrast matrix using a direct (Kronecker) product of either the $\ddot{C}_{Bi}$ matrix if the factor is included in the term or the J_i matrix when the factor is not in the term. Remember that the direct product is formed by multiplying each element of the second matrix by all members of the first matrix. Here is an example

$$\begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix} \otimes \begin{bmatrix} 1 & 0 & -1 \\ 0 & 2 & 0 \\ -1 & 0 & 3 \end{bmatrix} = \begin{bmatrix} 1 & 2 & 0 & 0 & -1 & -2 \\ 3 & 4 & 0 & 0 & -3 & -4 \\ 0 & 0 & 2 & 4 & 0 & 0 \\ 0 & 0 & 6 & 8 & 0 & 0 \\ -1 & -2 & 0 & 0 & 3 & 6 \\ -3 & -4 & 0 & 0 & 9 & 12 \end{bmatrix}$$

As an example, we will compute the *C* matrix suitable for testing factor *B*2

$$C_{B2} = J_2 \otimes \ddot{C}_{B2} \otimes J_4$$

Expanding the direct product results in

$$C_{B2} = J_2 \otimes \ddot{C}_{B2} \otimes J_4$$

$$= \begin{bmatrix} \frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} \end{bmatrix} \otimes \begin{bmatrix} \frac{-2}{\sqrt{6}} & \frac{1}{\sqrt{6}} & \frac{1}{\sqrt{6}} \\ 0 & \frac{-1}{\sqrt{2}} & \frac{1}{\sqrt{2}} \end{bmatrix} \otimes \begin{bmatrix} \frac{1}{\sqrt{4}} & \frac{1}{\sqrt{4}} & \frac{1}{\sqrt{4}} & \frac{1}{\sqrt{4}} \end{bmatrix}$$

$$= \begin{bmatrix} \frac{-2}{\sqrt{12}} & \frac{-2}{\sqrt{12}} & \frac{1}{\sqrt{12}} & \frac{1}{\sqrt{12}} & \frac{1}{\sqrt{12}} & \frac{1}{\sqrt{12}} \\ 0 & 0 & \frac{-1}{\sqrt{4}} & \frac{-1}{\sqrt{4}} & \frac{1}{\sqrt{4}} & \frac{1}{\sqrt{4}} \end{bmatrix} \otimes \begin{bmatrix} \frac{1}{\sqrt{4}} & \frac{1}{\sqrt{4}} & \frac{1}{\sqrt{4}} & \frac{1}{\sqrt{4}} \end{bmatrix}$$

$$= \begin{bmatrix} \frac{-2}{\sqrt{48}} & \frac{-2}{\sqrt{48}} & \frac{1}{\sqrt{48}} & \frac{1}{\sqrt{48}} & \frac{1}{\sqrt{48}} & \frac{1}{\sqrt{48}} & \frac{-2}{\sqrt{48}} & \frac{-2}{\sqrt{48}} & \frac{1}{\sqrt{48}} & \frac{1}{\sqrt{48}} & \frac{1}{\sqrt{48}} & \frac{1}{\sqrt{48}} & \frac{-2}{\sqrt{48}} & \frac{-2}{\sqrt{48}} & \frac{1}{\sqrt{48}} & \frac{1}{\sqrt{48}} & \frac{1}{\sqrt{48}} & \frac{1}{\sqrt{48}} & \frac{-2}{\sqrt{48}} & \frac{-2}{\sqrt{48}} & \frac{1}{\sqrt{48}} & \frac{1}{\sqrt{48}} & \frac{1}{\sqrt{48}} & \frac{1}{\sqrt{48}} \\ 0 & 0 & \frac{-1}{\sqrt{16}} & \frac{-1}{\sqrt{16}} & \frac{1}{\sqrt{16}} & \frac{1}{\sqrt{16}} & 0 & 0 & \frac{-1}{\sqrt{16}} & \frac{-1}{\sqrt{16}} & \frac{1}{\sqrt{16}} & \frac{1}{\sqrt{16}} & 0 & 0 & \frac{-1}{\sqrt{16}} & \frac{-1}{\sqrt{16}} & \frac{1}{\sqrt{16}} & \frac{1}{\sqrt{16}} & 0 & 0 & \frac{-1}{\sqrt{16}} & \frac{-1}{\sqrt{16}} & \frac{1}{\sqrt{16}} & \frac{1}{\sqrt{16}} \end{bmatrix}$$

