

Chapter 456

Non-Inferiority Tests for the Ratio of Two Poisson Rates

Introduction

This procedure may be used to calculate power and sample size for non-inferiority tests involving the ratio of two Poisson rates. This procedure includes the option of accounting for over-dispersion.

The calculation details upon which this procedure is based are found in Zhu (2016). Some of the details are summarized below.

Technical Details

Definition of Terms

The following table presents the various terms that are used.

Group	1 (Control)	2 (Treatment)
Sample size	N_1	N_2
Individual event rates	λ_1	λ_2

Dispersion parameter: φ ($\varphi > 1$ implies over-dispersion; $\varphi < 1$ implies under-dispersion)

Average exposure time: μ_t

Non-inferiority ratio: R_0 ($R_0 < 1$ when higher rates are better; $R_0 > 1$ when higher rates are worse)

Sample size ratio: $\theta = N_2/N_1$

Non-Inferiority Tests for the Ratio of Two Poisson Rates

Hypotheses

When higher rates are better, the non-inferiority test hypotheses are

$$H_0: \frac{\lambda_2}{\lambda_1} \leq R_0 \quad \text{vs.} \quad H_1: \frac{\lambda_2}{\lambda_1} > R_0$$

where $R_0 < 1$.

When higher rates are worse, the non-inferiority test hypotheses are

$$H_0: \frac{\lambda_2}{\lambda_1} \geq R_0 \quad \text{vs.} \quad H_1: \frac{\lambda_2}{\lambda_1} < R_0$$

where $R_0 > 1$.

Sample Size and Power Calculations

Sample Size Calculation

Zhu (2016) bases the sample size calculations on a non-inferiority test derived from a Poisson regression model. The sample size calculation is

$$N_1 \geq \frac{(z_\alpha \sqrt{V_0} + z_\beta \sqrt{V_1})^2}{(\log R_0 - \log(\lambda_2/\lambda_1))^2}$$

$$N_2 = \theta N_1$$

where

$$V_1 = \frac{\varphi}{\mu_t} \left(\frac{1}{\lambda_1} + \frac{1}{\theta \lambda_2} \right)$$

and V_0 may be calculated in either of two ways.

V_0 Calculation Method 1 (using assumed true rates)

$$V_{01} = \frac{\varphi}{\mu_t} \left(\frac{1}{\lambda_1} + \frac{1}{\theta \lambda_2} \right)$$

Using Method 1, V_0 and V_1 are equal.

V_0 Calculation Method 2 (fixed marginal total or restricted maximum likelihood estimation)

$$V_{02} = \frac{\varphi(1 + R_0\theta)^2}{\mu_t R_0 \theta (\lambda_1 + \theta \lambda_2)}$$

Zhu (2016) did not give a recommendation regarding whether Method 1 or Method 2 should be used, except to say that “sample sizes calculated using Method 2 are slightly larger compared to those calculated by Method 1 for most simulated scenarios...”.

Power Calculation

The corresponding power calculation to the sample size calculation above is

$$Power \geq 1 - \Phi \left(\frac{\sqrt{N_1}(\log R_0 - \log(\lambda_2/\lambda_1)) - z_\alpha \sqrt{V_0}}{\sqrt{V_1}} \right)$$

Procedure Options

This section describes the options that are specific to this procedure. These are located on the Design tab. For more information about the options of other tabs, go to the Procedure Window chapter.

Design Tab

The Design tab contains the parameters associated with this test such as the Poisson rates, sample sizes, alpha, and power.

Solve For

Solve For

This option specifies the parameter to be solved for from the other parameters.

Test

Higher Poisson Rates Are

Specify whether higher Poisson rates are better or worse. This selection determines the direction of the null and alternative hypotheses. When higher rates are better, the non-inferiority test hypotheses are

$H_0: \lambda_2 / \lambda_1 \leq R_0$ vs. $H_1: \lambda_2 / \lambda_1 > R_0$, and $R_0 < 1$.

When higher rates are worse (lower are better), the non-inferiority test hypotheses are

$H_0: \lambda_2 / \lambda_1 \geq R_0$ vs. $H_1: \lambda_2 / \lambda_1 < R_0$, and $R_0 > 1$.

Variance Calculation Method

Select among the two methods for calculating the V0 variance component (see the documentation above for details).

