Chapter 810

Tests for Intraclass Correlation

Introduction

The intraclass correlation coefficient is often used as an index of reliability in a measurement study. In these studies, there are K observations made on each of N individuals. These individuals represent a factor observed at random. This design arises when N subjects are each rated by K raters.

The intraclass correlation coefficient may be thought of as the correlation between any two observations made on the same subject. When this correlation is high, the observations on a subject tend to match, and the measurement reliability is 'high.'

Technical Details

Our formulation comes from Walter, Eliasziw, and Donner (1998) and Winer (1991). However, to be consistent within PASS, we have switched N and K. Denote response j of subject i by Y_{ij}, where $i = 1, 2, \ldots, N$ and $j = 1, 2, \ldots, K$. The model for this situation is

$$Y_{ij} = \mu + a_i + e_{ij}$$

where the random subject effects a_i are normally distributed with mean 0 and variance σ_a^2 and the measurement errors, e_{ij} are normally distributed with mean 0 and variance σ_e^2. We assume that the subject effects and the measurement errors are independent. The intraclass correlation is then defined as

$$\rho = \frac{\sigma_a^2}{\sigma_a^2 + \sigma_e^2}$$

The hypothesis test is stated formally as

$$H_0: \rho = \rho_0$$
$$H_1: \rho = \rho_1 > \rho_0$$

This hypothesis is tested from the data of a one-way analysis of variance table using the value: $\dfrac{MS_a}{MS_e}$. The critical value for the test statistic is

$$C\left(F_{1-\alpha/2, df1, df2}\right)$$

where

$$C = 1 + \frac{K\rho_0}{1 - \rho_0}$$

$$df1 = N - 1$$

$$df2 = N(K - 1)$$

The power of this test procedure is given by

$$Power = 1 - P(F \geq C_0 F_{1-\alpha/2, df1, df2})$$

where

$$C_0 = \frac{1 + K\rho_0/(1 - \rho_0)}{1 + K\rho_1/(1 - \rho_1)}$$

Procedure Options

This section describes the options that are specific to this procedure. These are located on the Design tab. For more information about the options of other tabs, go to the Procedure Window chapter.

Design Tab

The Design tab contains most of the parameters and options that you will be concerned with.

Solve For

Solve For

This option specifies the parameter to be solved for from the other parameters. The parameters that may be selected are $\rho0$, $\rho1$, N, K, *Alpha*, and *Power*.

Under most situations, you will select either *Power* to calculate power or N to calculate sample size.

Note that the value selected here always appears as the vertical axis on the charts.

The program is set up to evaluate power directly. For the other parameters, a search is made using an iterative procedure until an appropriate value is found.

Power and Alpha

Power

This option specifies one or more values for power. Power is the probability of rejecting a false null hypothesis, and is equal to one minus Beta. Beta is the probability of a type-II error, which occurs when a false null hypothesis is not rejected.

Values must be between zero and one. Historically, the value of 0.80 was used for power. Now, 0.90 is also commonly used.

A single value may be entered here or a range of values such as *0.8 to 0.95 by 0.05* may be entered.

Alpha

This option specifies one or more values for the probability of a type-I error. A type-I error occurs when you reject the null hypothesis when it is true.

Values of alpha must be between zero and one. Historically, the value of 0.05 has been used for alpha. This means that about one test in twenty will falsely reject the null hypothesis. You should pick a value for alpha that represents the risk of a type-I error you are willing to take in your experimental situation.

You may enter a range of values such as *0.01 0.05 0.10* or *0.01 to 0.10 by 0.01*.

Sample Size

N (Number of Subjects)

Enter a value (or range of values) for the number of subjects, *N*, that are measured.

You may enter a range of values such as *50,150,250* or *50 to 300 by 50*.

K (Observations per Subject)

Enter a value (or range of values) for the number of observations, *K*, per subject. In a reliability study, this is the number of raters (assuming each subject is rated by all raters).

You may enter a range of values such as *2,3,4* or *2 to 12 by 2*.

Effect Size

ρ0 (Intraclass Correlation 0)

This is the value(s) of the intraclass correlation coefficient when the null hypothesis is true. You may enter a single value or a list of values. The range of $\rho0$ is between zero and *R1*.

The intraclass correlation is calculated as $V(A)/[V(A)+V(E)]$ where $V(E)$ is the variation within a subject and $V(A)$ is the variation between subjects. It is a measure of the extent to which the observations within a subject are similar (or dependent) relative to observations from other subjects.

ρ1 (Intraclass Correlation 1) > ρ0

This is the value(s) of the intraclass correlation coefficient when the alternative hypothesis is true. You may enter a single value or a list of values. The range of $\rho1$ is between $\rho0$ and one.

Example 1 – Calculating Power

Suppose that a study is to be conducted in which ρ0 = 0.2; ρ1 = 0.3; N = 50 to 250 by 100; alpha = 0.05; and K = 2 to 5 by 1; and power is to be calculated.