Similarly, the *C* matrix suitable for testing interaction *B*2*B*3 is

$$C_{B2B3} = J_2 \otimes \ddot{C}_{B2} \otimes \ddot{C}_{B3}$$

Power Calculations

To calculate statistical power, we must determine distribution of the test statistic under the alternative hypothesis which specifies a different value for the regression parameter matrix *B*. The distribution theory in this case has not been worked out, so approximations must be used. We use the approximations given by Muller and Barton (1989) and Muller, LaVange, Ramey, and Ramey (1992). These approximations state that under the alternative hypothesis, F_U is distributed as a noncentral *F* random variable with degrees of freedom and noncentrality shown above. The calculation of the power of a particular test may be summarized as follows.

1. Specify values of X, M, Σ, C, and Θ_0.
2. Determine the critical value using $F_{crit} = FINV(1 - \alpha, df1, df2)$, where *FINV*( ) is the inverse of the central *F* distribution and α is the significance level.
3. Compute the noncentrality parameter λ.
4. Compute the power as

$$Power = 1 - NCFPROB(F_{crit}, df1, df2, \lambda)$$

where *NCFPROB*( ) is the noncentral *F* distribution.

Example 1 – Determining Power

Researchers are planning a study of the impact of a drug. They want to evaluate the differences in heart rate and blood pressure among three age groups: 20-40, 41-60, and over 60. They want to be able to detect a 10% change in heart rate and in blood pressure among the age groups. They plan to analyze the data using Wilks' lambda.

Previous studies have found an average heart rate of 93 with a standard deviation of 4 and an average blood pressure of 130 with a standard deviation of 5. The correlation between the two responses will be set at 0.7.

From a heart rate of 93, a 10% reduction gives 84. They want to be able to detect age-group heart-rate means the range from 93 to 84. From a blood pressure of 130, a 10% reduction gives 117. The want to be able to detect age-group blood-pressure means that range from 130 to 117. Hence, the means matrix that they will use is

C1	C2	C3
93	88	84
130	124	117

Based on the standard deviation settings that they chose to use, the variance-covariance matrix will be

C4	C5
16	14
14	25

In order to understand the relationship between power and sample size, they decide to calculate power values for sample sizes between 2 and 12, using a 0.05 significance level.

Setup

If the procedure window is not already open, use the PASS Home window to open it. The parameters for this example are listed below and are stored in the **Example 1** settings file. To load these settings to the procedure window, click **Open Example Settings File** in the Help Center or File menu.

Design: Means Tab

Solve For	**Power**
Wilks' Lambda	**Checked**
Pillai-Bartlett	**Unchecked**
Hotelling-Lawley	**Unchecked**
Alpha for All Terms	**0.05**
Number of Response Variables	**2**
Number of Factors	**1**
Number of Levels in Factor A	**3**
Columns Containing the Means	**C1-C3**
K's (Multipliers)	**0.5 1 1.5**
Group Allocation	**Equal (n1 = n2 = ··· = n)**
n (Size Per Group)	**2 4 6 8 10 12**

Design: Variance Tab

Input Type..**Variance-Covariance Matrix in Spreadsheet**
Columns Containing Covariance Matrix.........**C4-C5**

Input Spreadsheet Data

Row	C1	C2	C3	C4	C5
1	93	88	84	16	14
2	130	124	117	14	25

Output

Click the Calculate button to perform the calculations and generate the following output.

