Chapter 418

One-Sample T-Tests using Effect Size

Introduction

This procedure provides sample size and power calculations for a one- or two-sided one-sample t-test when the effect size is specified rather than the means and variance. The details of this procedure are given in Cohen (1988). In this design, a single population of independent, normally distributed data is sampled, and its mean is compared to a specified quantity by forming the difference scaled by the standard deviation.

Test Assumptions

When running a one-sample t-test, the basic assumptions are that the distribution is approximately normal, and the subjects are independent.

Test Procedure

If we assume that μ and μ_0 represent the population mean and the specified test value respectively and the (unknown) standard deviation is σ, then the effect size is represented by d where

$$d = \frac{\mu - \mu_0}{\sigma}$$

The null hypothesis is H_0: d = 0 and the alternative hypothesis depends on the number of "sides" of the test:

$$\text{Two-Sided:} \qquad H_1 : d \neq 0 \quad \text{or} \quad H_1 : \mu - \mu_0 \neq 0$$

$$\text{Upper One-Sided:} \qquad H_1 : d > 0 \quad \text{or} \quad H_1 : \mu - \mu_0 > 0$$

$$\text{Lower One-Sided:} \qquad H_1 : d < 0 \quad \text{or} \quad H_1 : \mu - \mu_0 < 0$$

A suitable Type I error probability (α) is chosen for the test, the data is collected, and a t-statistic is generated using the formula:

$$t = \frac{\bar{x} - \mu_0}{\sqrt{\dfrac{s^2}{N}}}$$

This t-statistic follows a t distribution with N - 1 degrees of freedom. The null hypothesis is rejected in favor of the alternative if,

for $H_1: d \neq 0$ or $H_1: \mu - \mu_0 \neq 0$

$$t < t_{\alpha/2} \text{ or } t > t_{1-\alpha/2},$$

for $H_1: d > 0$ or $H_1: \mu - \mu_0 > 0$

$$t > t_{1-\alpha},$$

or, for $H_1: d < 0$ or $H_1: \mu - \mu_0 < 0$

$$t < t_{\alpha}.$$

Comparing the t-statistic to the cut-off t-value (as shown here) is equivalent to comparing the p-value to α.

Power Calculation

The power is calculated using the same formulation as in the *Tests for One Mean* procedure with the modification that the σ used in that procedure is implicitly set equal to one.

The Effect Size

Suppose we assume that μ represents the mean of the population of interest and μ_0 represents a value to which the mean is to be compared. If the standard deviation is σ, the effect size is represented by d, where

$$d = \frac{\mu - \mu_0}{\sigma}$$

Cohen (1988) proposed the following interpretation of the d values. A d near 0.2 is a *small* effect, a d near 0.5 is a *medium* effect, and a d near 0.8 is a *large* effect. These values for small, medium, and large effects are popular in the social sciences. However, this convention is not as popular among the medical sciences since the scale of the effect is left unstated which makes interpretation difficult.

Example 1 – Finding the Sample Size

Researchers wish to compare a new anesthesia product to the value assumed to be typical of other products. Subjects in pain will receive the treatment and the subject's evaluation of pain intensity will be measured on a seven-point scale.

The researchers would like to determine the sample sizes required to detect a small, medium, and large effect size with a two-sided t-test when the power is 80% or 90% and the significance level is 0.05.

Setup

If the procedure window is not already open, use the PASS Home window to open it. The parameters for this example are listed below and are stored in the **Example 1** settings file. To load these settings to the procedure window, click **Open Example Settings File** in the Help Center or File menu.

Design Tab

Solve For ....**Sample Size**
Alternative Hypothesis ....**Two-Sided**
Power..**0.8 0.9**
Alpha..**0.05**
d...**0.2 0.5 0.8**

Output

Click the Calculate button to perform the calculations and generate the following output.

