Chapter 586

Studentized Range Tests

Introduction

This procedure computes power and sample size of tests of whether the means of two or more groups which are analyzed using a studentized range test are different. This is a two-sided tested.

Methodology for testing equality among three or more groups has received little attention. An article by Shieh (2018) gives results for two competing test procedures: The F-test and the studentized range test. Results for the F-test are available in **PASS** in another procedure. This procedure provides power and sample size results the studentized range test.

While the F-test is by far the most commonly used method for testing the equality of two or more means, Shieh (2018) showed that neither test is always optimal. In fact, the studentized range test is more powerful when the actual range is close to zero.

Technical Details for the Studentized Range Test

Suppose G groups each have a normal distribution and with means $\mu_1, \mu_2, \dots, \mu_G$ and common variance σ^2. Let $N_1, N_2, \dots, N_G = N_i$ denote the common sample size of all groups and let N denote the total sample size. In this case of equal group sizes, $N = GN_i$. The multigroup test problem requires one to show that the means are different. Shieh (2018) also considered whether the difference between the minimum and maximum means (the range of the means) is sufficiently small so that the differences among the means can be regarded as of no practical importance. These results are available in **PASS** in a companion procedure.

The One-Way Model

Consider the usual one-way fixed-effects model

$$Y_{gj} = \mu_g + \varepsilon_{gj}$$

where Y_{gj} is response, μ_g are the treatment means, and ε_{gj} are the independent, normally distributed error with zero mean and common variance σ^2. Here the subscript g indexes the G groups, and the subscript j indexes the N_i subjects in each group.

Cohen (1988) showed that hypotheses about the G means may be obtained using either the variance of the means in terms of the F-test or their range in terms of the studentized range.

Equality Hypothesis

The hypothesis of mean equality is

$$H_0: \frac{\delta}{\sigma} \leq 0 \quad \text{versus} \quad H_1: \frac{\delta}{\sigma} > 0$$

where $\delta = \mu_{Max} - \mu_{Min}$ represents the range.

Studentized Range Statistic

The studentized range statistic is defined as follows

$$Q = \frac{\left[\max_{g=1 \, to \, G}(\bar{Y}_g) - \min_{g=1 \, to \, G}(\bar{Y}_g)\right]\sqrt{N_i}}{S}$$

where $\bar{Y}_g$ are the sample means and S is the sample variance.

It turns out that the distribution of Q is a function of the pairwise mean differences $\mu_g - \mu_h$, not just the range (the maximum of these differences).

The cumulative distribution function, from which the power can be computed, is given by

$$\Theta(q) = P\{Q \leq q\} = E_K\left\{\sum_{g=1}^{G} E_{Z_g}\left[\prod_{\substack{h=1 \\ h \neq g}}^{G}(\Phi\{Z_g + \delta_{gh}\sqrt{N_i}\} - \Phi\{Z_g + \delta_{gh}\sqrt{N_i} - q\sqrt{K/(N-G)}\})\right]\right\}$$

where $\delta_{gh} = \mu_g - \mu_h$, K is a chi-square random variable with $N - G$ degrees of freedom, $\Phi\{z\}$ is the CDF of a standard normal distribution, Z_g are independent standard normal random variables, $E_K\{x\}$ is the expectation with respect to K, and $E_{Z_g}\{x\}$ is the expectation with respect to Z_g.

Note that the critical value is based on the set of group means. When testing mean equality, this set of means is just

$$\{\mu_1, \dots, \mu_G\} = \{0, \dots, 0\}$$

If a sample size is desired, it can be determined using a standard binary search algorithm.

Example 1 – Finding Power

An experiment is being designed to test whether the range of the maximum difference among four group means is greater than zero. The hypothesis test will use studentized range test at a significance level of 0.05. Previous studies have shown a standard deviation of 2. Power calculations assume that the actual range is 2.

To better understand the relationship between power and sample size, the researcher wants to compute the power for several group sample sizes between 10 and 50. The sample sizes will be equal across all groups.

Setup

If the procedure window is not already open, use the PASS Home window to open it. The parameters for this example are listed below and are stored in the **Example 1** settings file. To load these settings to the procedure window, click **Open Example Settings File** in the Help Center or File menu.