Numeric Reports

Results for Factor A (Levels = 3)

Solve For: Power

		Number of Subjects					Degrees of Freedom		
Test	Power	Group n	Total N	Multiply Means By	Test Statistic	Approximate F Statistic	DF1	DF2	Alpha
Wilks' Lambda	0.0729	2	6	0.5	0.622	0.27	4	4	0.05
Wilks' Lambda	0.1291	2	6	1.0	0.286	0.87	4	4	0.05
Wilks' Lambda	0.2046	2	6	1.5	0.146	1.62	4	4	0.05
Wilks' Lambda	0.1888	4	12	0.5	0.712	0.74	4	16	0.05
Wilks' Lambda	0.5749	4	12	1.0	0.378	2.51	4	16	0.05
Wilks' Lambda	0.8722	4	12	1.5	0.208	4.78	4	16	0.05
Wilks' Lambda	0.3191	6	18	0.5	0.733	1.17	4	28	0.05
Wilks' Lambda	0.8548	6	18	1.0	0.403	4.02	4	28	0.05
Wilks' Lambda	0.9916	6	18	1.5	0.226	7.71	4	28	0.05
Wilks' Lambda	0.4488	8	24	0.5	0.743	1.60	4	40	0.05
Wilks' Lambda	0.9603	8	24	1.0	0.415	5.51	4	40	0.05
Wilks' Lambda	0.9997	8	24	1.5	0.236	10.61	4	40	0.05
Wilks' Lambda	0.5678	10	30	0.5	0.748	2.03	4	52	0.05
Wilks' Lambda	0.9907	10	30	1.0	0.422	7.00	4	52	0.05
Wilks' Lambda	1.0000	10	30	1.5	0.241	13.49	4	52	0.05
Wilks' Lambda	0.6704	12	36	0.5	0.752	2.46	4	64	0.05
Wilks' Lambda	0.9981	12	36	1.0	0.427	8.48	4	64	0.05
Wilks' Lambda	1.0000	12	36	1.5	0.244	16.37	4	64	0.05

Power The probability of concluding that the means are different.
n The number of subjects per group.
N The total number of subjects in the study.
Multiply Means By The means were multiplied by this constant.
Test Statistic Wilks' Lambda (or other statistic) computed at the hypothesized values.
F Statistic The value of an approximate F from which the power is computed.
Df1 The numerator degrees of freedom of the approximate F statistic.
Df2 The denominator degrees of freedom of the approximate F statistic.
Alpha The probability of rejecting a true null hypothesis.

Summary Statements

A MANOVA design with 1 factor and 2 response variables has 3 groups with 2 subjects each for a total of 6 subjects. This design achieves 7% power to test factor A if a Wilks' Lambda Approximate F Test is used with a 5% significance level.

Dropout-Inflated Sample Size

Average Group Sample Size n	Group	Dropout Rate	Sample Size ni	Dropout-Inflated Enrollment Sample Size ni'	Expected Number of Dropouts Di
2	1 - 3	20%	2	3	1
	Total		6	9	3
4	1 - 3	20%	4	5	1
	Total		12	15	3
6	1 - 3	20%	6	8	2
	Total		18	24	6
8	1 - 3	20%	8	10	2
	Total		24	30	6
10	1 - 3	20%	10	13	3
	Total		30	39	9
12	1 - 3	20%	12	15	3
	Total		36	45	9

n The average group sample size.
Group Lists the group numbers.
Dropout Rate The percentage of subjects (or items) that are expected to be lost at random during the course of the study and for whom no response data will be collected (i.e., will be treated as "missing"). Abbreviated as DR.
ni The evaluable sample size for each group at which power is computed (as entered by the user). If ni subjects are evaluated out of the ni' subjects that are enrolled in the study, the design will achieve the stated power.
ni' The number of subjects that should be enrolled in each group in order to obtain ni evaluable subjects, based on the assumed dropout rate. ni' is calculated by inflating ni using the formula ni' = ni / (1 - DR), with ni' always rounded up. (See Julious, S.A. (2010) pages 52-53, or Chow, S.C., Shao, J., Wang, H., and Lokhnygina, Y. (2018) pages 32-33.)
Di The expected number of dropouts in each group. Di = ni' - ni.

