

Chapter 569

One-Way Repeated Measures

Introduction

This module calculates the power for a one-way repeated measures design. It computes power for both the univariate (F test and F test with Geisser-Greenhouse correction) and multivariate (Wilks' lambda, Pillai-Bartlett trace, and Hotelling-Lawley trace) approaches. It can also be used to calculate the power of *crossover* designs.

Repeated measures designs are popular because they allow a subject to serve as their own control. This usually improves the precision of the experiment. However, when the analysis of the data uses the traditional F tests, additional assumptions concerning the structure of the error variance must be made. When these assumptions do not hold, the Geisser-Greenhouse correction provides reasonable adjustments so that significance levels are accurate.

An alternative to using the F test with repeated measures designs is to use one of the multivariate tests: Wilks' lambda, Pillai-Bartlett trace, or Hotelling-Lawley trace. These alternatives are appealing because they do not make the strict, often unrealistic, assumptions about the structure of the variance-covariance matrix. Unfortunately, they may have less power than the F test and they cannot be used in all situations.

Impact of Treatment Order, Unequal Variances, and Unequal Correlations

Treatment Order

It is important to understand under what conditions the order of treatment application matters since the design will may include subjects with different treatment arrangements, although each subject will eventually receive all treatments. Usually, the design requires a *washout* period between each two treatment applications so that the effect of one treatment does not *carryover* to next treatment. It is always a good research practice to try several different treatment orders. **PASS** lets you study this by allowing you to set the means and standard deviations in any order you like.

A brief investigation showed that the order of treatment application only matters when the variances at each treatment are different. The pattern of the correlation matrix does have an impact, but only when the variances are different.

Unequal Variances

PASS lets you investigate the impact of unequal variances on power. Of course, different variance patterns lead to different powers. However, the special point to understand is that when the variances are different, different treatment orders result in different powers as well.

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Unequal Correlations

PASS lets you investigate the impact of unequal correlations on power. Of course, different correlation patterns lead to different powers. However, the special point to understand is that when the variances are equal, different treatment orders do not result in different powers. Thus, when the variances are the same, the power values for different treatment orders will remain constant across different treatment orderings.

Conclusions

The above considerations result in the following strategy when using this procedure to power analyze (find the sample size) for a particular design.

1. If the variance is constant across treatments, you can analyze any order and the results will stand for all orderings. For example, for the case $M = 3$, there are six possible orders: ABC, ACB, BAC, BCA, CAB, and CBA. In this case, the resulting sample size will be identical for any order.
2. If the variances are different across treatments (for example, if the variances are treatments with larger means are assumed to be proportionally larger), you should analyze each order separately and then use the average sample size or the maximum sample size. The point is, you cannot simply analyze one order.

Assumptions

The following assumptions are made when using the F test to analyze a factorial experimental design.

1. The response variable is continuous.
2. The residuals follow the normal probability distribution with mean equal to zero and constant variance.
3. The subjects are independent.

Since in a within-subject design responses coming from the same subject are not independent, assumption 3 must be modified for responses within a subject. Independence between subjects is still assumed.

4. The within-subject covariance matrix is equal for all subjects. In this type of experiment, the repeated measurements on a subject may be thought of as a multivariate response vector having a certain covariance structure.
5. When using an F test, the within-subject covariance matrix is assumed to be *circular*. One way of defining circularity is that the variances of differences between any two measurements within a subject are constant for all measurements. Since responses that are close together in time (or space) often have a higher correlation than those that are far apart, it is common for this assumption to be violated. This assumption is not necessary for the validity of the three multivariate tests: Wilks' lambda, Pillai-Bartlett trace, or Hotelling-Lawley trace.

Advantages of Within-Subjects Designs

Because the response to stimuli usually varies less within an individual than between individuals, the within-subject variability is usually less than (or at most equal to) the between-subject variability. By reducing the underlying variability, the same power can be achieved with a smaller number of subjects.

Disadvantages of Within-Subjects Designs

1. *Practice effect*. In some experiments, subjects systematically improve as they practice the task being studied. In other cases, subjects may systematically get worse as they get fatigued or bored with the experimental task. Note that only the treatment administered first is immune to practice effects. Hence, experimenters should make an effort to balance the number of subjects receiving each treatment first.

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2. *Carryover effect.* In many drug studies, it is important to wash out the influence of one drug completely before the next drug is administered. Otherwise, the influence of the first drug carries over into the response to the second drug.
3. *Statistical analysis.* The statistical model is more restrictive than in a one-way factorial design since the individual responses must have certain mathematical properties.

Even in the face of all these disadvantages, repeated measures (within-subject) designs are popular in many areas of research. It is important that you recognize these problems going in so you can make sure that the design is appropriate, rather than learning of them later after the research has been conducted.

Technical Details

General Linear Multivariate Model

This section provides the technical details of the repeated measures designs that can be analyzed by *PASS*. The approximate power calculations outlined in Muller, LaVange, Ramey, and Ramey (1992) are used. Using their notation, for N subjects, the usual general linear multivariate model is

$$Y = XM + R$$

where each row of the residual matrix R is distributed as a multivariate normal

$$\text{row}_k(R) \sim N_p(0, \Sigma)$$

Note that p is the number of levels of the within-subject factor, Y is the matrix of responses, X is the design matrix, M is the matrix of regression parameters (means), and R is the matrix of residuals.

Hypotheses about various sets of regression parameters are tested using

$$H_0: \Theta = \Theta_0$$

$$MD = \Theta$$

where D is an orthonormal contrast matrix and Θ_0 is a matrix of hypothesized values, usually zeros.

