

Chapter 597

One-Way Analysis of Variance F-Tests using Effect Size

Introduction

A common task in research is to compare the averages of two or more populations (groups). We might want to compare the income level of two regions, the nitrogen content of three lakes, or the effectiveness of four drugs. The one-way analysis of variance compares the means of two or more groups to determine if at least one mean is different from the others. The F test is used to determine statistical significance. F tests are non-directional in that the null hypothesis specifies that all means are equal, and the alternative hypothesis simply states that at least one mean is different from the rest.

The methods described here are usually applied to the one-way experimental design. This design is an extension of the design used for the two-sample t test. Instead of two groups, there are three or more groups.

In our more advanced one-way ANOVA procedure, you are required to enter hypothesized means and variances. This simplified procedure only requires the input of an *effect size*, usually f , as proposed by Cohen (1988).

Assumptions

Using the F test requires certain assumptions. One reason for the popularity of the F test is its robustness in the face of assumption violation. However, if an assumption is not even approximately met, the significance levels and the power of the F test are invalidated. Unfortunately, in practice it often happens that several assumptions are not met. This makes matters even worse. Hence, steps should be taken to check the assumptions before important decisions are made.

The assumptions of the one-way analysis of variance are:

1. The data are continuous (not discrete).
2. The data follow the normal probability distribution. Each group is normally distributed about the group mean.
3. The variances within the groups are equal.
4. The groups are independent. There is no relationship among the individuals in one group as compared to another.
5. Each group is a simple random sample from its population. Each individual in the population has an equal probability of being selected in the sample.

Technical Details for the One-Way ANOVA

Suppose G groups each have a normal distribution and equal means ($\mu_1 = \mu_2 = \dots = \mu_G$). Let $n_1 = \dots = n_G$ denote the number of subjects in each group and let N denote the total sample size of all groups. Let $\bar{\mu}_w$ denote the weighted mean of all groups. That is

$$\bar{\mu}_w = \sum_{i=1}^G \left(\frac{n_i}{N}\right) \mu_i$$

Let σ denote the common standard deviation of all groups.

Given the above terminology, the ratio of the mean square between groups to the mean square within groups follows a central F distribution with two parameters matching the degrees of freedom of the numerator mean square and the denominator mean square. When the null hypothesis of mean equality is rejected, the above ratio has a noncentral F distribution which also depends on the noncentrality parameter, λ . This parameter is calculated as

$$\lambda = N \frac{\sigma_m^2}{\sigma^2}$$

where

$$\sigma_m^2 = \sum_{i=1}^G \left(\frac{n_i}{N}\right) (\mu_i - \bar{\mu}_w)^2$$

Cohen (1988) proposed a further substitution of an *effect size* parameter f which is defined as

$$f = \sqrt{\frac{\sigma_m^2}{\sigma^2}}$$

Using this parameter, the noncentrality parameter simplifies to

$$\lambda = N f^2$$

Cohen (1988, 285-287) proposed the following interpretation of f : **$f = 0.1$** is a **small** effect, **$f = 0.25$** is a **medium** effect, and **$f = 0.4$** is a **large** effect.

Cohen (1988) also referenced another effect size parameter which he named η^2 (*eta-squared*). This parameter is defined as

$$\eta^2 = \frac{\sigma_m^2}{\sigma_m^2 + \sigma^2} = \frac{f^2}{1 + f^2}$$

η^2 is the proportion of the total variation in the dependent variable that is attributable to the groups. The above equation can be solved for f as follows

$$f^2 = \frac{\eta^2}{1 - \eta^2}$$

Power Calculations for One-Way ANOVA

The calculation of the power of a particular test proceeds as follows:

1. Determine the critical value, $F_{G-1, N-G, \alpha}$ where α is the probability of a type-I error and G and N are defined above. Note that this is a two-tailed test as no direction is assigned in the alternative hypothesis.
2. Choose an effect size parameter and its value.
3. Compute the power as the probability of being greater than $F_{G-1, N-G, \alpha}$ on a noncentral- F distribution with noncentrality parameter λ , where λ is calculated from f or η^2 using the formulas given above.

Example 1 – Finding Sample Size

An experiment is being designed to compare the means of four groups using an F test with a significance level of 0.05. The researchers would like to determine the sample size required to detect a small, medium, or large effect when the power is either 0.80 or 0.90.

Setup

If the procedure window is not already open, use the PASS Home window to open it. The parameters for this example are listed below and are stored in the **Example 1** settings file. To load these settings to the procedure window, click **Open Example Settings File** in the Help Center or File menu.

Design Tab

Solve For	Sample Size
Power.....	0.8 0.9
Alpha.....	0.05
G (Number of Groups)	4
Group Allocation Input Type	Equal (n1 = ... = nG)
Effect Size Type.....	f = σ_m/σ
f (σ_m / σ).....	0.1 0.25 0.4

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Output

Click the Calculate button to perform the calculations and generate the following output.