Dropout Summary Statements

Anticipating a 20% dropout rate, group sizes of 3, 3, and 3 subjects should be enrolled to obtain final group sample sizes of 2, 2, and 2 subjects.

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References

Muller, K. E., and Barton, C. N. 1989. 'Approximate Power for Repeated-Measures ANOVA Lacking Sphericity.' Journal of the American Statistical Association, Volume 84, No. 406, pages 549-555.
Muller, K. E., LaVange, L.E., Ramey, S.L., and Ramey, C.T. 1992. 'Power Calculations for General Linear Multivariate Models Including Repeated Measures Applications.' Journal of the American Statistical Association, Volume 87, No. 420, pages 1209-1226.

This report gives the power for each value of *n* and *K*. It is useful when you want to compare the powers of the terms in the design at a specific sample size.

In this example, for *K* = 1, the design goal of 0.95 power is achieved for *n* = 8.

The definitions of each of the columns of the report are as follows:

Test

This column identifies the test statistic. Since the power depends on the test statistic, you should make sure that this is the test statistic that you will use in your analysis.

Power

This is the computed power for the term.

n

The value of *n* is the number of subjects per group.

N

The value of *N* is the total number of subjects in the study.

Multiply Means By

This is the value of the means multiplier, *K*.

Test Statistic

This is the value of the test statistic computed at the hypothesized values. The name of the statistic is identified in the Test column. Possible values are Wilks' lambda, Pillai-Bartlett trace, or Hotelling-Lawley trace. The actual formulas used were given earlier in the Technical Details section.

Approximate F Statistic

This is the value of the *F* statistic that is used to compute the probability levels. This value is calculated using the hypothesized values. The actual formulas used were given earlier in the Technical Details section.

DF1, DF2

These are the numerator and denominator degrees of freedom of the approximating *F* distribution.

Alpha

Alpha is the significance level of the test.

Beta

Beta is the probability of failing to reject the null hypothesis when the alternative hypothesis is true.

Means Matrix

Means Matrix Section

Name	A1	A2	A3
Y1	93	88	84
Y2	130	124	117

This report shows the means matrix that was read in. It may be used to get an impression of the magnitude of the difference among the means that is being studied. When a Means Multiplier, *K*, is used, each value of *K* is multiplied times each value of this matrix.

Variance-Covariance Matrix Section

Variance-Covariance Matrix Section

Name	Y1	Y2
Y1	16	14
Y2	14	25

This report shows the variance-covariance matrix that was read in from the spreadsheet or generated by the settings of on the Covariance tab. The standard deviations are given on the diagonal and the correlations are given off the diagonal.