Tests of the main effect may be constructed with suitable choices for D . These tests are based on

$$\hat{M} = (X'X)^{-1} X'Y$$

$$\hat{\Theta} = \hat{M}D$$

$$H_{b \times b} = (\hat{\Theta} - \Theta_0)' [C(X'X)^{-1}C']^{-1} (\hat{\Theta} - \Theta_0)$$

$$E_{b \times b} = D' \hat{\Sigma} D \cdot (N - r)$$

$$T_{b \times b} = H + E$$

where r is the rank of X which is, in this case, $p - 1$. Also, C is the scalar 1 and can be ignored in this case.

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Geisser-Greenhouse F Test

Upon the assumption that Σ has compound symmetry, a size α test of $H_0: \Theta = \Theta_0$ is given by the F ratio

$$F = \frac{\text{tr}(H)/(p-1)}{\text{tr}(E)/[(p-1)(N-p+1)]}$$

with degrees of freedom given by

$$df_1 = p - 1$$

$$df_2 = (p-1)(N-p+1)$$

and noncentrality parameter

$$\lambda = df_1(F)$$

The assumption that Σ has compound symmetry is usually not viable. Box (1954a,b) suggested that adjusting the degrees of freedom of the above F -ratio could compensate for the lack of compound symmetry in Σ . His adjustment has become known as the Geisser-Greenhouse adjustment. Under this adjustment, the modified degrees of freedom and noncentrality parameter are given by

$$df_1 = (p-1)\varepsilon$$

$$df_2 = (p-1)(N-p+1)\varepsilon$$

$$\lambda = (df_1)F\varepsilon$$

where

$$\varepsilon = \frac{\text{tr}(D'\hat{\Sigma}D)^2}{b \text{tr}(D'\hat{\Sigma}DD'\hat{\Sigma}D)}$$

The range of ε is $\frac{1}{b-1}$ to 1. When $\varepsilon = 1$, the matrix is *spherical*. When $\varepsilon = \frac{1}{b-1}$, the matrix differs maximally from sphericity.

The critical value F_{Crit} is computed using the expected value of ε to adjust the degrees of freedom. That is, the degrees of freedom of F_{Crit} are given by

$$df_1 = (p-1)E(\varepsilon)$$

$$df_2 = (p-1)(N-p+1)E(\varepsilon)$$

where

$$E(\hat{\varepsilon}) = \begin{cases} \varepsilon + \frac{g_1}{N-r} & \text{if } \varepsilon > \frac{g_1}{N-r} \\ \varepsilon / 2 & \text{otherwise} \end{cases}$$

$$g_1 = \sum_{i=1}^T f_{ii}\xi_i^2 + \sum_{i \neq j} \frac{f_i \xi_i \xi_j}{(\xi_i - \xi_j)}$$

$$f_i = \frac{\partial \varepsilon}{\partial \xi_i} = \frac{2 \sum \xi_j}{df_1 \sum \xi_j^2} - \frac{2 \lambda_i (\sum \xi_j)^2}{df_1 (\sum \xi_j^2)^2}$$

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$$f_{ii} = \frac{\partial^{(2)} \varepsilon}{\partial \xi_i^{(2)}} \\ = 2h_1 - 8h_2 + 8h_3 - 2h_4$$

$$h_1 = \frac{2}{df_1 \sum \xi_j^2}$$

$$h_2 = \frac{\xi_i (\sum \xi_j)}{df_1 (\sum \xi_j^2)^2}$$

$$h_3 = \frac{\xi_i^2 (\sum \xi_j)^2}{df_1 (\sum \xi_j^2)^3}$$

$$h_4 = \frac{(\sum \xi_j)^2}{df_1 (\sum \xi_j^2)^2}$$

where the ξ_j 's are the ordered eigenvalues of $D' \Sigma D$.

Wilks' Lambda Approximate F Test

The hypothesis $H_0: \Theta = \Theta_0$ may be tested using Wilks' likelihood ratio statistic W . This statistic is computed using

$$W = |ET^{-1}|$$

An F approximation to the distribution of W is given by

$$F_{df_1, df_2} = \frac{\eta / df_1}{(1 - \eta) / df_2}$$

where

$$\lambda = df_1 F_{df_1, df_2}$$

$$\eta = 1 - W$$

$$df_1 = p - 1$$

$$df_2 = N - p + 1$$

Pillai-Bartlett Trace Approximate F Test

The hypothesis $H_0: \Theta = \Theta_0$ may be tested using the Pillai-Bartlett Trace. This statistic is computed using

$$T_{PB} = tr(HT^{-1})$$

A non-central F approximation to the distribution of T_{PB} is given by

$$F_{df_1, df_2} = \frac{\eta / df_1}{(1 - \eta) / df_2}$$

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where

$$\lambda = df_1 F_{df_1, df_2}$$

$$\eta = \frac{T_{PB}}{s}$$

$$s = 1$$

$$df_1 = p - 1$$

$$df_2 = N - p + 1$$

Hotelling-Lawley Trace Approximate F Test

The hypothesis $H_0: \Theta = \Theta_0$ may be tested using the Hotelling-Lawley Trace. This statistic is computed using

$$T_{HL} = tr(HE^{-1})$$

An F approximation to the distribution of T_{HL} is given by

$$F_{df_1, df_2} = \frac{\eta / df_1}{(1 - \eta) / df_2}$$

where

$$\lambda = df_1 F_{df_1, df_2}$$

$$\eta = \frac{\frac{T_{HL}}{s}}{1 + \frac{T_{HL}}{s}}$$

$$s = 1$$

$$df_1 = p - 1$$

$$df_2 = N - p + 1$$

The M (Mean) Matrix

In the general linear multivariate model presented above, M represents a matrix of regression coefficients. Since you must provide the elements of M , we will discuss its meaning in more detail. Although other structures and interpretations of M are possible, in this module we assume that the elements of M are the cell means. The row of M represents the single group and the columns of M represent the within-group categories.