Design Tab

Solve For ....**Power**
Alpha..**0.05**
G (Number of Groups) **4**
Ni (Sample Size Per Group) **10 20 30 40 50**
μ1i's Input Type ....**Enter Range of Means|H1**
δ1 (Range of Means|H1)...............................**2**
σ (Standard Deviation)..................................**2**

Output

Click the Calculate button to perform the calculations and generate the following output.

Numeric Reports

Numeric Results

Solve For: Power
Number of Groups: 4

	Sample Size		Range of Means|H1	Standard Deviation	
Power	**Total N**	**Group Ni**	**δ1**	**σ**	**Alpha**
0.3996	40	10	2	2	0.05
0.7406	80	20	2	2	0.05
0.9105	120	30	2	2	0.05
0.9736	160	40	2	2	0.05
0.9931	200	50	2	2	0.05

Power   The probability of rejecting a false null hypothesis when the alternative hypothesis is true.
N   The total number of subjects in the study.
Ni   The number of subjects sampled per group.
δ1   The range of the group means assumed by the alternative hypothesis. It is the value at which the power is computed.
σ   The standard deviation of the responses for all groups.
Alpha   The probability of rejecting a true null hypothesis.

Summary Statements

A one-way design with 4 groups will be used to test whether the range (maximum difference in means) is different from 0. The comparison will be made using a studentized range test with a Type I error rate (α) of 0.05. The common within-group standard deviation of responses for all groups is assumed to be 2. To detect a range of group means of 2, with group sample sizes of 10 subjects per group (for a total of 40 subjects), the power is 0.3996.

Dropout-Inflated Sample Size

Dropout Rate	Sample Size N	Dropout-Inflated Enrollment Sample Size N'	Expected Number of Dropouts D
20%	40	50	10
20%	80	100	20
20%	120	150	30
20%	160	200	40
20%	200	250	50

Dropout Rate   The percentage of subjects (or items) that are expected to be lost at random during the course of the study and for whom no response data will be collected (i.e., will be treated as "missing"). Abbreviated as DR.

N   The evaluable sample size at which power is computed (as entered by the user). If N subjects are evaluated out of the N' subjects that are enrolled in the study, the design will achieve the stated power.

N'   The total number of subjects that should be enrolled in the study in order to obtain N evaluable subjects, based on the assumed dropout rate. N' is calculated by inflating N using the formula N' = N / (1 - DR), with N' always rounded up. (See Julious, S.A. (2010) pages 52-53, or Chow, S.C., Shao, J., Wang, H., and Lokhnygina, Y. (2018) pages 32-33.)

D   The expected number of dropouts. D = N' - N.

Dropout Summary Statements

Anticipating a 20% dropout rate, 50 subjects should be enrolled to obtain a final sample size of 40 subjects.

References

Shieh, G. 2018. 'On Detecting a Minimal Important Difference among Standardized Means'. Current Psychology, Vol 37, Pages 640-647. Doi: 10.1007/s12144-016-9549-5

Cohen, Jacob. 1988. Statistical Power Analysis for the Behavioral Sciences. Lawrence Erlbaum Associates. Hillsdale, New Jersey.

This report shows the numeric results of this power study.