- **Using Assumed True Rates**
For this choice, the variance component V0 is based the values entered for λ_1 and λ_2 .
 - **Fixed Marginal Total or REML**
This method assumes a fixed number of events. This method gives the same result as that derived using restricted maximum likelihood estimation.
-

Power and Alpha

Power

This option specifies one or more values for power. Power is the probability of rejecting a false null hypothesis, and is equal to one minus Beta. Beta is the probability of a type-II error, which occurs when a false null hypothesis is not rejected. In this procedure, a type-II error occurs when you fail to reject the null hypothesis of inferiority when in fact the treatment mean is non-inferior.

Values must be between zero and one. Historically, the value of 0.80 (Beta = 0.20) was used for power. Now, 0.90 (Beta = 0.10) is also commonly used.

A single value may be entered here or a range of values such as *0.8 to 0.95 by 0.05* may be entered.

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Alpha

This option specifies one or more values for the probability of a type-I error. A type-I error occurs when a true null hypothesis is rejected. In this procedure, a type-I error occurs when rejecting the null hypothesis of inferiority when in fact the treatment group is not inferior to the reference group.

Values must be between zero and one. Historically, the value of 0.05 has been used for alpha. This means that about one test in twenty will falsely reject the null hypothesis. You should pick a value for alpha that represents the risk of a type-I error you are willing to take in your experimental situation.

You may enter a range of values such as *0.01 0.05 0.10* or *0.01 to 0.10 by 0.01*.

$\mu(t)$ (Average Exposure Time)

$\mu(t)$ (Average Exposure Time)

Enter a value (or range of values) for the average exposure (observation) time for each subject in each group. A value of one is commonly entered when exposure times are all equal. The range is $\mu(t) > 0$. You can enter a single value such as 1 or a series of values such as *0.8 0.9 1* or *0.8 to 1.2 by 0.1*.

Sample Size (When Solving for Sample Size)

Group Allocation

Select the option that describes the constraints on $N1$ or $N2$ or both.

The options are

- **Equal ($N1 = N2$)**
This selection is used when you wish to have equal sample sizes in each group. Since you are solving for both sample sizes at once, no additional sample size parameters need to be entered.
- **Enter $N2$, solve for $N1$**
Select this option when you wish to fix $N2$ at some value (or values), and then solve only for $N1$. Please note that for some values of $N2$, there may not be a value of $N1$ that is large enough to obtain the desired power.
- **Enter $R = N2/N1$, solve for $N1$ and $N2$**
For this choice, you set a value for the ratio of $N2$ to $N1$, and then PASS determines the needed $N1$ and $N2$, with this ratio, to obtain the desired power. An equivalent representation of the ratio, R , is

$$N2 = R * N1.$$
- **Enter percentage in Group 1, solve for $N1$ and $N2$**
For this choice, you set a value for the percentage of the total sample size that is in Group 1, and then PASS determines the needed $N1$ and $N2$ with this percentage to obtain the desired power.

$N2$ (Sample Size, Group 2)

This option is displayed if Group Allocation = "Enter $N2$, solve for $N1$ "

$N2$ is the number of items or individuals sampled from the Group 2 population.

$N2$ must be ≥ 2 . You can enter a single value or a series of values.

R (Group Sample Size Ratio)

This option is displayed only if Group Allocation = "Enter $R = N2/N1$, solve for $N1$ and $N2$."

R is the ratio of $N2$ to $N1$. That is,

$$R = N2 / N1.$$

Non-Inferiority Tests for the Ratio of Two Poisson Rates

Use this value to fix the ratio of $N2$ to $N1$ while solving for $N1$ and $N2$. Only sample size combinations with this ratio are considered.

$N2$ is related to $N1$ by the formula:

$$N2 = [R \times N1],$$

where the value $[Y]$ is the next integer $\geq Y$.

For example, setting $R = 2.0$ results in a Group 2 sample size that is double the sample size in Group 1 (e.g., $N1 = 10$ and $N2 = 20$, or $N1 = 50$ and $N2 = 100$).

R must be greater than 0. If $R < 1$, then $N2$ will be less than $N1$; if $R > 1$, then $N2$ will be greater than $N1$. You can enter a single or a series of values.