Consider now an example in which $p = 4$. That is, there is one group of subjects that are each measured four times. The matrix M would appear as follows.

$$M = [\mu_1 \quad \mu_2 \quad \mu_3 \quad \mu_4]$$

To calculate the power of this design, you would need to specify appropriate values of all four means under the alternative hypothesis.

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Specifying the M Matrix

When computing the power in a repeated measures analysis of variance, the specification of the M matrix is one of your main tasks. The program cannot do this for you. The calculated power is directly related to your choice. So your choice for the elements of M must be selected carefully and thoughtfully. When authorization and approval from a government organization is sought, you should be prepared to defend your choice of M . In this section, we will explain how you can specify M .

Before we begin, it is important that you have in mind exactly what M is. M is a row of means that represent the size of the differences among the means that you want the study or experiment to detect. M gives the means under the alternative hypothesis. Under the null hypothesis, these means are assumed to be equal.

The D Matrix for Within-Subject Contrasts

The D matrix is comprised of contrasts that are applied to the columns of M . The choice of D does not matter as long as it is orthogonal, so an appropriate matrix will be generated for you.

Power Calculations

To calculate statistical power, we must determine distribution of the test statistic under the alternative hypothesis which specifies a different value for the regression parameter matrix B . The distribution theory in this case has not been worked out, so approximations must be used. We use the approximations given by Mueller and Barton (1989) and Muller, LaVange, Ramey, and Ramey (1992). These approximations state that under the alternative hypothesis, F_U is distributed as a noncentral F random variable with degrees of freedom and noncentrality shown above. The calculation of the power of a particular test may be summarized as follows

1. Specify values of M, Σ .
2. Determine the critical value using $F_{crit} = \text{FINV}(1 - \alpha, df_1, df_2)$, where $\text{FINV}()$ is the inverse of the central F distribution and α is the significance level.
3. Compute the noncentrality parameter λ .
4. Compute the power as $\text{Power} = 1 - \text{NCFPROB}(F_{crit}, df_1, df_2, \lambda)$, where $\text{NCFPROB}()$ is the noncentral F distribution.

Covariance Matrix Assumptions

The following assumptions are made when using the F test. These assumptions are not needed when using one of the three multivariate tests.

In order to use the F ratio to test hypotheses, certain assumptions are made about the distribution of the residuals e_{ijk} . Specifically, it is assumed that the residuals for each subject, $e_{ij1}, e_{ij2}, \dots, e_{ijT}$, are distributed as a multivariate normal with means equal to zero and covariance matrix Σ_{ij} . Two additional assumptions are made about these covariance matrices. First, they are assumed to be equal for all subjects. That is, it is assumed that $\Sigma_{11} = \Sigma_{12} = \dots = \Sigma_{Gn} = \Sigma$. Second, the covariance matrix is assumed to have a particular form called *circularity*. A covariance matrix is *circular* if there exists a matrix A such that

$$\Sigma = A + A' + \lambda I_T$$

where I_T is the identity matrix of order T and λ is a constant.

This property may also be defined as

$$\sigma_{ii} + \sigma_{jj} - 2\sigma_{ij} = 2\lambda$$

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One type of matrix that is circular is one that has *compound symmetry*. A matrix with this property has all elements on the main diagonal equal and all elements off the main diagonal equal. An example of a covariance matrix with compound symmetry is

$$\Sigma = \begin{bmatrix} \sigma^2 & \rho\sigma^2 & \rho\sigma^2 & \cdots & \rho\sigma^2 \\ \rho\sigma^2 & \sigma^2 & \rho\sigma^2 & \cdots & \rho\sigma^2 \\ \rho\sigma^2 & \rho\sigma^2 & \sigma^2 & \cdots & \rho\sigma^2 \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ \rho\sigma^2 & \rho\sigma^2 & \rho\sigma^2 & \cdots & \sigma^2 \end{bmatrix}$$

or, with actual numbers,

$$\begin{bmatrix} 9 & 2 & 2 & 2 \\ 2 & 9 & 2 & 2 \\ 2 & 2 & 9 & 2 \\ 2 & 2 & 2 & 9 \end{bmatrix}$$

An example of a matrix which does not have compound symmetry but is still circular is

$$\begin{bmatrix} 1 & 1 & 1 & 1 \\ 2 & 2 & 2 & 2 \\ 3 & 3 & 3 & 3 \\ 4 & 4 & 4 & 4 \end{bmatrix} + \begin{bmatrix} 1 & 2 & 3 & 4 \\ 1 & 2 & 3 & 4 \\ 1 & 2 & 3 & 4 \\ 1 & 2 & 3 & 4 \end{bmatrix} + \begin{bmatrix} 2 & 0 & 0 & 0 \\ 0 & 2 & 0 & 0 \\ 0 & 0 & 2 & 0 \\ 0 & 0 & 0 & 2 \end{bmatrix} = \begin{bmatrix} 4 & 3 & 4 & 5 \\ 3 & 6 & 5 & 6 \\ 4 & 5 & 8 & 7 \\ 5 & 6 & 7 & 10 \end{bmatrix}$$

Needless to say, the need to have the covariance matrix circular is a very restrictive assumption.

