

## Chapter 247

# Superiority by a Margin Tests for the Ratio of Two Poisson Rates in a Cluster-Randomized Design

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### Introduction

This procedure calculates power and sample size for superiority by a margin tests of the ratio of two rates in a cluster-randomized design in which the outcome variable is a count. It uses the work of Wang, Zhang, and Ahn (2018) which give the power for the case of varying cluster sizes. The analysis uses a simple z-test comparing the two rates.

Cluster-randomized designs are those in which whole clusters of subjects (classes, hospitals, communities, etc.) are put into the treatment group or the control group. Generally speaking, the larger the cluster sizes and the higher the correlation among subjects within the same cluster, the larger will be the overall sample size necessary to detect an effect with the same power.

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### Hypotheses

When higher rates are better, the superiority by a margin test hypotheses are

$$H_0: \frac{\lambda_1}{\lambda_2} \leq R_0 \quad \text{vs.} \quad H_1: \frac{\lambda_1}{\lambda_2} > R_0$$

where  $R_0 > 1$ .

When higher rates are worse, the superiority by a margin test hypotheses are

$$H_0: \frac{\lambda_1}{\lambda_2} \geq R_0 \quad \text{vs.} \quad H_1: \frac{\lambda_1}{\lambda_2} < R_0$$

where  $R_0 < 1$ .

## Technical Details

The following discussion summarizes the results in Wang, Zhang, and Ahn (2018).

Suppose you are interested in comparing the incidence rates of two groups (treatment and control) with a superiority by a margin test. Further suppose that the response is known to be related to other covariates (such as age, race, or gender) and so their impact needs to be adjusted for. This may be accomplished by stratifying on the covariates and forming hypotheses about a common mean difference across all clusters and strata. Often, the stratification is based on cluster size, but this is not required.

Let  $Y_{jki}$  be the count outcome of the  $i$ th ( $i = 1, \dots, M_{jk}$ ) subject in the  $k$ th ( $k = 1, \dots, K_j$ ) cluster of the  $j$ th ( $j = 1, 2$ ) group. Assuming that  $Y_{jki}$  follows a Poisson model with

$$\lambda_j = E(Y_{jki}) = \text{Var}(Y_{jki})$$

and a common intracluster correlation coefficient (ICC)  $\rho = \text{corr}(Y_{jki}, Y_{jki'})$  for  $i \neq i'$ .

## Test Statistic

An unbiased estimator of  $\lambda_j$  is

$$\hat{\lambda}_j = \frac{\sum_{k=1}^{K_j} \sum_{i=1}^{M_{jk}} Y_{jki}}{\sum_{k=1}^{K_j} M_{jk}}$$

with estimated variance

$$s_j^2 = \frac{\hat{\lambda}_j \sum_{k=1}^{K_j} M_{jk} [1 + (M_{jk} - 1)\hat{\rho}]}{\left(\sum_{k=1}^{K_j} M_{jk}\right)^2}$$

Using a one-sided hypothesis  $H_0: \lambda_1 = \lambda_2$  versus  $H_1: \lambda_1 < \lambda_2$ ,  $H_0$  is rejected if

$$\frac{\hat{\lambda}_1 - \hat{\lambda}_2}{\sqrt{s_1^2 + s_2^2}} > z_{1-\alpha}$$

## Sample Size and Power

Wang, Zhang, and Ahn (2018) provide the following formula for estimating  $K_2$  (the number of clusters in the control group) when there are an unequal number of clusters in each group.

$$K_2 = \frac{\left(z_{1-\frac{\alpha}{2}} + z_{1-\beta}\right)^2 \left(\frac{\lambda_1}{R} + \lambda_2\right)}{(\lambda_1 - \lambda_2)^2} \left[ (1 - \rho) \frac{1}{\theta} + \rho + \rho\gamma^2 \right]$$

where  $R = K_1/K_2$ ,  $M = E(M_{jk})$ ,  $\tau^2 = \text{Var}(M_{jk})$ , and  $\gamma = \tau/M$ . Here  $M$  is the average cluster size of all clusters in the study and  $\gamma$  is the coefficient of variation of the cluster sizes. Note that  $z_x = \Phi(x)$  is the standard normal distribution function.

This equation can easily be rearranged to provide a formula for power  $(1 - \beta)$ .

## Example 1 – Finding the Number of Clusters

A superiority study is being planned to investigate whether a new intervention will decrease the incidence rate of a certain disease over the rate achieved by the current intervention. The response is a count. For a number of reasons, the researchers decide to administer the intervention to whole clusters (clinics) rather than randomize the treatment to individuals within a cluster. The number of clinics receiving each treatment will be balanced.

The average number of subjects per clinic is 21. The coefficient of variation of the cluster sizes is 0.42.