Percent in Group 1

This option is displayed only if Group Allocation = "Enter percentage in Group 1, solve for $N1$ and $N2$."

Use this value to fix the percentage of the total sample size allocated to Group 1 while solving for $N1$ and $N2$. Only sample size combinations with this Group 1 percentage are considered. Small variations from the specified percentage may occur due to the discrete nature of sample sizes.

The Percent in Group 1 must be greater than 0 and less than 100. You can enter a single or a series of values.

Sample Size (When Not Solving for Sample Size)

Group Allocation

Select the option that describes how individuals in the study will be allocated to Group 1 and to Group 2.

The options are

- **Equal ($N1 = N2$)**
This selection is used when you wish to have equal sample sizes in each group. A single per group sample size will be entered.
- **Enter $N1$ and $N2$ individually**
This choice permits you to enter different values for $N1$ and $N2$.
- **Enter $N1$ and R , where $N2 = R * N1$**
Choose this option to specify a value (or values) for $N1$, and obtain $N2$ as a ratio (multiple) of $N1$.
- **Enter total sample size and percentage in Group 1**
Choose this option to specify a value (or values) for the total sample size (N), obtain $N1$ as a percentage of N , and then $N2$ as $N - N1$.

Sample Size Per Group

This option is displayed only if Group Allocation = "Equal ($N1 = N2$)."

The Sample Size Per Group is the number of items or individuals sampled from each of the Group 1 and Group 2 populations. Since the sample sizes are the same in each group, this value is the value for $N1$, and also the value for $N2$.

The Sample Size Per Group must be ≥ 2 . You can enter a single value or a series of values.

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N1 (Sample Size, Group 1)

*This option is displayed if Group Allocation = “Enter N1 and N2 individually” or “Enter N1 and R, where $N2 = R * N1$.”*

$N1$ is the number of items or individuals sampled from the Group 1 population.

$N1$ must be ≥ 2 . You can enter a single value or a series of values.

N2 (Sample Size, Group 2)

This option is displayed only if Group Allocation = “Enter N1 and N2 individually.”

$N2$ is the number of items or individuals sampled from the Group 2 population.

$N2$ must be ≥ 2 . You can enter a single value or a series of values.

R (Group Sample Size Ratio)

*This option is displayed only if Group Allocation = “Enter N1 and R, where $N2 = R * N1$.”*

R is the ratio of $N2$ to $N1$. That is,

$$R = N2/N1$$

Use this value to obtain $N2$ as a multiple (or proportion) of $N1$.

$N2$ is calculated from $N1$ using the formula:

$$N2 = [R \times N1],$$

where the value $[Y]$ is the next integer $\geq Y$.

For example, setting $R = 2.0$ results in a Group 2 sample size that is double the sample size in Group 1.

R must be greater than 0. If $R < 1$, then $N2$ will be less than $N1$; if $R > 1$, then $N2$ will be greater than $N1$. You can enter a single value or a series of values.

Total Sample Size (N)

This option is displayed only if Group Allocation = “Enter total sample size and percentage in Group 1.”

This is the total sample size, or the sum of the two group sample sizes. This value, along with the percentage of the total sample size in Group 1, implicitly defines $N1$ and $N2$.

The total sample size must be greater than one, but practically, must be greater than 3, since each group sample size needs to be at least 2.

You can enter a single value or a series of values.

Percent in Group 1

This option is displayed only if Group Allocation = “Enter total sample size and percentage in Group 1.”

This value fixes the percentage of the total sample size allocated to Group 1. Small variations from the specified percentage may occur due to the discrete nature of sample sizes.

The Percent in Group 1 must be greater than 0 and less than 100. You can enter a single value or a series of values.

Non-Inferiority Tests for the Ratio of Two Poisson Rates

Effect Size

R0 (Non-Inferiority Ratio)

Enter a value (or range of values) for the ratio of the two mean event rates assumed by the null hypothesis of non-inferiority. The range of possible values depends on the test direction. When higher rates are better, the non-inferiority test hypotheses are $H_0: \lambda_2 / \lambda_1 \leq R_0$ vs. $H_1: \lambda_2 / \lambda_1 > R_0$, and $R_0 < 1$. When higher rates are worse (lower are better), the non-inferiority test hypotheses are $H_0: \lambda_2 / \lambda_1 \geq R_0$ vs. $H_1: \lambda_2 / \lambda_1 < R_0$, and $R_0 > 1$.