Standard Deviations and Autocorrelations Section

Standard Deviations and Autocorrelations Section

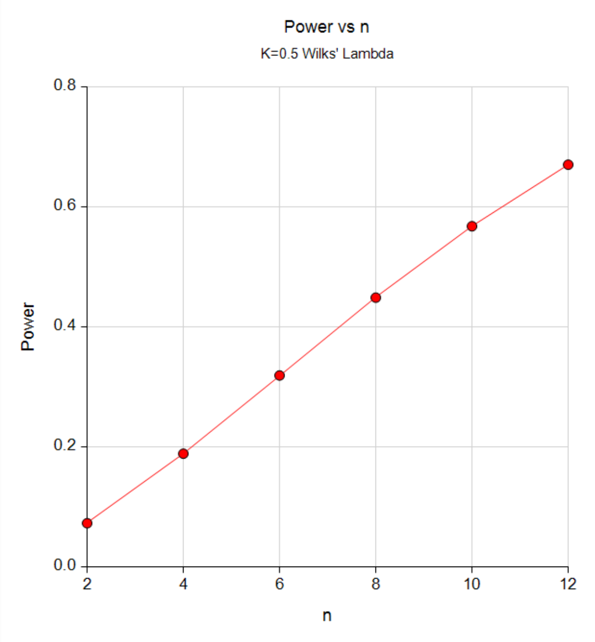
Response	Y1	Y2
Y1	4.0	0.7
Y2	0.7	5.0

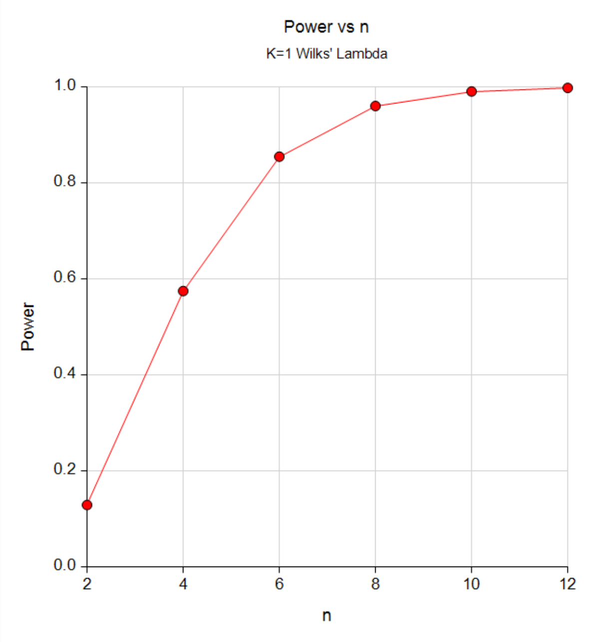
SD's on diagonal. Autocorrelations on off diagonal.

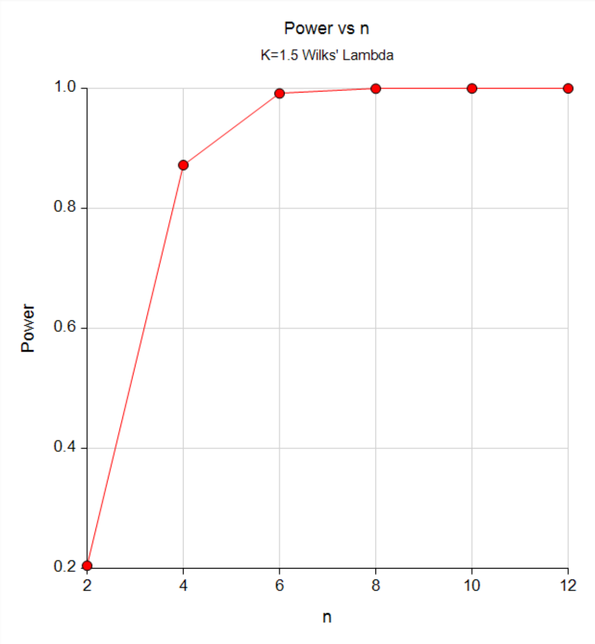
This report shows the variance-covariance matrix that was read in from the spreadsheet or generated by the settings of on the Covariance tab. The standard deviations are given on the diagonal and the correlations are given off the diagonal.