Numeric Reports

Numeric Results
───

Solve For: Sample Size
Test Type: One-Sample T-Test
Alternative Hypothesis: H1: d ≠ 0
───

| Power | | Sample Size | Effect Size | |
Target	Actual	N	d	Alpha
0.8	0.8017	199	0.2	0.05
0.9	0.9004	265	0.2	0.05
0.8	0.8078	34	0.5	0.05
0.9	0.9000	44	0.5	0.05
0.8	0.8213	15	0.8	0.05
0.9	0.9092	19	0.8	0.05

───
Target Power   The desired power. It may not be achieved because of integer N.
Actual Power   The achieved power. Because N is an integer, this value is often (slightly) larger than the target power.
N   The number of items sampled from the population.
d   Effect Size. Cohen recommended Low = 0.2, Medium = 0.5, and High = 0.8. d = (μ - μ0) / σ.
Alpha   The probability of rejecting a true null hypothesis.

Summary Statements

A single-group design will be used to test a mean against a null value. The comparison will be made using a two-sided, one-sample t-test, with a Type I error rate (α) of 0.05. To detect a population effect size (d = (μ - μ0) / σ) of 0.2 with 80% power, the number of needed subjects will be 199.

Dropout-Inflated Sample Size

Dropout Rate	Sample Size N	Dropout-Inflated Enrollment Sample Size N'	Expected Number of Dropouts D
20%	199	249	50
20%	265	332	67
20%	34	43	9
20%	44	55	11
20%	15	19	4
20%	19	24	5

Dropout Rate   The percentage of subjects (or items) that are expected to be lost at random during the course of the study and for whom no response data will be collected (i.e., will be treated as "missing"). Abbreviated as DR.

N   The evaluable sample size at which power is computed. If N subjects are evaluated out of the N' subjects that are enrolled in the study, the design will achieve the stated power.

N'   The total number of subjects that should be enrolled in the study in order to obtain N evaluable subjects, based on the assumed dropout rate. After solving for N, N' is calculated by inflating N using the formula N' = N / (1 - DR), with N' always rounded up. (See Julious, S.A. (2010) pages 52-53, or Chow, S.C., Shao, J., Wang, H., and Lokhnygina, Y. (2018) pages 32-33.)

D   The expected number of dropouts. D = N' - N.

Dropout Summary Statements

Anticipating a 20% dropout rate, 249 subjects should be enrolled to obtain a final sample size of 199 subjects.

References

Cohen, Jacob. 1988. Statistical Power Analysis for the Behavioral Sciences. Lawrence Erlbaum Associates. Hillsdale, New Jersey

Julious, S. A. 2010. Sample Sizes for Clinical Trials. Chapman & Hall/CRC. Boca Raton, FL.

Machin, D., Campbell, M., Tan, B. T., Tan, S. H. 2009. Sample Size Tables for Clinical Studies, 3rd Edition. Wiley-Blackwell.

Ryan, Thomas P. 2013. Sample Size Determination and Power. John Wiley & Sons. New Jersey.

This report shows the values of each of the parameters, one scenario per row.