λ_1 (Event Rate of Group 1)

Enter a value (or range of values) for the mean event rate per time unit in group 1 (control).

Example of Estimating λ_1

If 200 patients were exposed for 1 year (i.e. $t_1 = 1$ year) and 40 experienced the event of interest, then the mean event rate would be $\lambda_1 = 40/(200*1) = 0.2$ per patient-year. If 200 patients were exposed for 2 years (i.e. $t_1 = 2$ years) and 40 experienced the event of interest, then the mean event rate would be $\lambda_1 = 40/(200*2) = 0.1$ per patient-year.

λ_1 is used with λ_2 to calculate the event rate ratio as λ_2 / λ_1 . The range is $\lambda_1 > 0$. You can enter a single value such as 1 or a series of values such as 1 1.2 1.4 or 1 to 2 by 0.5.

Enter λ_2 or Ratio for Group 2

Indicate whether to enter the Group 2 event rate (λ_2) directly or the event rate ratio (λ_2 / λ_1) to specify λ_2 . The event rate ratio is calculated from λ_2 and λ_1 as λ_2 / λ_1 .

λ_2 (Event Rate of Group 2)

Enter a value (or range of values) for the mean event rate per time unit in group 2 (treatment).

Example of Estimating λ_2

If 200 patients were exposed for 1 year (i.e. $t_1 = 1$ year) and 40 experienced the event of interest, then the mean event rate would be $\lambda_2 = 40/(200*1) = 0.2$ per patient-year. If 200 patients were exposed for 2 years (i.e. $t_1 = 2$ years) and 40 experienced the event of interest, then the mean event rate would be $\lambda_2 = 40/(200*2) = 0.1$ per patient-year. λ_1 is used with λ_2 to calculate the event rate ratio as λ_2 / λ_1 . The range is $\lambda_2 > 0$. You can enter a single value such as 1 or a series of values such as 1 1.2 1.4 or 1 to 2 by 0.5.

λ_2 / λ_1 (Ratio of Event Rates)

This is the (assumed, known, true) value of the ratio of the two event rates, λ_1 and λ_2 , at which the power is to be calculated. The event rate ratio is calculated from λ_1 and λ_2 as λ_2 / λ_1 . The range is $\lambda_2 / \lambda_1 > 0$ and $\lambda_2 / \lambda_1 \neq R_0$. When higher rates are better, $\lambda_2 / \lambda_1 > R_0$ and $R_0 < 1$. When higher rates are worse (lower are better), $\lambda_2 / \lambda_1 < R_0$, and $R_0 > 1$. You can enter a single value such as 1 or a series of values such as 0.9 0.95 1 1.05 1.1 or 0.9 to 1.1 by 0.05.

ϕ (Dispersion)

Enter a value or series of values for the anticipated dispersion. Dispersion values lower than 1 indicate under-dispersion, which implies the variance is less than the mean. Dispersion values greater than 1 indicate over-dispersion, which implies the variance is greater than the mean. If over-dispersion or under-dispersion are not anticipated, enter a value of 1. You can enter a single value such as 1 or a series of values such as 0.9 0.95 1 1.05 1.1 or 0.9 to 1.1 by 0.05.

Non-Inferiority Tests for the Ratio of Two Poisson Rates

Example 1 – Calculating Sample Size

Researchers wish to determine whether the average Poisson rate of those receiving a new treatment is non-inferior to a current control. In the scenario, higher Poisson rates are worse than lower rates. The average exposure time for all subjects is 2.5 years. The event rate ratio at which the new treatment will be considered non-inferior is 1.2. The event rate of the control group is 2.2 events per year. The researchers would like to examine the effect on sample size of a range of treatment group event rates from 1.8 to 2.4. Over-dispersion is not anticipated.

The desired power is 0.9 and the significance level will be 0.025. The variance calculation method used will be the method where the assumed rates are used.

Setup

This section presents the values of each of the parameters needed to run this example. First load the **Non-Inferiority Tests for the Ratio of Two Poisson Rates** procedure window from the menus. You may then make the appropriate entries as listed below, or open **Example 1** by going to the **File** menu and choosing **Open Example Template**.

