Chapter 547

One-Way Analysis of Variance F-Tests

Introduction

A common task in research is to compare the averages of two or more populations (groups). We might want to compare the income level of two regions, the nitrogen content of three lakes, or the effectiveness of four drugs. The one-way analysis of variance compares the means of two or more groups to determine if at least one mean is different from the others. The *F* test is used to determine statistical significance. *F* tests are non-directional in that the null hypothesis specifies that all means are equal and the alternative hypothesis simply states that at least one mean is different.

The methods described here are usually applied to a one-way experimental design. This design is an extension of the design used for the two-sample *t* test. Instead of two groups, there are three or more groups.

Assumptions

Using the *F* test requires certain assumptions. One reason for the popularity of the *F* test is its robustness in the face of assumption violation. However, if an assumption is not even approximately met, the significance levels and the power of the *F* test are invalidated. Unfortunately, in practice it often happens that several assumptions are not met. This makes matters even worse. Hence, steps should be taken to check the assumptions before important decisions are made.

The assumptions of the one-way analysis of variance are:

1. The data are continuous (not discrete).
2. The data follow the normal probability distribution. Each group is normally distributed about the group mean.
3. The variances within the groups are equal.
4. The groups are independent. There is no relationship among the individuals in one group as compared to another.
5. Each group is a simple random sample from its population. Each individual in the population has an equal probability of being selected in the sample.

Technical Details for the One-Way ANOVA

Suppose G groups each have a normal distribution and equal means($\mu_1 = \mu_2 = \cdots = \mu_G$). Let $N_1 = N_2 = \cdots = N_G$ denote the number of subjects in each group and let N denote the total sample size of all groups. Let $\bar{\mu}_w$ denote the weighted mean of all groups. That is

$$\bar{\mu}_w = \sum_{i=1}^{G} \left(\frac{N_i}{N}\right) \mu_i$$

Let σ denote the common standard deviation of all groups.

Given the above terminology, the ratio of the mean square between groups to the mean square within groups follows a central F distribution with two parameters matching the degrees of freedom of the numerator mean square and the denominator mean square. When the null hypothesis of mean equality is rejected, the above ratio has a noncentral F distribution which also depends on the noncentrality parameter, λ. This parameter is calculated as

$$\lambda = N\frac{\sigma_m^2}{\sigma^2}$$

where

$$\sigma_m^2 = \sum_{i=1}^{G} \left(\frac{N_i}{N}\right) (\mu_i - \bar{\mu}_w)^2$$

Some authors use the symbol ϕ for the noncentrality parameter. The relationship between the two noncentrality parameters is

$$\phi = \sqrt{\frac{\lambda}{G}}$$

The process of planning an experiment should include the following steps:

1. Determine an estimate of the within group standard deviation, σ. This may be done from prior studies, from experimentation with the Standard Deviation Estimation module, from pilot studies, or from crude estimates based on the range of the data. See the chapter on estimating the standard deviation for more details.
2. Determine a set of means that represent the group differences that you want to detect.
3. Determine the appropriate group sample sizes that will ensure desired levels of α and β. Although it is tempting to set all group sample sizes equal, it is easy to show that putting more subjects in some groups than in others may have better power than keeping group sizes equal (see Example 4).

Power Calculations for One-Way ANOVA

The calculation of the power of a particular test proceeds as follows:

1. Determine the critical value, $F_{G-1,N-G,\alpha}$ where α is the probability of a type-I error and G and N are defined above. Note that this is a two-tailed test as no direction is assigned in the alternative hypothesis.
2. From a hypothesized set of $\mu_i's$, calculate the noncentrality parameter λ based on the values of $N, G, \sigma_m,$ and σ.
3. Compute the power as the probability of being greater than $F_{G-1,N-G,\alpha}$ on a noncentral-F distribution with noncentrality parameter λ.

Procedure Options

This section describes the options that are specific to this procedure. These are located on the Design tab. For more information about the options of other tabs, go to the Procedure Window chapter.

Design Tab

The Design tab contains most of the parameters and options that you will be concerned with.

Solve For

Solve For

This option specifies the parameter to be solved for from the other parameters. The parameters that may be selected are Power, Sample Size, and Effect Size. Under most situations, you will select either *Power* for a power analysis or *Sample Size* for a sample size determination.

Power and Alpha

Power

This option specifies one or more values for power. Power is the probability of rejecting a false null hypothesis and is equal to one minus Beta. Beta is the probability of a type-II error, which occurs when a false null hypothesis is not rejected. In this procedure, a type-II error occurs when you fail to reject the null hypothesis of equal means when in fact the means are different.

Values must be between zero and one. Historically, the value of 0.80 (Beta = 0.20) was used for power. Now, 0.90 (Beta = 0.10) is also commonly used.

A single value may be entered here or a range of values such as *0.8 to 0.95 by 0.05* may be entered.

