Chapter 565

Randomized Block Analysis of Variance

Introduction

This module analyzes a randomized block analysis of variance with up to two treatment factors and their interaction. It provides tables of power values for various configurations of the randomized block design.

The Randomized Block Design

The randomized block design (*RBD*) may be used when a researcher wants to reduce the experimental error among observations of the same treatment by accounting for the differences among blocks. If three treatments are arranged in two blocks, the *RBD* might appear as follows:

Block A
Treatment 1
Treatment 3
Treatment 2

Block B
Treatment 2
Treatment 1
Treatment 3

This diagram shows the main features of an *RBD*:

1. Each block is divided into *k* sub-blocks, where *k* is the number of treatments.
2. Each block receives all the treatments.
3. The treatments are assigned to the sub-blocks in random order.
4. There is some reason to believe that the blocks are the same internally, but different from each other.

RBD Reduces Random Error

The random error component of a completely randomized design (such as a one-way or a fixed-effects factorial design) represents the influence of all possible variables in the universe on the response except for the controlled (treatment) variables. This random error component is called the standard deviation or σ (sigma).

As we have discussed, the sample size required to meet alpha and beta error requirements depends directly on the standard deviation. As the standard deviation increases, the sample size increases. Hence, researchers are always looking for ways to reduce the standard deviation. Since the random error component contains the variation due to all possible variables other than treatment variables, one of the most obvious ways to reduce the standard deviation is to remove one or more of these *nuisance* variables from the random error component. One of the simplest ways of doing this is by blocking on them.

For example, an agricultural experiment is often blocked on fields so that differences among fields are explicitly accounted for and removed from the error component. Since these field differences are caused by variations in variables such as soil type, sunlight, temperature, and water, blocking on fields removes the influence of several variables.

Blocks are constructed so that the response is as alike (homogeneous) as possible within a block, but as different as possible between blocks. In many situations, there are obvious natural blocking factors such as schools, seasons, individual farms, families, times of day, etc. In other situations, the blocks may be somewhat artificially constructed.

Once the blocks are defined, they are divided into *k* smaller sections called *subblocks,* where *k* is the number of treatment levels. The *k* treatments are randomly assigned to the subblocks, one block at a time. Hence the order of treatment application will be different from block to block.

Measurement of Random Error

The measurement of the random error component (σ) is based on the assumption that there is no fundamental relationship between the treatment variable and the blocking variable. When this is true, the interaction component between blocks and treatment is zero. If the interaction component is zero, then the amount measured by the interaction is actually random error and can be used as an estimate of σ.

Hence, the randomized block design makes the assumption that there is no interaction between treatments and blocks. The block by treatment mean square is still calculated, but it is used as the estimated standard deviation. This means that the degrees of freedom associated with the block-treatment interaction are the degrees of freedom of the error estimate. If the experimental design has *k* treatments and *b* blocks, the interaction degrees of freedom are equal to *(k-1)(b-1)*. Hence the sample size of this type of experiment is measured in terms of the number of blocks.

Treatment Effects

Either one or two treatment variables may be specified. If two are used, their interaction may also be measured. The null hypothesis in the *F* test states that the effects of the treatment variable are zero. The magnitude of the alternative hypothesis is represented as the size of the standard deviation (σ_m) of these effects. The larger the size of the effects, the larger their standard deviation.

When there are two factors, the block-treatment interaction may be partitioned just as the treatment may be partitioned. For example, if we let *C* and *D* represent two treatments, an analysis of variance will include the terms *C*, *D*, and *CD*. If we represent the blocking factor as *B*, there will be three interactions with blocks: *BC*, *BD*, and *BCD*. Since all three of these terms are assumed to measure the random error, the overall estimate of random error is found by averaging (or *pooling*) these three interactions. The pooling of these interactions increases the power of the experiment by effectively increasing the sample size on which the estimate of σ is based. However, it is based on the assumption that $\sigma = \sigma_{BC} = \sigma_{BD} = \sigma_{BCD}$, which may or may not be true.

An Example

Following is an example of data from a randomized block design. The block factor has four blocks (*B1*, *B2*, *B3*, *B4*) while the treatment factor has three levels (low, medium, and high). The response is shown within the table.

Randomized Block Example			
	Treatments		
Blocks	**Low**	**Medium**	**High**
B1	16	19	20
B2	18	20	21
B3	15	17	22
B4	14	17	19

Analysis of Variance Hypotheses

The *F* test for treatments in a randomized block design tests the hypothesis that the treatment effects are zero. (See the beginning of the Fixed-Effects Analysis of Variance chapter for a discussion of the meaning of effects.)