Option	Value
Design Tab	
Solve For	Sample Size
Higher Poisson Rates Are	Worse
Variance Calculation Method.....	Using Assumed True Rates
Power.....	0.90
Alpha.....	0.025
$\mu(t)$ (Average Exposure Time)	2.5
Group Allocation	Equal (N1 = N2)
R0 (Non-Inferiority Ratio).....	1.2
λ_1 (Event Rate of Group 1).....	2.2
Enter λ_2 or Ratio for Group 2	λ_2 (Event Rate of Group 2)
λ_2 (Event Rate of Group 2).....	1.8 to 2.4 by 0.1
ϕ (Dispersion)	1

Annotated Output

Click the Calculate button to perform the calculations and generate the following output.

Numeric Results

Numeric Results for Non-Inferiority Tests of the Ratio of Two Poisson Rates

Test Direction Assumption: Higher Poisson Rates Are Worse

H0: $\lambda_2 / \lambda_1 \geq R_0$ vs. H1: $\lambda_2 / \lambda_1 < R_0$

Variance Calculation Method: Using Assumed True Rates

Power	N1	N2	N	Average	Grp 1	Grp 2	Event	Non-Inf-	Disper-	Alpha
				Exposure	Cntrl	Trt				
				Time	Event	Event	Rate	Ratio	ϕ	
				$\mu(t)$	Rate	Rate	λ_2 / λ_1	R0		
					λ_1	λ_2				
0.90056	29	29	58	2.50	2.20	1.80	0.818	1.200	1.00	0.025
0.90649	39	39	78	2.50	2.20	1.90	0.864	1.200	1.00	0.025
0.90507	53	53	106	2.50	2.20	2.00	0.909	1.200	1.00	0.025
0.90114	75	75	150	2.50	2.20	2.10	0.955	1.200	1.00	0.025
0.90014	115	115	230	2.50	2.20	2.20	1.000	1.200	1.00	0.025
0.90051	197	197	394	2.50	2.20	2.30	1.045	1.200	1.00	0.025
0.90064	404	404	808	2.50	2.20	2.40	1.091	1.200	1.00	0.025

Non-Inferiority Tests for the Ratio of Two Poisson Rates

References

Zhu, H. 2016. 'Sample Size Calculation for Comparing Two Poisson or Negative Binomial Rates in Non-Inferiority or Equivalence Trials.' Statistics in Biopharmaceutical Research, Accepted Manuscript.

Report Definitions

Power is the probability of rejecting the null hypothesis when it is false.

N_1 and N_2 are the number of subjects in groups 1 and 2, respectively.

N is the total sample size. $N = N_1 + N_2$.

$\mu(t)$ is the average exposure (observation) time across subjects in both groups.

λ_1 is the event rate per time unit in Group 1 (control).

λ_2 is the event rate per time unit in Group 2 (treatment).

λ_2 / λ_1 is the (known, true, assumed) ratio of the two event rates.

R_0 is the non-inferiority (boundary) ratio.

Dispersion (ϕ) is the dispersion parameter ($\phi > 1$ implies over-dispersion, $\phi < 1$ implies under-dispersion).

Alpha is the probability of rejecting the null hypothesis when it is true.

Summary Statements

For a test of $H_0: \lambda_2 / \lambda_1 \geq 1.200$ vs. $H_1: \lambda_2 / \lambda_1 < 1.200$ (assuming higher Poisson rates are worse), and using the variance calculation method with assumed true rates, samples of 29 and 29 subjects with average exposure time 2.50 achieve 90.056% power to detect an event rate ratio λ_2 / λ_1 of 0.818 when the event rate in group 1 (λ_1) is 2.20, the event rate in group 2 (λ_2) is 1.80, the dispersion is 1.00, and the significance level (alpha) is 0.025.

Dropout-Inflated Sample Size

Dropout Rate	Sample Size			Dropout-Inflated Enrollment Sample Size			Expected Number of Dropouts		
	N1	N2	N	N1'	N2'	N'	D1	D2	D
20%	29	29	58	37	37	74	8	8	16
20%	39	39	78	49	49	98	10	10	20
20%	53	53	106	67	67	134	14	14	28
20%	75	75	150	94	94	188	19	19	38
20%	115	115	230	144	144	288	29	29	58
20%	197	197	394	247	247	494	50	50	100
20%	404	404	808	505	505	1010	101	101	202

Definitions

Dropout Rate (DR) is the percentage of subjects (or items) that are expected to be lost at random during the course of the study and for whom no response data will be collected (i.e. will be treated as "missing").