Alpha

This option specifies one or more values for the probability of a type-I error. A type-I error occurs when a true null hypothesis is rejected. In this procedure, a type-I error occurs when you reject the null hypothesis of equal means when in fact the means are equal.

Values must be between zero and one. Historically, the value of 0.05 has been used for alpha. This means that about one test in twenty will falsely reject the null hypothesis. You should pick a value for alpha that represents the risk of a type-I error you are willing to take in your experimental situation.

You may enter a range of values such as *0.01 0.05 0.10* or *0.01 to 0.10 by 0.01*.

Sample Size and Group Allocation

G (Number of Groups)

This is the number of groups (arms) whose means are being compared. The number of items used in the Group Allocation boxes is controlled by this number.

This value must be an integer greater than or equal to two.

Group Allocation Input Type (when Solve For = Power or Effect Size)

Specify how you want to enter the information about how the subjects are allocated to each of the G groups.

Possible options are:

- **Equal (N1 = N2 = ... = NG)**
  The sample size of all groups is Ni. Enter one or more values for the common group sample size.

- **Enter group multipliers**
  Enter a list of group multipliers (r1, r2, ..., rG) and one or more values of Ni. The individual group sample sizes are found by multiplying the multipliers by Ni. For example, N1 = r1 x Ni.

- **Enter N1, N2, ..., NG**
  Enter a list of group sample sizes, one for each group.

- **Enter columns of Ni's**
  Select one or more columns of the spreadsheet that each contain a set of group sample sizes going down the column. Each column is analyzed separately.

Ni (Subjects Per Group)

Enter *Ni*, the number of subjects in each group. The total sample size, *N*, is equal to *Ni* x G.

You can specify a single value or a list.

Single Value

Enter a value for the individual sample size of all groups. If you enter '10' here and there are five groups, then each group will be assigned 10 subjects and the total sample size will be 50.

List of Values

A separate power analysis is calculated for each value of Ni in the list. All analyses assume that the common, group sample size is Ni.

Range of Ni

$Ni > 1$

Group Multipliers (r1, r2, ..., rG)

Enter a set of G multipliers, one for each group.

The individual group sample sizes is computed as *Ng* = ceiling[*rg* x *Ni*], where ceiling[y] is the first integer greater than or equal to *y*. For example, the multipliers {1, 1, 2, 2.95} and *base Ni* of 10 would result in the sample sizes {10, 10, 20, 30}.

Incomplete List

If the number of items in the list is less than *G*, the missing multipliers are set equal to the last entry in the list.

Range

The items in the list must be positive. The resulting sample sizes must be at least 1.

Ni (Base Subjects Per Group)

Enter *Ni*, the base sample size of each group. The number of subjects in the group is found by multiplying this number by the corresponding group multiplier, {*r1*, *r2*, ..., *rG*}, and rounding up to the next integer.

You can specify a single value or a list.

Single Value

Enter a value for the base group subject count.

List of Values

A separate power analysis is calculated for each value of *Ni* in the list.

Range

Ceiling[*Ni* x *ri*] ≥ 1.

N1, N2, …, NG (List)

Enter a list of G subject counts, one for each group.

Incomplete List

If the number of items in the list is less than G, the missing subject counts are set equal to the last entry in the list.

Range

The items in the list must be positive. At least one item in the list must be greater than 1.

Columns of Ni's

Enter one or more spreadsheet columns containing vertical lists of group subject counts.

Press the Spreadsheet icon (directly to the right) to select the columns and then enter the values.

Press the Input Spreadsheet icon (to the right and slightly up) to view/edit the spreadsheet. Also note that you can obtain the spreadsheet by selecting "Tools", then "Input Spreadsheet", from the menus.

On the spreadsheet, the group subject counts are entered going down.

Examples (assuming G = 3)

C1	C2	C3
111	1	28
115	20	68
100	30	46

Definition of a Single Column

Each column gives one list. Each column results in a new scenario. The columns are not connected, but all should have exactly G rows.

Each entry in the list is the subject count of that group.

Incomplete List

If the number of items in the list is less than G, the missing entries are set equal to the last entry in the list.

Valid Entries

All values should be positive integers. At least one value must be greater than one.

Note

The column names (C1, C2, ...) can be changed by right-clicking on them in the spreadsheet.

Group Allocation Input Type (when Solve For = Sample Size)

Specify how you want to enter the information about how the subjects are allocated to each of the G groups.

Options

- **Equal (N1 = N2 = ... = NG)**
  All group subject counts are equal to Ni. The value of Ni will be found by conducting a search.

- **Enter group allocation pattern**
  Enter an allocation pattern (r1, r2, ..., rG). The pattern consists of a set of G numbers. These numbers will be rescaled into proportions by dividing each item by the sum of all items. The individual group subject counts are found by multiplying these proportions by N (the total subject count) and rounding up.