Single-Factor Repeated Measures Designs

The randomized block design is often confused with a single-factor repeated measures design because the analysis of each is similar. However, the randomization pattern is different. In a randomized block design, the treatments are applied in random order within each block. In a repeated measures design, however, the treatments are usually applied in the same order through time. You should not mix the two. If you are analyzing a repeated measures design, we suggest that you use that module of **PASS** to do the sample size and power calculations.

Example 1 – Power after a Study

This example will explain how to calculate the power of *F* tests from data that have already been collected and analyzed. We will analyze the power of the experiment that was given at the beginning of this chapter. These data were analyzed using the analysis of variance procedure in **NCSS** and the following results were obtained.

Analysis of Variance Table

Source Term	DF	Sum of Squares	Mean Square	F-Ratio	Prob Level	Power (Alpha=0.05)
A: Blocks	3	13.66667	4.555555			
B: Treatment	2	45.16667	22.58333	19.83	0.002269*	0.991442
AB	6	6.833333	1.138889			
S	0	0				
Total (Adjusted)	11	65.66666				
Total	12					

* Term significant at alpha = 0.05

Means and Standard Error Section

Term	Count	Mean	Standard Error	Effect
B: Treatment				
High	4	20.5	0.5335937	2.333333
Low	4	15.75	0.5335937	-2.416667
Medium	4	18.25	0.5335937	8.333334E-02

We will now calculate the power of the *F* test. Note that factor *B* in this printout becomes factor *A* on the **PASS** input.

To analyze these data, we enter the means for factor *A*. The value of σ is estimated as the square root of the mean square error:

$$\sigma = \sqrt{1.138889} = 1.0672$$

Setup

If the procedure window is not already open, use the PASS Home window to open it. The parameters for this example are listed below and are stored in the **Example 1** settings file. To load these settings to the procedure window, click **Open Example Settings File** in the Help Center or File menu.

Design Tab

Solve For	**Power**
Alpha for All Terms	**0.05**
Number of Factors	**1**
A - Levels	**3**
A - Means	**List of Means**
A - List of Means	**15.75 18.25 20.50**
σ (Block-Treatment Interaction)	**1.0672**
Number of Blocks	**2 3 4 5**

Output

Click the Calculate button to perform the calculations and generate the following output.

Numeric Results

Solve For: Power

		Number of		Degrees of Freedom		Standard Deviation of Means	Block-Treatment Interaction	Effect Size	
Term	**Power**	**Blocks**	**Units**	**df1**	**df2**	**σm**	**σ**	**σm/σ**	**Alpha**
A	0.42132	2	6	2	2	1.94007	1.0672	1.81791	0.05
A	0.89376	3	9	2	4	1.94007	1.0672	1.81791	0.05
A	0.99144	4	12	2	6	1.94007	1.0672	1.81791	0.05
A	0.99956	5	15	2	8	1.94007	1.0672	1.81791	0.05

Power	The probability of rejecting a false null hypothesis when the alternative hypothesis is true.
Blocks	The number of blocks in the design.
Units	The number of experimental units in the design.
df1	The numerator degrees of freedom.
df2	The denominator degrees of freedom.
σm	The standard deviation of the group means or effects.
σ	The pooled block-treatment interaction.
σm/σ	The Effect Size.
Alpha	The probability of rejecting a true null hypothesis.

Summary Statements

A randomized block design with one blocking factor and one treatment (fixed) factor with 3 levels will be used to test whether there is a difference among the levels of the factor. The factor will be tested using a randomized block analysis of variance F-test with a Type I error rate (α) of 0.05. The common standard deviation (square root of the block-by-treatment interaction mean square) is assumed to be 1.0672. The number of blocks is 2. For factor A, to detect a standard deviation among level means of 1.94007 (an effect size of 1.81791), the power is 0.42132.

References

Odeh, R.E. and Fox, M. 1991. Sample Size Choice. Marcel Dekker, Inc. New York, NY.
Winer, B.J. 1991. Statistical Principles in Experimental Design. Third Edition. McGraw-Hill. New York, NY.

This report shows the power for each of the four block counts. We see that adequate power of about 0.9 would have been achieved by three blocks.

It is important to emphasize that these power values are for the case when the effects associated with the alternative hypotheses are equal to those given by the data. It will often be informative to calculate the power for other values as well.