Numeric Reports

Numeric Results

Solve For: **Sample Size**

Number of Groups: 4

Power	Sample Size		Effect Size		Alpha
	Total N	Group ni	f	η^2	
0.80073	1096	274	0.10	0.00990	0.05
0.80399	180	45	0.25	0.05882	0.05
0.82340	76	19	0.40	0.13793	0.05
0.90065	1424	356	0.10	0.00990	0.05
0.90181	232	58	0.25	0.05882	0.05
0.91155	96	24	0.40	0.13793	0.05

Power The probability of rejecting a false null hypothesis when the alternative hypothesis is true.

N The total number of subjects in the study.

ni The Sample Size per Group is the number of items sampled from each group in the study.

f The Effect Size. $f = \sigma_m / \sigma$, where σ_m is the (sample size weighted) standard deviation of the means and σ is the standard deviation within a group.

η^2 The Effect Size. $\eta^2 = \sigma_m^2 / (\sigma_m^2 + \sigma^2)$, where σ_m is the (sample size weighted) standard deviation of the means and σ is the standard deviation within a group. It is the proportion of the total variation ($\sigma_m^2 + \sigma^2$) that is explained by the variation in the means (σ_m^2). It is analogous to R^2 in regression.

Alpha The probability of rejecting a true null hypothesis.

Group Sample Size Details

n	N	Group Sample Sizes	Group Allocation Proportions
n(1)	1096	274, 274, 274, 274	0.25, 0.25, 0.25, 0.25
n(2)	180	45, 45, 45, 45	0.25, 0.25, 0.25, 0.25
n(3)	76	19, 19, 19, 19	0.25, 0.25, 0.25, 0.25
n(4)	1424	356, 356, 356, 356	0.25, 0.25, 0.25, 0.25
n(5)	232	58, 58, 58, 58	0.25, 0.25, 0.25, 0.25
n(6)	96	24, 24, 24, 24	0.25, 0.25, 0.25, 0.25

Summary Statements

A one-way ANOVA design with 4 groups will be used to test whether there is a difference among the 4 group means. The comparison will be made using an F-test with a Type I error rate (α) of 0.05. To detect an effect size f of 0.1 (where $f = (\sigma_m / \sigma)$, and σ_m depends on both the means and the sample sizes), with 80% power, the needed group sample sizes are 274, 274, 274, 274 (for a total of 1096 subjects).

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Dropout-Inflated Sample Size

Dropout Rate	Sample Size N	Dropout- Inflated Enrollment Sample Size N'	Expected Number of Dropouts D
20%	1096	1370	274
20%	180	225	45
20%	76	95	19
20%	1424	1780	356
20%	232	290	58
20%	96	120	24

Dropout Rate	The percentage of subjects (or items) that are expected to be lost at random during the course of the study and for whom no response data will be collected (i.e., will be treated as "missing"). Abbreviated as DR.
N	The evaluable sample size at which power is computed. If N subjects are evaluated out of the N' subjects that are enrolled in the study, the design will achieve the stated power.
N'	The total number of subjects that should be enrolled in the study in order to obtain N evaluable subjects, based on the assumed dropout rate. After solving for N, N' is calculated by inflating N using the formula $N' = N / (1 - DR)$, with N' always rounded up. (See Julious, S.A. (2010) pages 52-53, or Chow, S.C., Shao, J., Wang, H., and Lohknygina, Y. (2018) pages 32-33.)
D	The expected number of dropouts. $D = N' - N$.

Dropout Summary Statements

Anticipating a 20% dropout rate, 1370 subjects should be enrolled to obtain a final sample size of 1096 subjects.