Covariance Patterns

In a repeated measures design with N subjects, each measured M times, observations within a single subject may be correlated, and a pattern for their covariance must be specified. In this case, the overall covariance matrix will have the block-diagonal form:

$$\mathbf{V} = \begin{pmatrix} \mathbf{V}_1 & \mathbf{0} & \mathbf{0} & \cdots & \mathbf{0} \\ \mathbf{0} & \mathbf{V}_2 & \mathbf{0} & \cdots & \mathbf{0} \\ \mathbf{0} & \mathbf{0} & \mathbf{V}_3 & \cdots & \mathbf{0} \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ \mathbf{0} & \mathbf{0} & \mathbf{0} & \cdots & \mathbf{V}_N \end{pmatrix},$$

where \mathbf{V}_i is the $M \times M$ covariance submatrices corresponding to the i^{th} subject. The $\mathbf{0}$'s represent $M \times M$ matrices of zeros giving zero covariances for observations on different subjects. This routine allows the specification of two different covariance matrix types: All ρ 's Equal and AR(1), Banded(1), and Banded(2).

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All ρ 's Equal (Compound Symmetry)

A compound symmetry covariance model assumes that all covariances are equal, and all variances on the diagonal are equal. That is

$$\Sigma = \sigma^2 \begin{pmatrix} 1 & \rho & \rho & \rho & \cdots & \rho \\ \rho & 1 & \rho & \rho & \cdots & \rho \\ \rho & \rho & 1 & \rho & \cdots & \rho \\ \rho & \rho & \rho & 1 & \cdots & \rho \\ \vdots & \vdots & \vdots & \vdots & \ddots & \vdots \\ \rho & \rho & \rho & \rho & \cdots & 1 \end{pmatrix}_{M \times M}$$

where σ^2 is the subject variance and ρ is the correlation between observations on the same subject.

AR(1)

An AR(1) (autoregressive order 1) covariance model assumes that all variances on the diagonal are equal and that covariances t time periods apart are equal to $\rho^t \sigma^2$. **This choice is recommended.** That is

$$\Sigma = \sigma^2 \begin{pmatrix} 1 & \rho & \rho^2 & \rho^3 & \cdots & \rho^{M-1} \\ \rho & 1 & \rho & \rho^2 & \cdots & \rho^{M-2} \\ \rho^2 & \rho & 1 & \rho & \cdots & \rho^{M-3} \\ \rho^3 & \rho^2 & \rho & 1 & \cdots & \rho^{M-4} \\ \vdots & \vdots & \vdots & \vdots & \ddots & \vdots \\ \rho^{M-1} & \rho^{M-2} & \rho^{M-3} & \rho^{M-4} & \cdots & 1 \end{pmatrix}_{M \times M}$$

where σ^2 is the residual variance and ρ is the correlation between observations on the same subject.

Banded(1)

A Banded(1) (banded order 1) covariance model assumes that all variances on the diagonal are equal, covariances for observations one time period apart are equal to $\rho \sigma^2$, and covariances for measurements greater than one time period apart are equal to zero. That is

$$\Sigma = \sigma^2 \begin{pmatrix} 1 & \rho & 0 & 0 & \cdots & 0 \\ \rho & 1 & \rho & 0 & \cdots & 0 \\ 0 & \rho & 1 & \rho & \cdots & 0 \\ 0 & 0 & \rho & 1 & \cdots & 0 \\ \vdots & \vdots & \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & 0 & 0 & \cdots & 1 \end{pmatrix}_{M \times M}$$

where σ^2 is the residual variance and ρ is the correlation between observations on the same subject.

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Banded(2)

A Banded(2) (banded order 2) covariance model assumes that all variances on the diagonal are equal, covariances for observations one or two time periods apart are equal to $\rho\sigma^2$, and covariances for measurements greater than two time period apart are equal to zero. That is

$$\Sigma = \sigma^2 \begin{pmatrix} 1 & \rho & \rho & 0 & \cdots & 0 \\ \rho & 1 & \rho & \rho & \cdots & 0 \\ \rho & \rho & 1 & \rho & \cdots & 0 \\ 0 & \rho & \rho & 1 & \cdots & 0 \\ \vdots & \vdots & \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & 0 & 0 & \cdots & 1 \end{pmatrix}_{M \times M}$$

where σ^2 is the residual variance and ρ is the correlation between observations on the same subject.

Procedure Options

This section describes the options that are specific to this procedure. These are located on the Design tab. For more information about the options of other tabs, go to the Procedure Window chapter.

Design Tab

The Design tab contains most of the parameters and options that you will be concerned with.

Solve For

Solve For

This option specifies the parameter to be solved for. When you choose to solve for *Sample Size*, the program searches for the lowest sample size that meets the alpha and power criterion you have specified.

This parameter is displayed on the vertical axis of the plot.

Tests

Test Statistic

You should select the test statistic that will be used to analyze the data. There are two general types: univariate and multivariate. The univariate repeated measures ANOVA approach (also known as the mixed model approach) analyzes the power of the corresponding F-test. These tests make strong assumptions about the covariance matrix. The multivariate approach uses any of three common multivariate tests. For the one-way repeated measures design, all three multivariate tests give the same result.

The test statistics are

Univariate: F Test

This test assumes that all variances are equal and all correlations are equal (a condition called compound symmetry). If you want to use this option, you must choose equal σ 's and constant ρ 's across time. Because of the strong assumptions, this test is not recommended.

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Univariate: Geisser-Greenhouse F Test

The Geisser-Greenhouse adjustment corrects the degrees of freedom of the F-Test so that the actual significance levels are closer to the stated levels when there is a lack of compound symmetry in the variance-covariance matrix (which is the usual situation).

Multivariate: Wilks Lambda Test

Wilks' Lambda is the multivariate generalization of $1 - R^2$ in multiple regression. Because of its easy interpretation, it is the most popular of the three tests.