N_1 , N_2 , and N are the evaluable sample sizes at which power is computed. If N_1 and N_2 subjects are evaluated out of the N_1' and N_2' subjects that are enrolled in the study, the design will achieve the stated power.

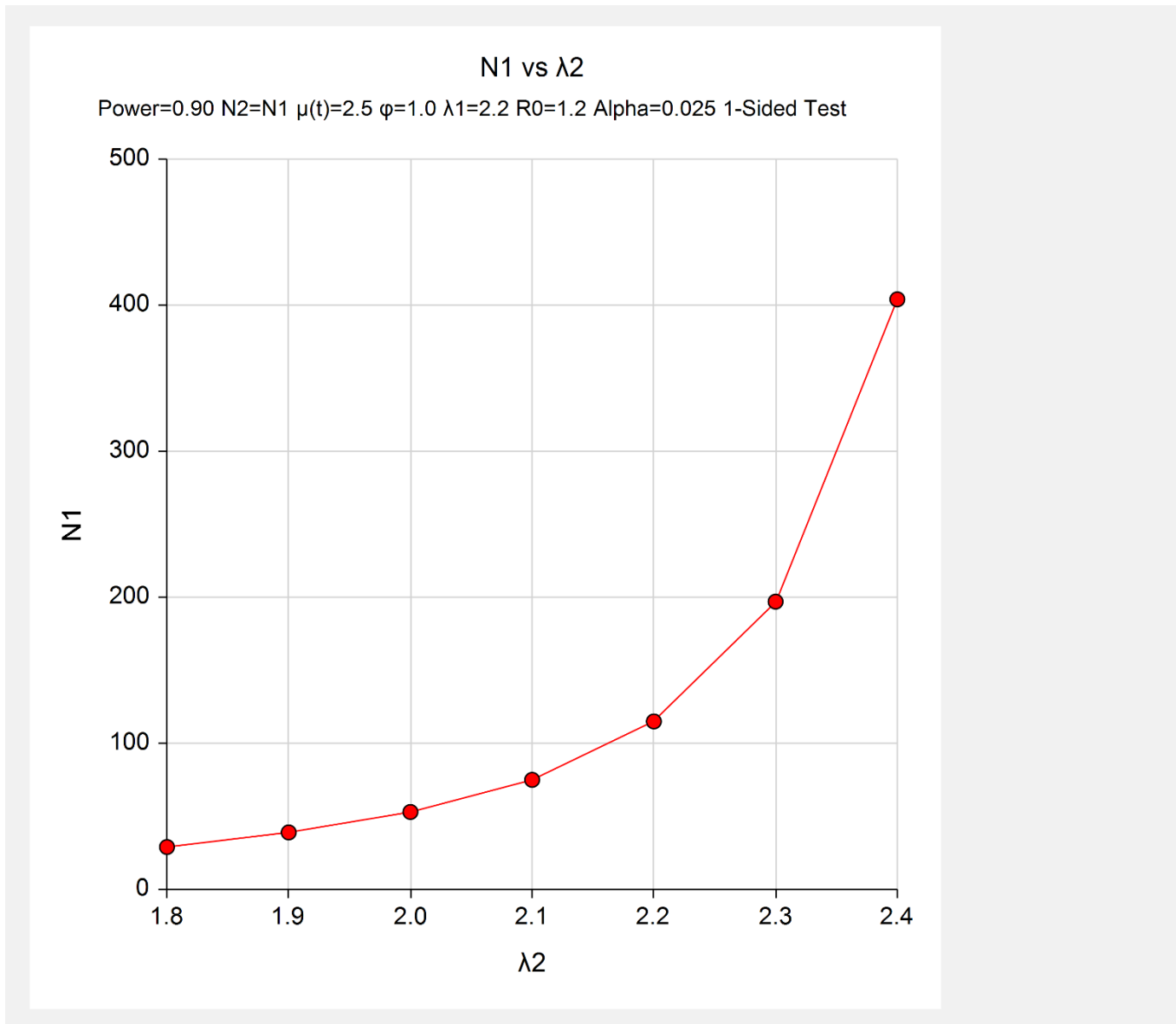
N_1' , N_2' , and N' are the number of subjects that should be enrolled in the study in order to end up with N_1 , N_2 , and N evaluable subjects, based on the assumed dropout rate. After solving for N_1 and N_2 , N_1' and N_2' are calculated by inflating N_1 and N_2 using the formulas $N_1' = N_1 / (1 - DR)$ and $N_2' = N_2 / (1 - DR)$, with N_1' and N_2' always rounded up. (See Julious, S.A. (2010) pages 52-53, or Chow, S.C., Shao, J., and Wang, H. (2008) pages 39-40.)