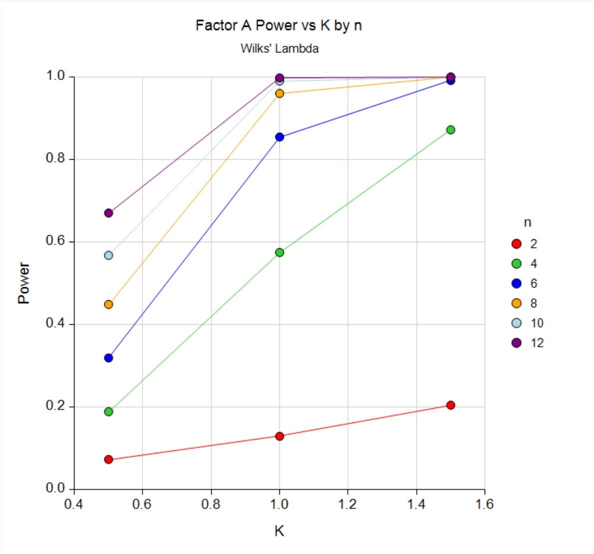
Plots Section

Plots

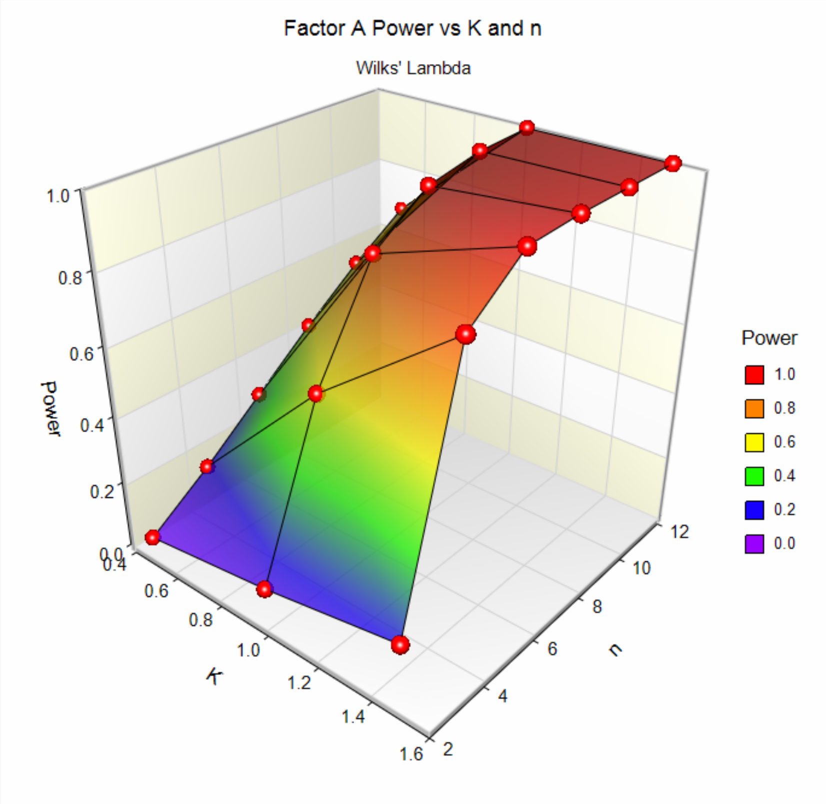








3D Plots



These charts show the relationship between power and *n* for each value of *K*. Note that high-order interactions may be omitted from the plot by reducing the Maximum Term-Order Plotted option on the Reports tab. Remember that *K* is the mean multiplier. It changes the effect size.

Example 2 – Validation

In this example, we will set p = 2, q = 3, alpha = 0.05, and n = 4. The mean and covariance matrices are

$$M = \begin{bmatrix} 1 & 1 \\ 2 & 1 \\ 3 & 2 \end{bmatrix} \text{ and } \Sigma = \begin{bmatrix} 4 & 1 \\ 1 & 4 \end{bmatrix}$$

The contrast matrix C is

$$C = \begin{bmatrix} \frac{-2}{\sqrt{6}} & \frac{1}{\sqrt{6}} & \frac{1}{\sqrt{6}} \\ 0 & \frac{-1}{\sqrt{2}} & \frac{1}{\sqrt{2}} \end{bmatrix}$$

The $X'X$ matrix is

$$X'X = \begin{bmatrix} 4 & 0 & 0 \\ 0 & 4 & 0 \\ 0 & 0 & 4 \end{bmatrix}$$

The matrix Θ is

$$\Theta = CM = \begin{bmatrix} \frac{3}{\sqrt{6}} & \frac{1}{\sqrt{6}} \\ \frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} \end{bmatrix}$$