Plots Section

Plots
_____
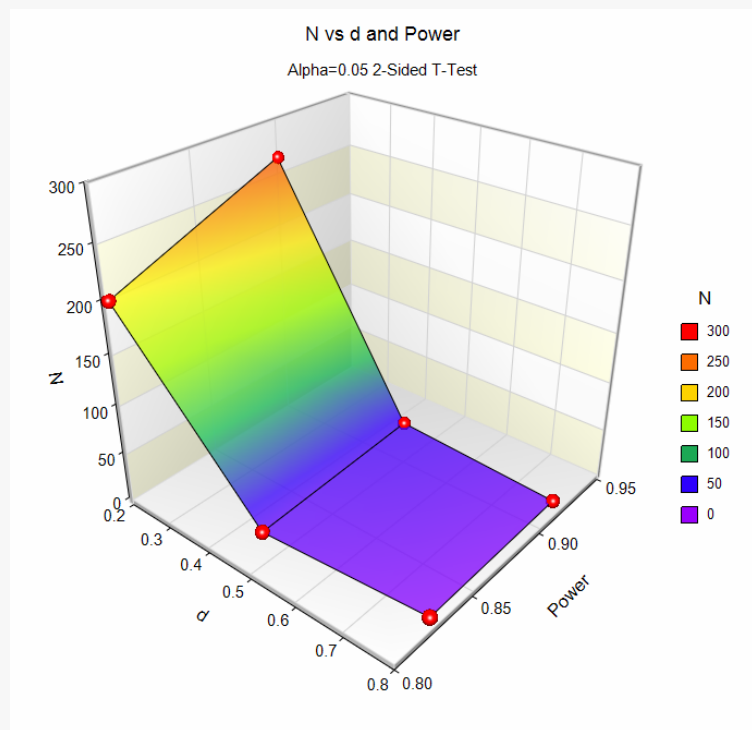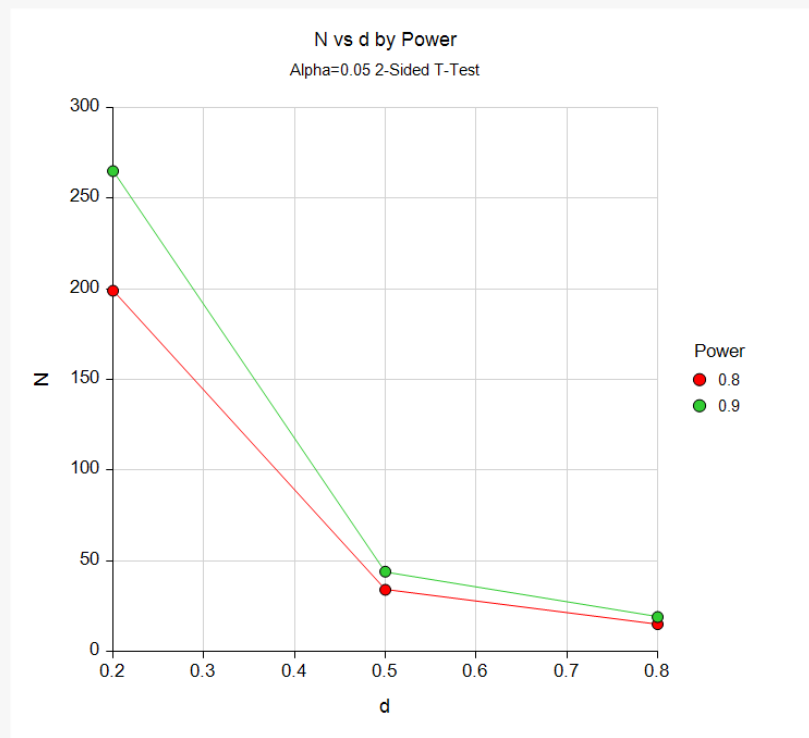
These plots show the relationship between effect size, power, and sample size.

Example 2 – Validation using Another Procedure

This procedure should give identical results to the **Tests for One Mean** procedure when the value of σ there is set to one. We will use this fact to provide a validation problem for this procedure.

If we run that routine with power = 0.90, alpha = 0.05, μ_1 = 0.5, μ_2 = 0, σ = 1, and solve for sample size. The result is N = 44.

Setup

If the procedure window is not already open, use the PASS Home window to open it. The parameters for this example are listed below and are stored in the **Example 2** settings file. To load these settings to the procedure window, click **Open Example Settings File** in the Help Center or File menu.

Design Tab

Solve For ....**Sample Size**
Alternative Hypothesis ....**Two-Sided**
Power...**0.9**
Alpha...**0.05**
d...**0.5**

Output

Click the Calculate button to perform the calculations and generate the following output.

Numeric Results
───
Solve For: Sample Size
Test Type: One-Sample T-Test
Alternative Hypothesis: H1: d ≠ 0
───

Power		Sample Size	Effect Size	
Target	Actual	N	d	Alpha
0.9	0.9	44	0.5	0.05

───

This procedure also calculated N = 44, thus the procedure is validated.