D_1 , D_2 , and D are the expected number of dropouts. $D_1 = N_1' - N_1$, $D_2 = N_2' - N_2$, and $D = D_1 + D_2$.

This report shows the sample sizes for the indicated scenarios.

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Plots Section



This plot represents the required sample sizes for various values of λ_2 .

Non-Inferiority Tests for the Ratio of Two Poisson Rates

Example 2 – Validation using Zhu (2016)

Zhu (2016) presents an example of solving for sample size where lower Poisson rates are better, the event rates are both 1.5, the (over-)dispersion is 1.35, the average duration is 0.85, the non-inferiority ratio is 1.1, the power is 0.9, and the Type I error rate is 0.025.

The calculated sample sizes are 2450 and 2453 per group for the Assumed True Rate and Fixed Marginal Total or REML variance calculation methods, respectively.

Setup

This section presents the values of each of the parameters needed to run this example. First load the **Non-Inferiority Tests for the Ratio of Two Poisson Rates** procedure window from the menus. You may then make the appropriate entries as listed below, or open **Example 2 (a or b)** by going to the **File** menu and choosing **Open Example Template**.

<u>Option</u>	<u>Value</u>
Design Tab	
Solve For	Sample Size
Higher Poisson Rates Are	Worse
Variance Calculation Method.....	Using Assumed True Rates (2 nd run: Fixed Marginal Total or REML)
Power.....	0.90
Alpha.....	0.025
$\mu(t)$ (Average Exposure Time)	0.85
Group Allocation	Equal (N1 = N2)
R0 (Non-Inferiority Ratio).....	1.1
λ_1 (Event Rate of Group 1).....	1.5
Enter λ_2 or Ratio for Group 2	λ_2 (Event Rate of Group 2)
λ_2 (Event Rate of Group 2).....	1.5
ϕ (Dispersion)	1.35

Output

Click the Calculate button to perform the calculations and generate the following output.

Numeric Results

Numeric Results for Non-Inferiority Tests of the Ratio of Two Poisson Rates											
Test Direction Assumption: Higher Poisson Rates Are Worse											
H0: $\lambda_2 / \lambda_1 \geq R_0$ vs. H1: $\lambda_2 / \lambda_1 < R_0$											
Variance Calculation Method: Using Assumed True Rates											
Power	N1	N2	N	Average Exposure Time $\mu(t)$	Grp 1 Cntrl Event Rate λ_1	Grp 2 Trt Event Rate λ_2	Event Rate Ratio λ_2 / λ_1	Non-Inferiority Ratio R0	Dispersion ϕ	Alpha	
0.90006	2450	2450	4900	0.85	1.50	1.50	1.000	1.100	1.35	0.025	
Variance Calculation Method: Fixed Marginal Total or REML											
0.90002	2453	2453	4906	0.85	1.50	1.50	1.000	1.100	1.35	0.025	

The sample sizes calculated in **PASS** match those of Zhu (2016) exactly.

Non-Inferiority Tests for the Ratio of Two Poisson Rates

Example 3 – Validation using Stucke and Kieser (2013)

Stucke and Kieser (2013) present a table of sample size calculations on page 211. The table assumes a power of 0.8, a Type I error rate of 0.025, an exposure time of 1, and no over- or under-dispersion.