Multivariate: Pillai-Bartlett Test

The Pillai-Bartlett statistic calculates the amount of variance in the dependent variable accounted for by the greatest separation of the independent (design) variables. This test is considered most reliable in small sample sizes.

Multivariate: Hotelling-Lawley Trace Test

This test is usually converted to Hotelling's T^2 .

Power and Alpha

Power

This option specifies one or more values for power. Power is the probability of rejecting a false null hypothesis, and is equal to one minus Beta. Beta is the probability of a type-II error, which occurs when a false null hypothesis is not rejected. In this procedure, a type-II error occurs when you fail to reject the null hypothesis of equal means when in fact the means are different.

Values must be between zero and one. Historically, the value of 0.80 (Beta = 0.20) was used for power. Now, 0.90 (Beta = 0.10) is also commonly used.

A single value may be entered here or a range of values such as *0.8 to 0.95 by 0.05* may be entered.

Alpha

This option specifies one or more values for the probability of a type-I error. A type-I error occurs when a true null hypothesis is rejected. In this procedure, a type-I error occurs when you reject the null hypothesis of equal means when in fact the means are equal.

Values must be between zero and one. Historically, the value of 0.05 has been used for alpha. This means that about one test in twenty will falsely reject the null hypothesis. You should pick a value for alpha that represents the risk of a type-I error you are willing to take in your experimental situation.

Note that this is a two-sided test.

You may enter a range of values such as *0.01 0.05 0.10* or *0.01 to 0.10 by 0.01*.

Sample Size

N (Subjects)

Enter a value for the sample size (N), the number of subjects in the study. Each subject is measured two or more times. You may enter a single value or a range of values. A separate power calculation will be made for each value of N you enter.

Examples

10 to 100 by 10

10 30 80 90

10, 30, 80, 90

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Effect Size – Number of Repeated Measurements

M (Measurements)

Enter a value for the number of time points (repeated measurements) at which each subject is observed. You must enter a single value 2 or greater.

Effect Size – Means

These options specify the means under the alternative hypothesis, one for each time point. The test statistic tests whether the differences among these means are all zero.

The means represent the alternative hypothesis. They are not necessarily the means you expect. Rather, they represent the magnitude of the differences that you want to detect.

μ_i 's ($\mu_1, \mu_2, \dots, \mu_M$)

This option specifies how you want to enter the M means for the alternative hypothesis. Your choices are List, Range, or Sequence.

μ_i 's ($\mu_1, \mu_2, \dots, \mu_M$) = List

Enter a set of M hypothesized means, μ_i , one mean for each time point. Enter numbers separated by blanks or commas. If the number of means entered is less than M, the last mean entered will be copied forward. If too many means are entered, the extra will be ignored.

μ_i 's ($\mu_1, \mu_2, \dots, \mu_M$) = Range

Enter a range of values by specifying the first and the last. The other means will be generated equi-spaced between these values.

μ_i 's ($\mu_1, \mu_2, \dots, \mu_M$) = Sequence

Enter a set of means by specifying the first mean and a step-size (increment) to be added for each succeeding mean.

K (Mean Multiplier)

Enter one or more values for K, the mean multiplier. A separate power calculation is conducted for each value of K. In each analysis, all means are multiplied by K. In this way, you can determine how sensitive the power values are to the magnitude of the means without the need to change them all individually.

For example, if the original means are 0, 1, and 2, setting this option to '1 2' results in two sets of means used in separate analyses: 0, 1, and 2 in the first analysis and 0, 2, and 4 in the second analysis.

If you want to ignore this option, enter a one.

Effect Size – σ (Standard Deviation)

Pattern of σ 's Across Time

Specify whether the σ_i 's vary across the measurement points or are the same.

Equal: $\sigma = \sigma_1 = \sigma_2 = \dots = \sigma_M$

The σ_i 's are constant across time. This assumption is required by the univariate F-test.

Unequal: $\sigma_1 \neq \sigma_2 \neq \dots \neq \sigma_M$

The σ_i 's can vary across time. Most researchers would agree that this is a more reasonable assumption in most cases. Because of this, the multivariate tests are easier to justify.

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σ (Standard Deviation) – Equal Pattern

This is the between subject standard deviation of the response variable (Y) at a particular time point. It is assumed to be the same for all time points. As a standard deviation, the number(s) must be greater than zero.

This represents the variability from subject to subject that occurs when the subjects are treated identically.

You can enter a list of values separated by blanks or commas, in which case, a separate analysis will be calculated for each value.

σ Button

You can press the σ button and select 'Covariance Matrix' to obtain help on estimating the standard deviation from an ANOVA table.

Examples

1,4,7,10

1 4 7 10

1 to 10 by 3

σ_i 's ($\sigma_1, \sigma_2, \dots, \sigma_M$) – Unequal Pattern

Specify how you want to enter the M σ_i 's. Your choices are List, Range, or Sequence.

σ_i 's ($\sigma_1, \sigma_2, \dots, \sigma_M$) = List (Unequal Pattern)

Enter a list of σ_i 's in the box that appears to the right.

σ_i 's ($\sigma_1, \sigma_2, \dots, \sigma_M$) = Range (Unequal Pattern)

Enter a range of σ_i 's by specifying the first and the last. The other σ_i 's will be generated between these values using a straight-line trend.