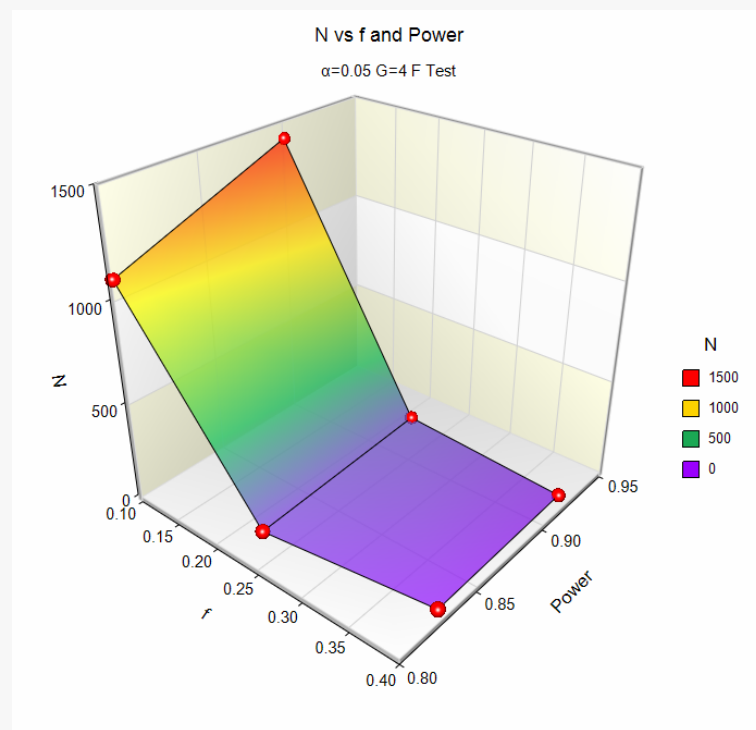
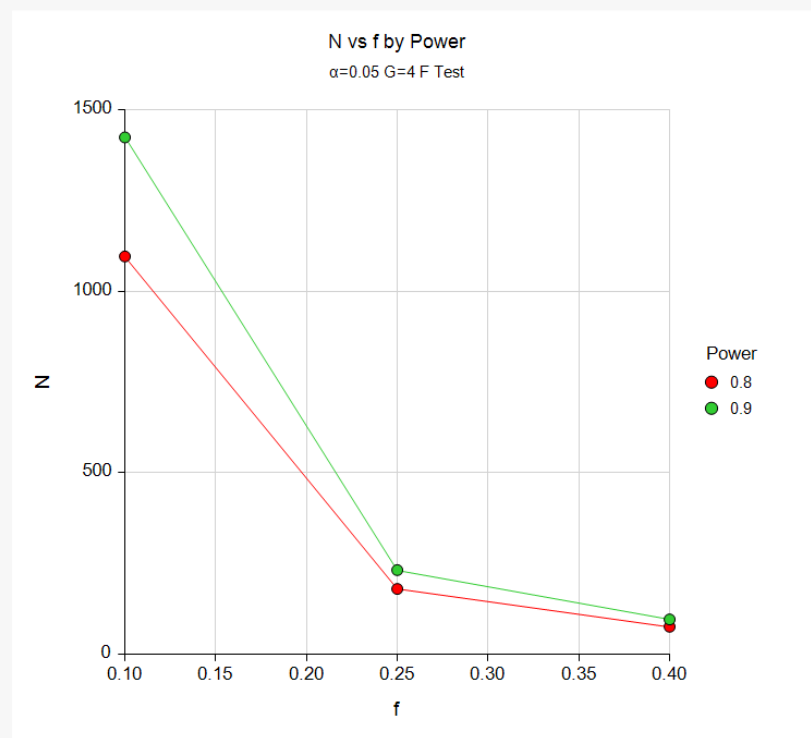
References

- Cohen, Jacob. 1988. *Statistical Power Analysis for the Behavioral Sciences*. Lawrence Erlbaum Associates. Hillsdale, New Jersey
- Desu, M. M. and Raghavarao, D. 1990. *Sample Size Methodology*. Academic Press. New York.
- Fleiss, Joseph L. 1986. *The Design and Analysis of Clinical Experiments*. John Wiley & Sons. New York.
- Kirk, Roger E. 1982. *Experimental Design: Procedures for the Behavioral Sciences*. Brooks/Cole. Pacific Grove, California.

This report shows the numeric results of this power study.

Plots Section

Plots



These plots give a visual presentation to the results in the Numeric Report.

Example 2 – Validation using Fleiss (1986)

Fleiss (1986) page 374 presents an example of determining a sample size in an experiment with 4 groups; means of 9.775, 12, 12, and 14.225; standard deviation of 3; alpha of 0.05, and beta of 0.20. He determines that a sample size of 11 per group is needed—for a total sample size of 44. These means and standard deviation result in a value of 0.5244 for f .

Setup

If the procedure window is not already open, use the PASS Home window to open it. The parameters for this example are listed below and are stored in the **Example 2** settings file. To load these settings to the procedure window, click **Open Example Settings File** in the Help Center or File menu.

Design Tab	
Solve For	Sample Size
Power.....	0.8
Alpha.....	0.05
G (Number of Groups)	4
Group Allocation Input Type	Equal (n1 = ... = nG)
Effect Size Type.....	f = σ_m/σ
f (σ_m / σ).....	0.5244

Output

Click the Calculate button to perform the calculations and generate the following output.

Numeric Results					
Solve For:	Sample Size				
Number of Groups:	4				
	Sample Size		Effect Size		
	Total	Group			
Power	N	ni	f	η^2	Alpha
0.80266	44	11	0.5244	0.21568	0.05
Group Sample Size Details					
n	N	Group Sample Sizes	Group Allocation Proportions		
n(1)	44	11, 11, 11, 11	0.25, 0.25, 0.25, 0.25		

PASS also found N = 44. Note that Fleiss used calculations based on a normal approximation, but **PASS** uses exact calculations based on the non-central F distribution.