Plots Section

Plots
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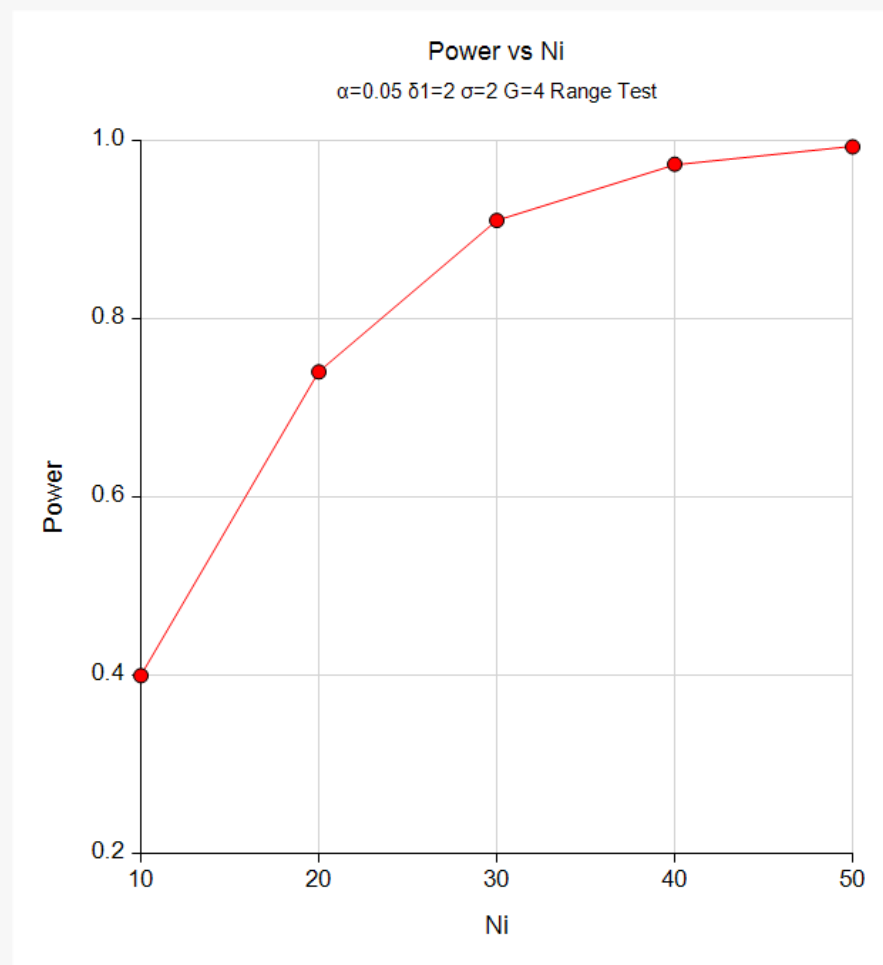
This plot gives a visual presentation of the results in the Numeric Report.

Example 2 – Finding the Sample Size Necessary to Reject

Continuing with the last example, we will determine how large the sample size would need to have been for alpha = 0.05 and power = 0.80 or 0.9.

Setup

If the procedure window is not already open, use the PASS Home window to open it. The parameters for this example are listed below and are stored in the **Example 2** settings file. To load these settings to the procedure window, click **Open Example Settings File** in the Help Center or File menu.

Design Tab

Solve For ....**Sample Size**
Power...**0.8 0.9**
Alpha...**0.05**
G (Number of Groups) **4**
μ1i's Input Type ....**Enter Range of Means|H1**
δ1 (Range of Means|H1)................................**2**
σ (Standard Deviation)..................................**2**

Output

Click the Calculate button to perform the calculations and generate the following output.

Numeric Results
───
Solve For: Sample Size
Number of Groups: 4
───

| | Sample Size | | Range of Means|H1 | Standard Deviation | |
Power	Total N	Group Ni	δ1	σ	Alpha
0.8079	92	23	2	2	0.05
0.9105	120	30	2	2	0.05

This report shows the necessary sample sizes for achieving powers of 0.8 and 0.9.

Example 3 – Validation using Shieh (2018)

Shieh (2018) page 644 presents an example in which alpha = 0.05, G = 3, σ = 3.189, means under alternative hypothesis are {7.77, 9.77, 6.68}, and power = 0.8. The sample size was stated as being 22 per group for a total of 66.

Setup

If the procedure window is not already open, use the PASS Home window to open it. The parameters for this example are listed below and are stored in the **Example 3** settings file. To load these settings to the procedure window, click **Open Example Settings File** in the Help Center or File menu.

Design Tab

Solve For ....**Sample Size**
Power...**0.8**
Alpha..**0.05**
G (Number of Groups) **3**
μ1i's Input Type ....**Enter μ11, μ12, ..., μ1G**
μ11, μ12, ..., μ1G..**7.77 9.77 6.68**
σ (Standard Deviation)..................................**3.189**

Output

Click the Calculate button to perform the calculations and generate the following output.

Numeric Results

Solve For: Sample Size
Number of Groups: 3

| | Sample Size | | Group Means|H1 | | | |
| | Total | Group | Means | Range | Standard Deviation | |
Power	N	Ni	μ1i	δ1	σ	Alpha
0.8187	66	22	μ1i(1)	3.09	3.189	0.05

Item	Values
μ1i(1)	7.77, 9.77, 6.68

PASS also found Ni = 22 and N = 66.