σ_i 's ($\sigma_1, \sigma_2, \dots, \sigma_M$) = Sequence (Unequal Pattern)

Enter a range of σ_i 's by specifying the first and the step-size (increment) to be added for each succeeding σ_i .

h (σ_i Multiplier)

Enter a list of h values. A separate analysis is made for each value of h. For each analysis, the σ_i 's entered above are all multiplied by h. Thus $\sigma_1, \sigma_2, \dots, \sigma_M$ become $h\sigma_1, h\sigma_2, \dots, h\sigma_M$. Hence, using this parameter, you can perform a sensitivity analysis about the value(s) of the standard deviation.

Note that the resulting values must all be positive, so all h's must be greater than 0.

If you want to ignore this option, enter a 1.

Effect Size – ρ (Correlation Between Measurements)

Pattern of ρ 's Across Time

Specify the correlation structure of the covariance matrix. The number of diagonal elements in the matrix is equal to M.

Possible options are

- **All ρ 's Equal**

All variances on the diagonal of the within-subject variance-covariance matrix are equal to σ^2 , and all covariances are equal to $\rho\sigma^2$.

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$$\Sigma = \sigma^2 \begin{pmatrix} 1 & \rho & \rho & \rho & \cdots & \rho \\ \rho & 1 & \rho & \rho & \cdots & \rho \\ \rho & \rho & 1 & \rho & \cdots & \rho \\ \rho & \rho & \rho & 1 & \cdots & \rho \\ \vdots & \vdots & \vdots & \vdots & \ddots & \vdots \\ \rho & \rho & \rho & \rho & \cdots & 1 \end{pmatrix}_{M \times M}$$

- **AR(1)**

All variances on the diagonal of the within-subject variance-covariance matrix are equal to σ^2 , and the covariance between observations t time periods apart is $\rho^t \sigma^2$.

$$\Sigma = \sigma^2 \begin{pmatrix} 1 & \rho & \rho^2 & \rho^3 & \cdots & \rho^{M-1} \\ \rho & 1 & \rho & \rho^2 & \cdots & \rho^{M-2} \\ \rho^2 & \rho & 1 & \rho & \cdots & \rho^{M-3} \\ \rho^3 & \rho^2 & \rho & 1 & \cdots & \rho^{M-4} \\ \vdots & \vdots & \vdots & \vdots & \ddots & \vdots \\ \rho^{M-1} & \rho^{M-2} & \rho^{M-3} & \rho^{M-4} & \cdots & 1 \end{pmatrix}_{M \times M}$$

- **Banded(1)**

All variances on the diagonal of the within-subject variance-covariance matrix are equal to σ^2 , and the covariance between observations one time period apart is $\rho \sigma^2$. Covariances between observations more than one time period apart are equal to zero.

$$\Sigma = \sigma^2 \begin{pmatrix} 1 & \rho & 0 & 0 & \cdots & 0 \\ \rho & 1 & \rho & 0 & \cdots & 0 \\ 0 & \rho & 1 & \rho & \cdots & 0 \\ 0 & 0 & \rho & 1 & \cdots & 0 \\ \vdots & \vdots & \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & 0 & 0 & \cdots & 1 \end{pmatrix}_{M \times M}$$

- **Banded(2)**

All variances on the diagonal of the within-subject variance-covariance matrix are equal to σ^2 , and the covariance between observations one or two time periods apart are $\rho \sigma^2$. Covariances between observations more than two time periods apart are equal to zero.

$$\Sigma = \sigma^2 \begin{pmatrix} 1 & \rho & \rho & 0 & \cdots & 0 \\ \rho & 1 & \rho & \rho & \cdots & 0 \\ \rho & \rho & 1 & \rho & \cdots & 0 \\ 0 & \rho & \rho & 1 & \cdots & 0 \\ \vdots & \vdots & \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & 0 & 0 & \cdots & 1 \end{pmatrix}_{M \times M}$$

One-Way Repeated Measures

ρ (Correlation)

This is the correlation, ρ , between measurements on the same subject taken at the first and second time points.

At least one value must be entered. If multiple values are entered, a separate analysis is performed for each value.

Range

$0 < \rho < 1$ (negative values are not used). A value near 0 indicates low correlation. A value near 1 indicates high correlation.

Recommended

The value of this parameter depends on time or location pattern at which measurements are taken. In their book on sample size, Machin and Campbell comment that values between 0.60 and 0.75 are typical. Some authors recommend using 0.2 when nothing is known about the actual value.

Examples

0.5

0.5 0.6 0.7

0 to 0.9 by 0.1

One-Way Repeated Measures

Example 1 – Determining Sample Size

Researchers are planning a study of the impact of a new drug on heart rate. They want to evaluate the differences in heart rate within subjects after this drug is administered. Each subject will be measured 5 minutes before exercise, 5 minutes after exercise, 10 minutes after exercise, and finally 15 minutes after exercise. Two days later, each subject will be measured in the same way again, except that, this time, the drug will not be administered. The differences between these two sets of measurements will be used as the basic data for the analysis.

They expect a quadratic pattern in the mean differences and want to be able to detect a 10% shift in heart rate between the two treatments. The data analyzed will be the difference between the with-drug and without-drug measurements at each time point.

Similar studies have found a standard deviation of the difference between scores at each time point to be between 7 and 9, and a correlation between adjacent differences on the same individual to be 0.6. The researchers assume that a first-order autocorrelation pattern adequately models the data. Since the covariances will not be equal, they decide to use the Wilks' Lambda test statistic.

They decided to use a mean pattern of 0, -4, -3, and 0 to represent the differences at the four time points. They decide to look at three values of K: 1, 2, and 3.

The test will be conducted at the 0.05 significance level. What sample size is necessary to achieve 90% power over a range of possible means and standard deviations?