- **Enter columns of allocation patterns**
  Select one or more columns of the spreadsheet that each contain a group allocation pattern going down the column. Each column is analyzed separately.

Group Allocation Pattern (r1, r2, ..., rG)

Enter an allocation pattern (*r1, r2, ..., rG*). The pattern consists of a set of *G* numbers. These numbers will be rescaled into proportions of N by dividing each item by the sum of all items. The individual group subject counts are found by multiplying these proportions by *N* (the total subject count) and rounding up.

For example, the pattern {1, 3, 4} will be rescaled to {0.125, 0.375, 0.5}. The group subject counts will be constrained to these proportions (within rounding) during the search for the subject count configuration that meets the power requirement.

Incomplete List

If the number of items in the list is less than *G*, the missing numbers are set equal to the last entry in the list.

Range

The items in the list must be positive. The resulting subject counts must be at least 1.

Columns of Group Allocation Patterns

Enter one or more spreadsheet columns containing vertical lists of group allocation patterns.

Press the Spreadsheet icon (directly to the right) to select the columns and then enter the values.

Press the Input Spreadsheet icon (to the right and slightly up) to view/edit the spreadsheet. Also note that you can obtain the spreadsheet by selecting "Tools", then "Input Spreadsheet", from the menus.

On the spreadsheet, the group allocation patterns are entered going down.

Examples (assuming G = 3)

C1	C2	C3
1	1	3
1	2	3
3	2	1

Definition of a Single Column

Each column gives one allocation pattern. Each column results in a new scenario. The columns are not connected, but all should have exactly G rows.

Incomplete List

If the number of items in a list is less than G, the missing numbers are set equal to the last entry in the list before they are rescaled.

Valid Entries

All values should be positive numbers. You can enter decimal values.

Note

The column names (C1, C2, ...) can be changed by right-clicking on them in the spreadsheet.

Effect Size

μi's Input Type

Specify how you want to enter the G group means μ1, μ2, ..., μG assumed by the alternative hypothesis. The power is calculated for these values.

Note that under the null hypothesis, these means are all equal.

Options

- **μ1, μ2, ..., μG**
  Specify the values of the group means. The SD of these values is proportional to the effect size that you want to detect.
- **μ1, μ2, ..., μG and Multipliers**
  Specify the values of the group means as well as one or more multipliers for quickly generating sets of means.
- **Columns Containing Sets of μi's**
  Select one or more columns of the spreadsheet that each contain a set of μi's going down the column. Each column is analyzed separately.
- **σm (SD of μi's)**
  Enter one or more values of the standard deviation of the μi's.

  (The individual values of the μi's are not needed. Only their SD is used in the power calculations.)

μ1, μ2, ..., μG

Enter the values of the G group means under the alternative hypothesis. The effect size that the study will detect is a function of the differences among these values.

The mean for a particular group is the average response of all subjects in that group.

Range

Each μi should be numeric and at least one of the values must be different from the rest.

Example

10 10 10 40

Incomplete List

If the number of items in a list is less than G, the missing numbers are set equal to the last entry.

K (Means Multiplier)

Enter one or more values for K, the means multiplier. A separate power calculation is conducted for each value of K. In each analysis, all means (μi's) are multiplied by K. In this way, you can determine how sensitive the power values are to the magnitude of the means without the need to change them individually.

For example, if the original means are '0 1 2', setting this option to '1 2' results in two sets of means used in separate analyses: '0 1 2' in the first analysis and '0 2 4' in the second analysis.

Examples

1

0.5 1 1.5

0.8 to 1.2 by 0.1

Columns Containing Sets of μi's

Enter one or more spreadsheet columns containing vertical lists of μ1, μ2, ..., μG.

Press the Spreadsheet icon (directly to the right) to select the columns and then enter the values.

Press the Input Spreadsheet icon (to the right and slightly up) to view/edit the spreadsheet. Also note that you can obtain the spreadsheet by selecting "Tools", then "Input Spreadsheet", from the menus.

On the spreadsheet, the μi's are entered going down.

Examples (assuming G = 3)

C1	C2	C3
10	10	30
10	20	30
30	20	10

Definition of a Single Column

Each column gives one set of means. Each column results in a new scenario. The columns are not connected, but all should have exactly G rows.

Incomplete List

If the number of items in a list is less than G, the missing numbers are set equal to the last entry in the list.

Valid Entries

You can enter any numeric value.

Note

The column names (C1, C2, ...) can be changed by clicking on them in the spreadsheet.

σm (SD of μi's)

Enter one or more values of σm, the standard deviation of the group means. This value approximates the average size of the differences among the means that is to be detected. By detected, we mean that if σm is this large, the null hypothesis of mean equality will likely be rejected.

The value of σm is calculated using the formula shown earlier in this chapter.

Since this is a standard deviation, it must be greater than zero. It is in the same scale as σ, the within group standard deviaton.