Term

This is the term (main effect or interaction) from the analysis of variance model being displayed on this line.

Power

This is the power of the *F* test for this term. Note that since adding and removing terms changes the denominator degrees of freedom (*df2*), the power depends on which other terms are included in the model.

Blocks

This is the number of blocks in the design.

Units

This is the number of sub-blocks (plots) in the design. It is the product of the number of treatment levels and the number of blocks.

df1

This is the numerator degrees of freedom of the *F* test.

df2

This is the denominator degrees of freedom of the *F* test. This value depends on which terms are included in the AOV model.

Standard Deviation of Means (σm)

This is the standard deviation of the means (or effects). It represents the size of the differences among the effects that is to be detected by the analysis. If you have entered hypothesized means, only their standard deviation is displayed here.

Block-Treatment Interaction (σ)

This is the pooled (averaged) block-treatment interaction mean square.

Effect Size (σm/ σ)

This is the standard deviation of the means divided by the block-treatment interaction. It provides an index of the magnitude of the difference among the means that can be detected by this design.

Alpha

This is the significance level of the *F* test. This is the probability of a type-I error given the null hypothesis of equal means and zero effects.

Example 2 – Validation using Prihoda (1983)

Prihoda (1983) presents details of an example that is given in Odeh and Fox (1991). In this example, *Alpha* is 0.025, *Sm* of *A* is 0.577, the number of treatments in factor *A* is 6, the number of treatments in factor *B* is 3, S is 1.0, and the Number of Blocks is 2, 3, 4, 5, 6, 7, and 8. Prihoda gives the power values for the *F* test on factor *A* as 0.477, 0.797, 0.935, 0.982, 0.995, 0.999, and 1.000.

Setup

If the procedure window is not already open, use the PASS Home window to open it. The parameters for this example are listed below and are stored in the **Example 2** settings file. To load these settings to the procedure window, click **Open Example Settings File** in the Help Center or File menu.

Design Tab

Solve For .... **Power**
Alpha for All Terms **0.025**
Number of Factors **2**
A - Levels... **6**
A - Means .... **As a Std Dev**
A - Std Dev **0.577**
B - Levels... **3**
B - Means .... **As a Std Dev**
B - Std Dev **1**
A*B... **Checked**
A*B - Effects .... **Std Dev of Effects**
A*B - Std Dev... **1**
σ (Block-Treatment Interaction) **1.0**
Number of Blocks .... **2 to 8 by 1**

Output

Click the Calculate button to perform the calculations and generate the following output.

Numeric Results

Solve For: Power

		Number of		Degrees of Freedom		Standard Deviation of Means	Block-Treatment Interaction	Effect Size	
Term	Power	Blocks	Units	df1	df2	σm	σ	σm/σ	Alpha
A	**0.47622**	2	36	5	17	0.577	1	0.577	0.025
B	0.99697	2	36	2	17	1.000	1	1.000	0.025
A*B	0.85337	2	36	10	17	1.000	1	1.000	0.025
A	**0.79521**	3	54	5	34	0.577	1	0.577	0.025
B	0.99999	3	54	2	34	1.000	1	1.000	0.025
A*B	0.99615	3	54	10	34	1.000	1	1.000	0.025
A	**0.93479**	4	72	5	51	0.577	1	0.577	0.025
B	1.00000	4	72	2	51	1.000	1	1.000	0.025
A*B	0.99995	4	72	10	51	1.000	1	1.000	0.025
A	**0.98226**	5	90	5	68	0.577	1	0.577	0.025
B	1.00000	5	90	2	68	1.000	1	1.000	0.025
A*B	1.00000	5	90	10	68	1.000	1	1.000	0.025
A	**0.99573**	6	108	5	85	0.577	1	0.577	0.025
B	1.00000	6	108	2	85	1.000	1	1.000	0.025
A*B	1.00000	6	108	10	85	1.000	1	1.000	0.025
A	**0.99907**	7	126	5	102	0.577	1	0.577	0.025
B	1.00000	7	126	2	102	1.000	1	1.000	0.025
A*B	1.00000	7	126	10	102	1.000	1	1.000	0.025
A	**0.99981**	8	144	5	119	0.577	1	0.577	0.025
B	1.00000	8	144	2	119	1.000	1	1.000	0.025
A*B	1.00000	8	144	10	119	1.000	1	1.000	0.025

We have bolded the power values on this report that should match Prihoda’s results. You see that they do match.