Setup

This section presents the values of each of the parameters needed to run this example. First, from the PASS Home window, load the **One-Way Repeated Measures** procedure window by expanding **Means**, then **Repeated Measures**, and then clicking on **One-Way Repeated Measures**. You may then make the appropriate entries as listed below, or open **Example 1** by going to the **File** menu and choosing **Open Example Template**.

<u>Option</u>	<u>Value</u>
Design Tab	
Solve For	Sample Size
Test Statistic	Multivariate: Wilks Lambda Test
Power	0.90
Alpha	0.05
M (Measurements).....	4
Means	List
Means – List Values	0 -4 -3 0
K (Means Multiplier).....	1 2 3
Pattern of σ 's Across Time	Equal
σ (Standard Deviation)	7 9
Pattern of ρ 's Across Time	AR(1)
ρ (Correlation).....	0.6

One-Way Repeated Measures

Annotated Output

Click the Calculate button to perform the calculations and generate the following output.

Numeric Results

Numeric Results for a One-Way Repeated Measures Test

Test: Wilks Lambda Test

μ 's: 0 -4 -3 0

σ 's: All Equal

ρ 's: AR(1)

Power	N	Time Points M	Mean Multiplier K	Standard Deviation of Means	Standard Deviation	Corr	Alpha
				σ_m	σ	ρ	
0.9040	26	4	1.0	1.79	7.00	0.60	0.05
0.9025	39	4	1.0	1.79	9.00	0.60	0.05
0.9176	11	4	2.0	3.57	7.00	0.60	0.05
0.9314	15	4	2.0	3.57	9.00	0.60	0.05
0.9298	8	4	3.0	5.36	7.00	0.60	0.05
0.9433	10	4	3.0	5.36	9.00	0.60	0.05

References

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Report Definitions

Power is the probability of rejecting a false null hypothesis.

N: The total number of subjects in the study.

M is the number of time points at which each subject is measured.

K is the effect size multiplier. The original means are all multiplied by this value, resulting in a corresponding change in the effect size.

Std Dev of Means (σ_m): This value represents the magnitude of differences among the hypothesized means.

σ is the standard deviation across subjects at a given time point.

ρ is the (auto)correlation between observations made on a subject at the first and second time points.

Alpha: The significance level of the test. The probability of rejecting the null hypothesis when the null hypothesis is true.

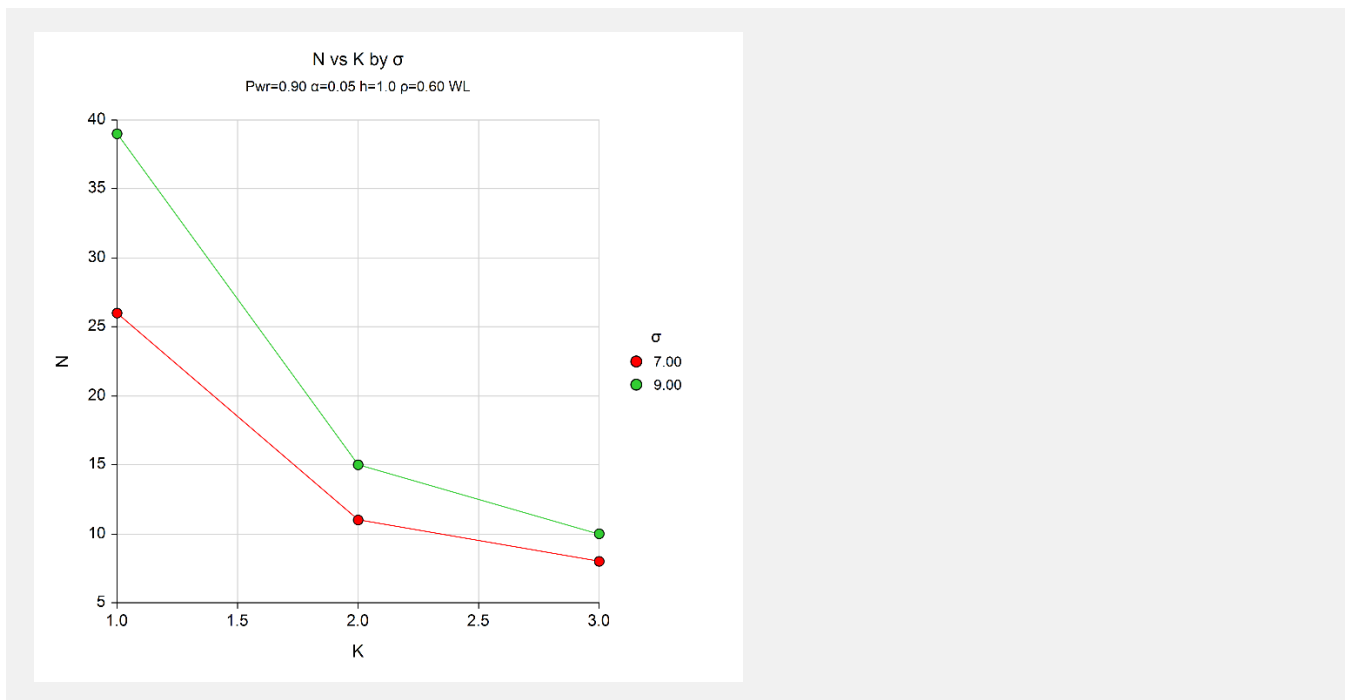
Summary Statements

A single-factor, repeated measures design with a sample of 26 subjects, measured at 4 time points, achieves 90% power to detect differences among the means using a Wilks Lambda Test at a 0.05 significance level. The standard deviation across subjects at the same time point is assumed to be 7.00. The correlation matrix from which the covariance matrix is generated follows an AR(1) pattern across time with a correlation of 0.60 between the first and second time point measurements. The standard deviation of the hypothesized means is 1.79.