A common measure of the effect size is σm/σ.

σ (Standard Deviation)

This is σ, the standard deviation between subjects within a group. It represents the variability from subject to subject that occurs when the subjects are treated identically. It is assumed to be the same for all groups. This value is approximated in an analysis of variance table by the square root of the mean square error.

Since they are positive square roots, the numbers must be strictly greater than zero. You can press the σ button to obtain further help on estimating the standard deviation.

Note that if you are using this procedure to test a factor (such as an interaction) from a more complex design, the value of standard deviation is estimated by the square root of the mean square of the term that is used as the denominator in the F test.

You can enter a single value such as '10' or a series of values such as '10 20 30 40 50' or '1 to 5 by 0.5'.

Example 1 – Finding power

An experiment is being designed to compare the means of four groups using an *F* test with a significance level of either 0.01 or 0.05. Previous studies have shown that the standard deviation is 18. Treatment means of 40, 10, 10, and 10 represent clinically important treatment differences. To better understand the relationship between power and sample size, the researcher wants to compute the power for several group sample sizes between 2 and 14. The sample sizes will be equal across all groups.

Setup

This section presents the values of each of the parameters needed to run this example. First, from the PASS Home window, load the procedure window. You may then make the appropriate entries as listed below, or open **Example 1** by going to the **File** menu and choosing **Open Example Template**.

Option	Value
Design Tab	
Solve For	**Power**
Alpha	**0.01 0.05**
G (Number of Groups)	**4**
Group Allocation Input Type	**Equal (N1 = N2 = ⋯ = NG)**
Ni (Subjects Per Group)	**2 4 6 8 10 12 14**
μi's Input Type	**μ1, μ2, ..., μG**
μ1, μ2, ..., μG	**40 10 10 10**
σ (Standard Deviation)	**18**

Annotated Output

Click the Calculate button to perform the calculations and generate the following output.

Numeric Results

Numeric Results
Number of Groups: 4

Power	Total Sample Size N	Subjects Per Group Ni	Group Means Set μi	SD of Group Means σm	Std Dev σ	Effect Size σm/σ	Alpha
0.0424	8	2	μi(1)	12.99	18.00	0.722	0.010
0.2389	16	4	μi(1)	12.99	18.00	0.722	0.010
0.5058	24	6	μi(1)	12.99	18.00	0.722	0.010
0.7269	32	8	μi(1)	12.99	18.00	0.722	0.010
0.8670	40	10	μi(1)	12.99	18.00	0.722	0.010
0.9414	48	12	μi(1)	12.99	18.00	0.722	0.010
0.9762	56	14	μi(1)	12.99	18.00	0.722	0.010
0.1751	8	2	μi(1)	12.99	18.00	0.722	0.050
0.5216	16	4	μi(1)	12.99	18.00	0.722	0.050
0.7733	24	6	μi(1)	12.99	18.00	0.722	0.050
0.9064	32	8	μi(1)	12.99	18.00	0.722	0.050
0.9651	40	10	μi(1)	12.99	18.00	0.722	0.050
0.9880	48	12	μi(1)	12.99	18.00	0.722	0.050
0.9961	56	14	μi(1)	12.99	18.00	0.722	0.050

Set(Set Number): Values
μi(1): 40.00, 10.00, 10.00, 10.00

References

Desu, M. M. and Raghavarao, D. 1990. Sample Size Methodology. Academic Press. New York.
Fleiss, Joseph L. 1986. The Design and Analysis of Clinical Experiments. John Wiley & Sons. New York.
Kirk, Roger E. 1982. Experimental Design: Procedures for the Behavioral Sciences. Brooks/Cole. Pacific Grove, California.

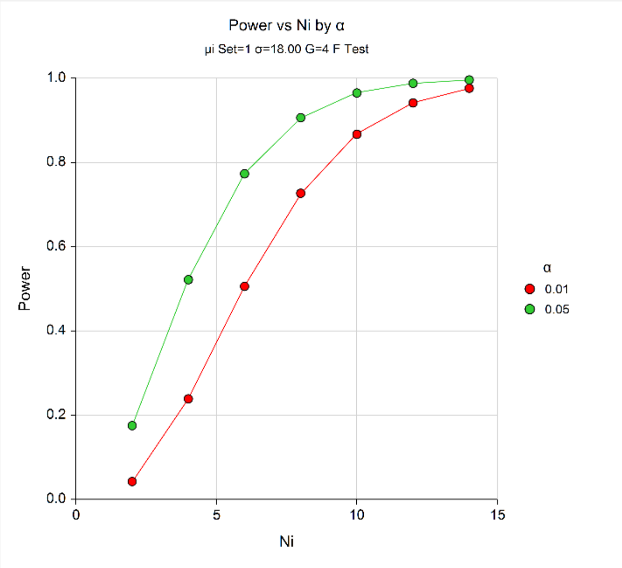
Report Definitions
Power is the probability of rejecting a false null hypothesis.
Total Sample Size N is the total number of subjects in the study.
Subjects Per Group Ni is the number of subjects per group.
Group Means Set μi gives the name and number of the set containing the mean responses for each group.
SD of Group Means σm is the population standard deviation of the group means.
Std Dev σ is the common standard deviation of the responses within a group.
Effect Size σm/σ is a measure of the effect size. It is the ratio of σm and σ.
Alpha is the significance level of the test: the probability of rejecting the null hypothesis of equal means when it is true.