Prior studies have obtained an incidence rate of 0.35 for the current treatment and an ICC of 0.07. The superiority ratio is set to 0.86. The researchers want to compare the necessary sample size when the actual ratio is 0.43, 0.57, and 0.71.

The one-sided significance level is set to 0.025 and the power is set to 0.8.

### Setup

This section presents the values of each of the parameters needed to run this example. First, from the PASS Home window, load this procedure. You may then make the appropriate entries as listed below, or open **Example 1** by going to the **File** menu and choosing **Open Example Template**.

<u>Option</u>	<u>Value</u>
<b>Design Tab</b>	
Solve For .....	<b>K1 (Number of Clusters in Group 1)</b>
Higher Rates Are .....	<b>Worse (H1: <math>\lambda_1 / \lambda_2 &lt; R_0</math>, where <math>R_0 &lt; 1</math>)</b>
Power.....	<b>0.80</b>
Alpha.....	<b>0.025</b>
K2 (Clusters in Group 2).....	<b>K1</b>
M (Average of Cluster Sizes).....	<b>21</b>
CV (Coef of Variation of Cluster Sizes) .....	<b>0.42</b>
Incidence Rate Input Type.....	<b>Ratios (R0 and R1)</b>
R0 (Superiority Ratio) .....	<b>0.86</b>
R1 (Actual Ratio) .....	<b>0.43 0.57 0.71</b>
$\lambda_2$ (Incidence Rate of Control Group).....	<b>0.35</b>
$\rho$ (Intracluster Correlation, ICC).....	<b>0.07</b>

### Annotated Output

Click the Calculate button to perform the calculations and generate the following output.

### Numeric Results

#### Numeric Results

Higher Rates Are: Worse

Hypotheses:  $H_0: \lambda_1 / \lambda_2 \geq R_0$  vs.  $H_1: \lambda_1 / \lambda_2 < R_0$

	Sample Size	Clusters		Cluster Size		Incidence Rates			Rate Ratios			Alpha
		Grp 1	Grp 2	Ave	COV	Sup	Act	Cntl	Sup	Act	ICC	
Power	N	K1	K2	M	CV	$\lambda_{1.0}$	$\lambda_{1.1}$	$\lambda_2$	R0	R1	$\rho$	
0.80020	7854	187	187	21	0.42	0.301	0.1505	0.35	0.86	0.43	0.07	0.025
0.80153	8736	208	208	21	0.42	0.301	0.1995	0.35	0.86	0.57	0.07	0.025
0.80090	9576	228	228	21	0.42	0.301	0.2485	0.35	0.86	0.71	0.07	0.025

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### References

Wang, J., Zhang, S., and Ahn, C. 2018. 'Sample size calculation for count outcomes in cluster randomization trials with varying cluster sizes.' Communications in Statistics--Theory and Methods, DOI: 10.1080/03610926.2018.1532004.

### Report Definitions

Power is the probability of rejecting a false null hypothesis. It should be close to one.

N is the total sample size, i.e., the total number of subjects from all clusters.

K1 is the number of clusters assigned to group 1, the treatment group.

K2 is the number of clusters assigned to group 2, the control group.

M is the average cluster size. It is the anticipated average number of subjects per cluster.

CV is the coefficient of variation of the cluster sizes. It is the standard deviation of cluster sizes divided by the average cluster size.

$\lambda_{1.0}$  is the superiority bound on incidence rate in the treatment group.

$\lambda_{1.1}$  is the actual incidence rate of the treatment group at which the power is computed.

$\lambda_2$  is the incidence rate in the control group.

R0 is the incidence rate ratio assumed by the null hypothesis. This is often called the superiority boundary.  $R0 = \lambda_{1.0} / \lambda_2$ .

R1 is the incidence rate ratio assumed by the alternative hypothesis. This is the ratio at which the power is computed.  $R1 = \lambda_{1.1} / \lambda_2$ .

$\rho$  is the intracluster correlation coefficient. This is the correlation between any two subjects within a particular cluster.

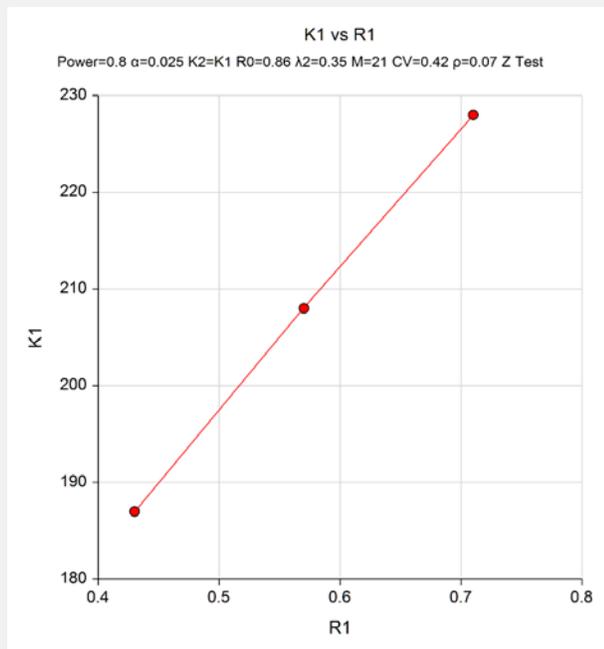
Alpha is the probability of rejecting a true null hypothesis.