The event rates, the sample size ratios, and the non-inferiority ratios are varied, giving the following sample sizes:

Event Rate	N1/N2	R0	N1	N2	N
0.1	1	2	327	327	654
0.1	2/3	2	409	273	682
0.1	3/2	2	273	409	682
0.2	1	2	164	164	328
0.2	2/3	2	205	137	342
0.2	3/2	2	137	205	342
0.6	1	3/2	160	160	320
0.6	2/3	3/2	199	133	332
0.6	3/2	3/2	133	199	332
1	1	3/2	96	96	192
1	2/3	3/2	80	120	200
1	3/2	3/2	120	80	200
3	1	3/2	32	32	64
3	2/3	3/2	40	27	67
3	3/2	3/2	27	40	682

Setup

This section presents the values of each of the parameters needed to run this example. First load the **Non-Inferiority Tests for the Ratio of Two Poisson Rates** procedure window from the menus. You may then make the appropriate entries as listed below, or open **Example 3 (a or b)** by going to the **File** menu and choosing **Open Example Template**.

<u>Option</u>	<u>Value</u>
Design Tab	
Solve For	Sample Size
Higher Poisson Rates Are	Worse
Variance Calculation Method.....	Using Assumed True Rates
Power.....	0.80
Alpha.....	0.025
$\mu(t)$ (Average Exposure Time)	1.0
Group Allocation	Enter R = N2/N1, solve for N1 and N2
R	0.666666667 1 1.5
R0 (Non-Inferiority Ratio).....	2 (for first 6 table entries); 1.5 (for last 9 table entries)
λ_1 (Event Rate of Group 1).....	0.1 0.2 (for first 6 table entries); 0.6 1 3 (for last 9 table entries)
Enter λ_2 or Ratio for Group 2	λ_2 / λ_1 (Ratio of Event Rates)
λ_2 (Event Rate of Group 2).....	1
ϕ (Dispersion).....	1.0

Non-Inferiority Tests for the Ratio of Two Poisson Rates

Output

Click the Calculate button to perform the calculations and generate the following output.

Numeric Results

Numeric Results for Non-Inferiority Tests of the Ratio of Two Poisson Rates

Test Direction Assumption: Higher Poisson Rates Are Worse

H0: $\lambda_2 / \lambda_1 \geq R_0$ vs. H1: $\lambda_2 / \lambda_1 < R_0$

Variance Calculation Method: Using Assumed True Rates

Power	N1	N2	N	R	Average Exposure Time $\mu(t)$	Grp 1 Cntrl Event Rate λ_1	Grp 2 Trt Event Rate λ_2	Event Rate Ratio λ_2 / λ_1	Non-Inferiority Ratio R_0	Dispersion ϕ	Alpha
0.80057	409	273	682	0.67	1.00	0.10	0.10	1.000	2.000	1.00	0.025
0.80152	205	137	342	0.67	1.00	0.20	0.20	1.000	2.000	1.00	0.025
0.80033	327	327	654	1.00	1.00	0.10	0.10	1.000	2.000	1.00	0.025
0.80152	164	164	328	1.00	1.00	0.20	0.20	1.000	2.000	1.00	0.025
0.80104	273	410	683	1.50	1.00	0.10	0.10	1.000	2.000	1.00	0.025
0.80247	137	206	343	1.50	1.00	0.20	0.20	1.000	2.000	1.00	0.025
[second calculation]											
0.80015	199	133	332	0.67	1.00	0.60	0.60	1.000	1.500	1.00	0.025
0.80211	120	80	200	0.67	1.00	1.00	1.00	1.000	1.500	1.00	0.025
0.80211	40	27	67	0.67	1.00	3.00	3.00	1.000	1.500	1.00	0.025
0.80211	160	160	320	1.00	1.00	0.60	0.60	1.000	1.500	1.00	0.025
0.80211	96	96	192	1.00	1.00	1.00	1.00	1.000	1.500	1.00	0.025
0.80211	32	32	64	1.00	1.00	3.00	3.00	1.000	1.500	1.00	0.025
0.80113	133	200	333	1.50	1.00	0.60	0.60	1.000	1.500	1.00	0.025
0.80211	80	120	200	1.50	1.00	1.00	1.00	1.000	1.500	1.00	0.025
0.80694	27	41	68	1.50	1.00	3.00	3.00	1.000	1.500	1.00	0.025

The sample sizes calculated in **PASS** match the table of Stucke and Kieser (2013).