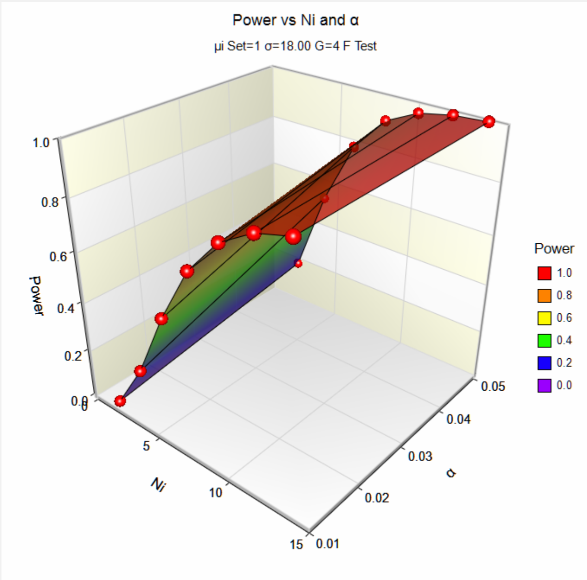
Summary Statements

In a one-way ANOVA study, a sample of 8 subjects, divided among 4 groups, achieves a power of 0.0424. This power assumes an F test is used with a significance level of 0.010. The group subject counts are 2, 2, 2, 2. The group means under the alternative hypothesis are 40.00, 10.00, 10.00, 10.00. The standard deviation of these means is 12.99. The common standard deviation of the responses is 18.00. The effect size is 0.722.

This report shows the numeric results of this power study.

Plots Section





These plots give a visual presentation to the results in the Numeric Report. We can quickly see the impact on the power of increasing the sample size and the increase in the significance level.

When you create one of these plots, it is important to use trial and error to find an appropriate range for the horizontal variable so that you have results with both low and high power.

Example 2 – Power after a study

This example will cover the situation in which you are calculating the power of a one-way analysis of variance *F* test on data that have already been collected and analyzed.

An experiment included a control group and two treatment groups. Each group had seven individuals. A single response was measured for each individual and recorded in the following table.

Control	T1	T2
452	646	685
674	547	658
554	774	786
447	465	536
356	759	653
654	665	669
558	767	557

When analyzed using the one-way analysis of variance procedure in NCSS, the following results were obtained.

Analysis of Variance Table

Source Term	DF	Sum of Squares	Mean Square	F-Ratio	Prob Level
A (...)	2	75629.8	37814.9	3.28	0.061167
S(A)	18	207743.4	11541.3		
Total (Adjusted)	20	283373.3			
Total	21				

Means Section

Group	Count	Mean
Control	7	527.8571
T1	7	660.4286
T2	7	649.1429

The significance level (Prob Level) was 0.061—not enough for statistical significance. The researcher had hoped to show that the treatment groups had higher response levels than the control group. He could see that the group means followed this pattern since the mean for *T1* was about 25% higher than the control mean and the mean for *T2* was about 23% higher than the control mean. He decided to calculate the power of the experiment using these values of the means. (We do not recommend this approach because the power should be calculated for the minimum difference among the means that is of interest, not at the values of the sample means.)

The data entry for this problem is simple. The only entry that is not straight forward is finding an appropriate value for the standard deviation. Since the standard deviation is estimated by the square root of the mean square error, it is calculated as $\sqrt{11541.3} = 107.4304$.

Setup

This section presents the values of each of the parameters needed to run this example. First, from the PASS Home window, load the procedure window. You may then make the appropriate entries as listed below, or open **Example 2** by going to the **File** menu and choosing **Open Example Template**.

Option	Value
Design Tab	
Solve For	**Power**
Alpha	**0.05**
G (Number of Groups)	**3**
Group Allocation Input Type	**Equal (N1 = N2 = ⋯ = NG)**
Ni (Subjects Per Group)	**7**
μi's Input Type	**μ1, μ2, ..., μG**
μ1, μ2, ..., μG	**527.8571 660.4286 649.1429**
σ (Standard Deviation)	**107.4304**

Output

Click the Calculate button to perform the calculations and generate the following output.