### Summary Statements

A cluster-randomized, superiority by a margin design consists of 187 treatment clusters and 187 control clusters. Each cluster is assumed to have an average of 21 subjects per cluster. The outcome of the study will be analyzed by comparing the ratio of incidence rates of a treatment and a control group to a superiority margin. Higher values of these incidence rates are assumed to be worse. The expected sample size of 7854 subjects achieves 80% power to reject non-superiority when the actual ratio is 0.43. The superiority bound of the incidence rate ratio is 0.86. The incidence rate of the control group 0.35. The incidence rate of the treatment group assumed by the alternative hypothesis is 0.1505. The incidence rate of the treatment group which is the superiority bound is 0.301. The intracluster correlation coefficient is 0.07. The significance level of the test is 0.025. The individual cluster sizes (the number of subjects per cluster) are assumed to vary according to a discrete distribution with mean 21 and coefficient of variation 0.42.

## Chart Section

### Chart Section



The values from the Numeric Results report are displayed in this plot.

## Example 2 – Validation using a Previously Validated Procedure

We could not find a validation example in the literature so we will use a previously validated procedure in **PASS** to validate this procedure.

Suppose in a superiority design, higher rates are better, power = 0.9, alpha = 0.025, K2 = K1, M = 50, CV = 0.2,  $\lambda_{1.0} = 0.6$ ,  $\lambda_{1.1} = 0.5$ ,  $\lambda_2 = 0.5$ , and  $\rho = 0.002$ . Solve for K1.

This scenario can be solved using the procedure “Tests for the Difference Between Two Poisson Rates in a Cluster-Randomized Design” with the following settings.

Set the alternative hypothesis to “One-Sided ( $H_1: \delta > 0$ )”, power = 0.9, alpha = 0.025, K2 = K1, M = 50, CV = 0.2,  $\lambda_1 = 0.6$ ,  $\lambda_2 = 0.5$ , and  $\rho = 0.002$ . The solution is K1 = K2 = 26.

### Setup

This section presents the values of each of the parameters needed to run this example. First, from the PASS Home window, load this procedure. You may then make the appropriate entries as listed below, or open **Example 2** by going to the **File** menu and choosing **Open Example Template**.

Option	Value
<b>Design Tab</b>	
Solve For .....	<b>K1 (Number of Clusters in Group 1)</b>
Higher Rates Are .....	<b>Better (H1: <math>\lambda_1 / \lambda_2 &gt; R_0</math>, where <math>R_0 &gt; 1</math>)</b>
Power.....	<b>0.9</b>
Alpha.....	<b>0.025</b>
K2 (Clusters in Group 2).....	<b>K1</b>
M (Average of Cluster Sizes).....	<b>50</b>
CV (Coef of Variation of Cluster Sizes) .....	<b>0.2</b>
Incidence Rate Input Type.....	<b>Incidence Rates (<math>\lambda_{1.0}</math> and <math>\lambda_{1.1}</math>)</b>
$\lambda_{1.0}$ (Superiority Incidence Rate) .....	<b>0.6</b>
$\lambda_{1.1}$ (Actual Incidence Rate) .....	<b>0.5</b>
$\lambda_2$ (Incidence Rate of Control Group).....	<b>0.5</b>
$\rho$ (Intracluster Correlation, ICC).....	<b>0.002</b>

### Output

Click the Calculate button to perform the calculations and generate the following output.

#### Numeric Results

<b>Numeric Results</b>													
Higher Rates Are: Better													
Hypotheses: H0: $\lambda_1 / \lambda_2 \leq R_0$ vs. H1: $\lambda_1 / \lambda_2 > R_0$													
	Sample Size	Clusters		Cluster Size		Incidence Rates			Rate Ratios		ICC	Alpha	
		Grp 1	Grp 2	Ave	COV	Sup	Act	Cntl	Sup	Act			
Power	N	K1	K2	M	CV	$\lambda_{1.0}$	$\lambda_{1.1}$	$\lambda_2$	R0	R1	$\rho$		
0.90572	2600	26	26	50	0.2	0.6	0.5	0.5	1.2	1	0.002	0.025	

**PASS** has also obtained K1 = K2 = 26. Thus, the procedure is validated.