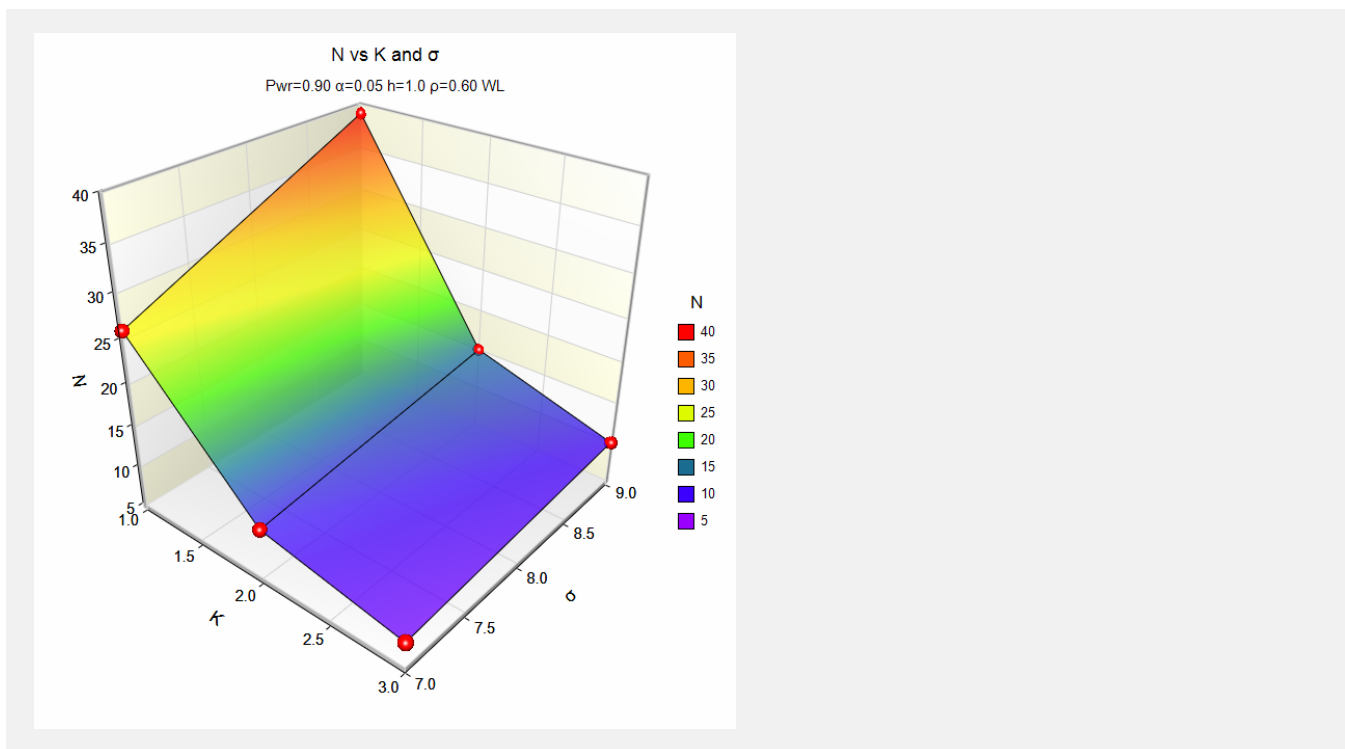
This report gives the power for each value of the other parameters. The definitions are shown in the report.

One-Way Repeated Measures

Plots Section



The chart shows the relationship between N, K, and σ when the other parameters in the design are held constant.



The chart shows a 3D view of the relationship between N, K, and σ when the other parameters in the design are held constant.

One-Way Repeated Measures

Example 2 – Validation using PASS’s Repeated Measures Procedure

To validate this procedure, we will run the first scenario through **PASS’s** Repeated Measures procedure. We did this and obtained the following report.

Design Report – Repeated Measures										
Term	Test	Power	n	N	Multiply Means By K	Std Dev of Effects (σ_m)	Standard Deviation (σ)	Effect Size (σ_m/σ)	Alpha	Beta
W1(4)	Wilks	0.904016	26	26	1.0	1.79	2.58	0.692	0.05	0.10

Setup

This section presents the values of each of the parameters needed to run this example. First, from the PASS Home window, load the **One-Way Repeated Measures** procedure window by expanding **Means**, then **Repeated Measures**, and then clicking on **One-Way Repeated Measures**. You may then make the appropriate entries as listed below, or open **Example 1** by going to the **File** menu and choosing **Open Example Template**.

Option	Value
Design Tab	
Solve For	Sample Size
Test Statistic	Multivariate: Wilks Lambda Test
Power.....	0.90
Alpha.....	0.05
M (Measurements).....	4
Means	List
Means – List Values	0 -4 -3 0
K (Means Multiplier).....	1
Pattern of σ 's Across Time	Equal
σ (Standard Deviation)	7
Pattern of ρ 's Across Time	AR(1)
ρ (Correlation).....	0.6

Output

Click the Calculate button to perform the calculations and generate the following output.

Numeric Results

Numeric Results for a One-Way Repeated Measures Test								
Test: Wilks Lambda Test								
μ 's: 0 -4 -3 0								
σ 's: All Equal								
ρ 's: AR(1)								
Power	N	Time Points M	Mean Multiplier K	Standard Deviation of Means σ_m	Standard Deviation σ	Corr ρ	Alpha	
0.9040	26	4	1.0	1.79	7.00	0.60	0.05	

This new, simplified procedure has obtained identical results with the previously validated procedure.