Numeric Results

Numeric Results
Number of Groups: 3

Power	Total Sample Size N	Subjects Per Group Ni	Group Means Set μi	SD of Group Means σm	Std Dev σ	Effect Size σm/σ	Alpha
0.5479	21	7	μi(1)	60.01	107.43	0.559	0.050

The power is only 0.55. That is, there was only a 55% chance of rejecting a false null hypothesis. It is important to understand this power statement is conditional, so we will state it in detail. Given that the population means are equal to the sample means (that σm is 60.01) and the population standard deviation is equal to 107.43, the probability of rejecting the false null hypothesis is 0.55. If the population means are different from the sample means (which they must be), the power is different.

Example 3 – Finding the sample size necessary to reject

Continuing with the last example, we will determine how large the sample size would need to have been for alpha = 0.05 and power = 0.80.

Setup

This section presents the values of each of the parameters needed to run this example. First, from the PASS Home window, load the procedure window. You may then make the appropriate entries as listed below, or open **Example 3** by going to the **File** menu and choosing **Open Example Template**.

Option	Value
Design Tab	
Solve For	**Sample Size**
Power	**0.80**
Alpha	**0.05**
G (Number of Groups)	**3**
Group Allocation Input Type	**Equal (N1 = N2 = ··· = NG)**
μi's Input Type	**μ1, μ2, ..., μG**
μ1, μ2, ..., μG	**527.8571 660.4286 649.1429**
σ (Standard Deviation)	**107.4304**

Output

Click the Calculate button to perform the calculations and generate the following output.

Numeric Results

Numeric Results
Number of Groups: 3

Power	Total Sample Size N	Group Alloc Set ri Set	Group Means Set μi	SD of Group Means σm	Std Dev σ	Effect Size σm/σ	Alpha
0.8251	36	ri(1)	μi(1)	60.01	107.43	0.559	0.050

Set(Set Number): Values
ri(1): 0.333, 0.333, 0.333
μi(1): 527.86, 660.43, 649.14

The required sample size is 36 which is 12 per group.

Example 4 – Power using unequal sample sizes

Continuing with the last example, consider the impact of allowing the group sample sizes to be unequal. Since the control group is being compared to two treatment groups, the mean of the control group is assumed to be different from those of the treatment groups. In this situation, adding extra subjects to the control group can increase power because the value of σm is increased.

In this example, we will evaluate to designs, both of which have a total of 33 subjects. In the first design, all three groups are assigned 11 subjects. In the second design, the sample size of the control group is set to 15 and to 9 in each of the treatment groups.

In order to carry out this comparison, we will enter the two sets of sample sizes in the spreadsheet. The resulting spreadsheet will appear as

C1	C2
11	15
11	9
11	9

Setup

This section presents the values of each of the parameters needed to run this example. First, from the PASS Home window, load the procedure window. You may then make the appropriate entries as listed below, or open **Example 4** by going to the **File** menu and choosing **Open Example Template**.

Option	Value
Design Tab	
Solve For	**Power**
Alpha	**0.05**
G (Number of Groups)	**3**
Group Allocation Input Type	**Enter columns of Ni's**
Columns of Ni's	**C1 C2**
μi's Input Type	**μ1, μ2, ..., μG**
μ1, μ2, ..., μG	**527.8571 660.4286 649.1429**
σ (Standard Deviation)	**107.4304**

Output

Click the Calculate button to perform the calculations and generate the following output.

Numeric Results

Numeric Results
Number of Groups: 3

Power	Total Sample Size N	Group Sample Size Set Ni Set	Group Means Set μi	SD of Group Means σm	Std Dev σ	Effect Size σm/σ	Alpha
0.7851	33	C1(1)	μi(1)	60.01	107.43	0.559	0.050
0.8297	33	C2(2)	μi(1)	63.34	107.43	0.590	0.050

Set(Set Number): Values
C1(1): 11, 11, 11
C2(2): 15, 9, 9
μi(1): 527.86, 660.43, 649.14

The power of 0.8297 achieved with the second design is slightly higher than the power of 0.7851 that was achieved by the first design. As was mentioned above, this occurs because the value of σm was increased with the second design.

Example 5 – Minimum detectable difference

It may be useful to determine the minimum detectable difference among the means that can be found at the experimental conditions. This amounts to finding *σm*.

Continuing with the previous example, find *σm* for a range of sample sizes from 15 to 240 when alpha is 0.05 and power is 0.80 or 0.90.

Setup

This section presents the values of each of the parameters needed to run this example. First, from the PASS Home window, load the procedure window. You may then make the appropriate entries as listed below, or open **Example 5** by going to the **File** menu and choosing **Open Example Template**.