The matrix H is

$$H = \left(\hat{\Theta} - \Theta_0\right)' [C(X'X)^- C']^{-1} \left(\hat{\Theta} - \Theta_0\right) = \begin{bmatrix} 8 & 4 \\ 4 & 8/3 \end{bmatrix}$$

The matrix E is

$$E = \hat{\Sigma} \cdot (N - r) = \begin{bmatrix} 36 & 9 \\ 9 & 36 \end{bmatrix}$$

The matrix T is

$$T = H + E = \begin{bmatrix} 44 & 13 \\ 13 & 38\frac{2}{3} \end{bmatrix}$$

Using these matrices, we can calculate the values of the test statistics. We will only calculate the results for Wilks' lambda. We have

$$W = \det(ET^{-1}) = 0.79290842$$

$$a = q - 1 = 2$$

$$g = \left(\frac{a^2p^2 - 4}{a^2 + p^2 - 5}\right)^{\frac{1}{2}} = \left(\frac{2^2 2^2 - 4}{2^2 + 2^2 - 5}\right)^{\frac{1}{2}} = 2$$

$$\eta = 1 - W^{1/g} = 1 - \sqrt{0.79290842} = 0.10954595$$

$$df1 = ap = 4$$

$$df2 = g[(N - r) - (p - a + 1)/2] - (ap - 2)/2$$

$$= 2[(12 - 3) - (2 - 2 + 1)/2] - (4 - 2)/2 = 16$$

$$F_{df_1,df_2} = \frac{\eta \;/\; df_1}{(1 - \eta) \;/\; df_2} = \frac{0.10954595 \;/\; 4}{(1 - 0.10954595) \;/\; 16} = 0.49209030$$

$$\lambda = df_1 F_{df_1,df_2} = 4(0.49209030) = 1.96836120$$

For an *F* with 4 and 16 degrees of freedom, the 5% critical value is 3.0069172799. Finally, compute the power using a noncentral *F* with 4 and 16 degrees of freedom and noncentrality parameter

$$Power = \Pr(f > F | df_1 = 4, df_2 = 16, \lambda = 1.96836120) = 0.1370631884$$

In order to run this example in **PASS**, the values of the means and the covariance matrix (given above) must be entered on a spreadsheet. The instructions below assume that the means are in columns C1-C3, while the covariance matrix is in columns C4-C5.

Setup

If the procedure window is not already open, use the PASS Home window to open it. The parameters for this example are listed below and are stored in the **Example 2** settings file. To load these settings to the procedure window, click **Open Example Settings File** in the Help Center or File menu.

Design: Means Tab

Solve For ....**Power**
Wilks' Lambda ....**Checked**
Pillai-Bartlett ....**Unchecked**
Hotelling-Lawley ....**Unchecked**
Alpha for All Terms **0.05**
Number of Response Variables **2**
Number of Factors **1**
Number of Levels in Factor A **3**
Columns Containing the Means....................**C1-C3**
K's (Multipliers) **1.0**
Group Allocation ....**Equal (n1 = n2 = ··· = n)**
n (Size Per Group)..**4**

Design: Variance Tab

Input Type..**Variance-Covariance Matrix in Spreadsheet**
Columns Containing Covariance Matrix.........**C4-C5**

Input Spreadsheet Data

Row	C1	C2	C3	C4	C5
1	1	2	3	4	1
2	1	1	2	1	4

Output

Click the Calculate button to perform the calculations and generate the following output.

Results for Factor A (Levels = 3)

Solve For: Power

		Number of Subjects					Degrees of Freedom		
Test	Power	Group n	Total N	Multiply Means By	Test Statistic	Approximate F Statistic	DF1	DF2	Alpha
Wilks' Lambda	0.1371	4	12	1	0.793	0.49	4	16	0.05

As you can see, the power computed here matches the results we computed manually.