Option	Value
Design Tab	
Solve For	**Effect Size**
Power	**0.8 0.9**
Alpha	**0.05**
G (Number of Groups)	**3**
Group Allocation Input Type	**Equal (N1 = N2 = ··· = NG)**
Ni (Subjects Per Group)	**5 10 15 20 40 60 80**
σ (Standard Deviation)	**107.4304**

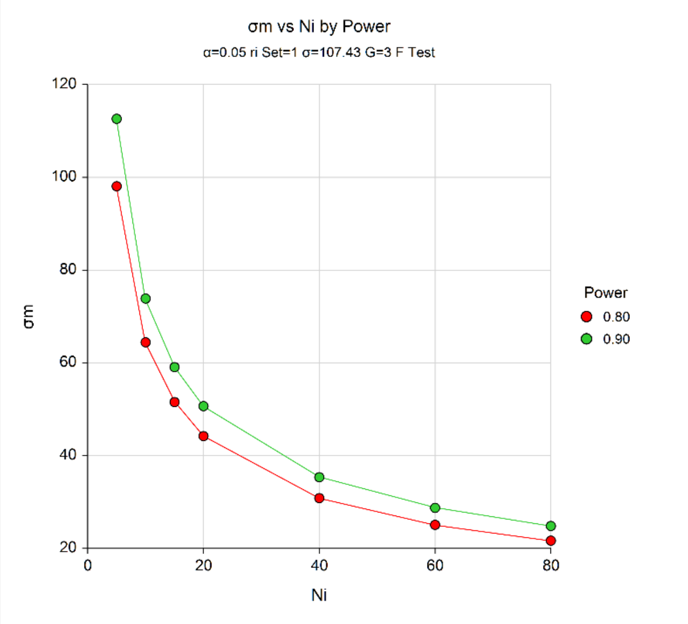
Output

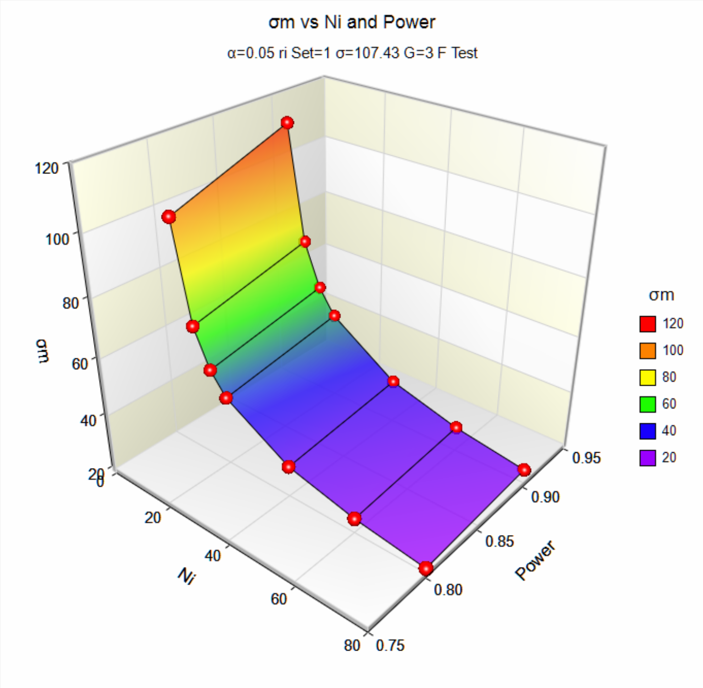
Click the Calculate button to perform the calculations and generate the following output.

Numeric Results and Plots

Numeric Results
Number of Groups: 3

Power	Total Sample Size N	Subjects Per Group Ni	SD of Group Means σm	Std Dev σ	Effect Size σm/σ	Alpha
0.8000	15	5	98.08	107.43	0.913	0.050
0.8000	30	10	64.42	107.43	0.600	0.050
0.8000	45	15	51.54	107.43	0.480	0.050
0.8000	60	20	44.21	107.43	0.411	0.050
0.8000	120	40	30.83	107.43	0.287	0.050
0.8000	180	60	25.07	107.43	0.233	0.050
0.8000	240	80	21.66	107.43	0.202	0.050
0.9000	15	5	112.62	107.43	1.048	0.050
0.9000	30	10	73.86	107.43	0.688	0.050
0.9000	45	15	59.07	107.43	0.550	0.050
0.9000	60	20	50.67	107.43	0.472	0.050
0.9000	120	40	35.34	107.43	0.329	0.050
0.9000	180	60	28.73	107.43	0.267	0.050
0.9000	240	80	24.82	107.43	0.231	0.050





These plots show the relationships between power, sample size, and effect size. Several conclusions are possible, but the most impressive is the sharp elbow in the curve that occurs near Ni = 10 when *σm* is about 60.

How do you to interpret a *σm* of 60? One way is to find a set of means that have a standard deviation of 60. To do this, press the σ button in the lower right corner of the panel to load the Standard Deviation Estimator module. Under the *Data* tab, enter the three values 0, 0, and 64 as a starting point. These values have a standard deviation of 30. Doubling the 64 to 128 results in an SD of 60. The difference between the minimum and the maximum of these three values is 128. Hence the minimum detectable difference is about 128 for a *σm* of 60 when Ni is 10 and the power is 80%.

Example 6 – Validation using Fleiss (1986)

Fleiss (1986) page 374 presents an example of determining a sample size in an experiment with 4 equal sized groups; means of 9.775, 12, 12, and 14.225; standard deviation of 3; alpha of 0.05, and beta of 0.20. He finds a sample size of 11 per group which amounts to a total sample size of 44.

Setup

This section presents the values of each of the parameters needed to run this example. First, from the PASS Home window, load the procedure window. You may then make the appropriate entries as listed below, or open **Example 6** by going to the **File** menu and choosing **Open Example Template**.

Option	Value
Design Tab	
Solve For	**Sample Size**
Power	**0.8**
Alpha	**0.05**
G (Number of Groups)	**4**
Group Allocation Input Type	**Equal (N1 = N2 = ⋯ = NG)**
μi's Input Type	**μ1, μ2, ..., μG**
μ1, μ2, ..., μG	**9.775 12 12 14.225**
σ (Standard Deviation)	**3**

Output

Click the Calculate button to perform the calculations and generate the following output.

Numeric Results

Numeric Results
Number of Groups: 4

Power	Total Sample Size N	Group Alloc Set ri Set	Group Means Set μi	SD of Group Means σm	Std Dev σ	Effect Size σm/σ	Alpha
0.8027	44	ri(1)	μi(1)	1.57	3.00	0.524	0.050

Set(Set Number): Values
ri(1): 0.250, 0. 250, 0. 250, 0.250
μi(1): 9.78, 12.00, 12.00, 14.23

PASS also found N = 44. Note that Fleiss used calculations based on a normal approximation, but **PASS** uses exact calculations based on the non-central *F* distribution.

Example 7 – Validation using Desu (1990)

Desu (1990) page 48 presents an example of determining a sample size in an experiment with 3 groups; means of 0, -0.2553, and 0.2553; standard deviation of 1; alpha of 0.05, and beta of 0.10. He finds a sample size of 99 per group for a total of 297.

Setup

This section presents the values of each of the parameters needed to run this example. First, from the PASS Home window, load the procedure. You may then make the appropriate entries as listed below, or open **Example 7** by going to the **File** menu and choosing **Open Example Template**.

Option	Value
Design Tab	
Solve For	**Sample Size**
Power	**0.9**
Alpha	**0.05**
G (Number of Groups)	**3**
Group Allocation Input Type	**Equal (N1 = N2 = ··· = NG)**
μi's Input Type	**μ1, μ2, ..., μG**
μ1, μ2, ..., μG	**0 -.2553 0.2553**
σ (Standard Deviation)	**1**

Output

Click the Run button to perform the calculations and generate the following output.

Numeric Results

Numeric Results
Number of Groups: 3

Power	Total Sample Size N	Group Alloc Set ri Set	Group Means Set μi	SD of Group Means σm	Std Dev σ	Effect Size σm/σ	Alpha
0.9028	297	ri(1)	μi(1)	0.21	1.00	0.208	0.050

Set(Set Number): Values
ri(1): 0.333, 0.333, 0.333
μi(1): 0.00, -0.26, 0.26

PASS also found N = 297.

Example 8 – Validation using Kirk (1982)

Kirk (1982) pages 140-144 presents an example of determining a sample size in an experiment with 4 groups; means of 2.75, 3.50, 6.25, and 9.0; standard deviation of 1.20995; alpha of 0.05, and beta of 0.05. He finds a sample size of 3 per group for a total sample size of 12.

Setup

This section presents the values of each of the parameters needed to run this example. First, from the PASS Home window, load the procedure. You may then make the appropriate entries as listed below, or open **Example 8** by going to the **File** menu and choosing **Open Example Template**.

Option	Value
Design Tab	
Solve For	**Sample Size**
Power	**0.95**
Alpha	**0.05**
G (Number of Groups)	**4**
Group Allocation Input Type	**Equal (N1 = N2 = ··· = NG)**
μi's Input Type	**μ1, μ2, ..., μG**
μ1, μ2, ..., μG	**2.75 3.5 6.25 9**
σ (Standard Deviation)	**1.20995**

Output

Click the Calculate button to perform the calculations and generate the following output.

Numeric Results

Numeric Results
Number of Groups: 3

Power	Total Sample Size N	Group Alloc Set ri Set	Group Means Set μi	SD of Group Means σm	Std Dev σ	Effect Size σm/σ	Alpha
0.9977	12	ri(1)	μi(1)	2.47	1.21	2.038	0.050

Set(Set Number): Values
ri(1): 0.250, 0.250, 0.250, 0.250
μi(1): 2.75, 3.50, 6.25, 9.00

PASS also found N = 12.