Setup

This section presents the values of each of the parameters needed to run this example. First, from the PASS Home window, load the **Tests for Intraclass Correlation** procedure window by expanding **Correlation**, then clicking on **Intraclass Correlation**, and then clicking on **Tests for Intraclass Correlation**. You may then make the appropriate entries as listed below, or open **Example 1** by going to the **File** menu and choosing **Open Example Template**.

Option	Value
Design Tab	
Solve For	**Power**
Alpha..	**0.05**
N (Number of Subjects)	**50 to 250 by 100**
K (Observations per Subject)	**2 to 5 by 1**
ρ0 (Intraclass Correlation 0)	**0.2**
ρ1 (Intraclass Correlation 1) > ρ0	**0.3**

Annotated Output

Click the Calculate button to perform the calculations and generate the following output.

Numeric Results

Power	N Number of Subjects	K Observations Per Subject	ρ0 Intraclass Correlation 0	ρ1 Intraclass Correlation 1	Alpha	Beta
0.18333	50	2	0.200	0.300	0.05000	0.81667
0.29534	50	3	0.200	0.300	0.05000	0.70466
0.38528	50	4	0.200	0.300	0.05000	0.61472
0.45522	50	5	0.200	0.300	0.05000	0.54478
0.36558	150	2	0.200	0.300	0.05000	0.63442
0.60094	150	3	0.200	0.300	0.05000	0.39906
0.74538	150	4	0.200	0.300	0.05000	0.25462
0.83005	150	5	0.200	0.300	0.05000	0.16995
0.51549	250	2	0.200	0.300	0.05000	0.48451
0.78790	250	3	0.200	0.300	0.05000	0.21210
0.90522	250	4	0.200	0.300	0.05000	0.09478
0.95403	250	5	0.200	0.300	0.05000	0.04597

Report Definitions
Power is the probability of rejecting a false null hypothesis. It should be close to one.
N is the number of subjects.
K is the number of observations per subject in the sample.
ρ0 is intraclass correlation assuming the null hypothesis.
ρ1 is intraclass correlation assuming the alternative hypothesis.
Alpha is the probability of rejecting a true null hypothesis. It should be small.
Beta is the probability of accepting a false null hypothesis. It should be small.

Summary Statements
A sample size of 50 subjects with 2 observations per subject achieves 18% power to detect an intraclass correlation of 0.300 under the alternative hypothesis when the intraclass correlation under the null hypothesis is 0.200 using an F-test with a significance level of 0.05000.

This report shows the power for each of the scenarios.

Plots Section
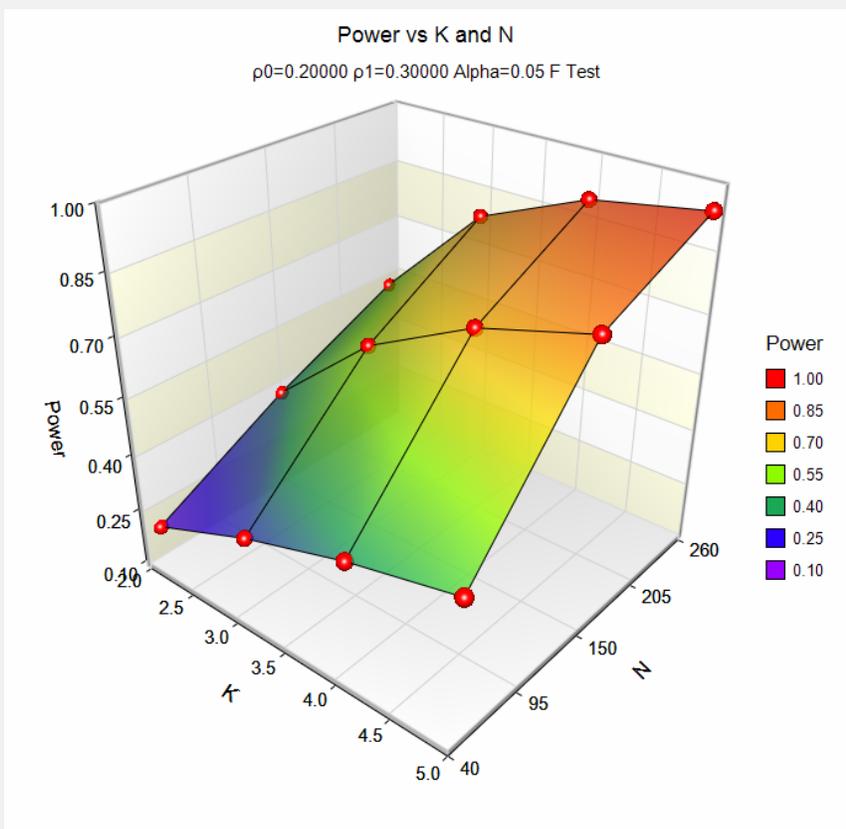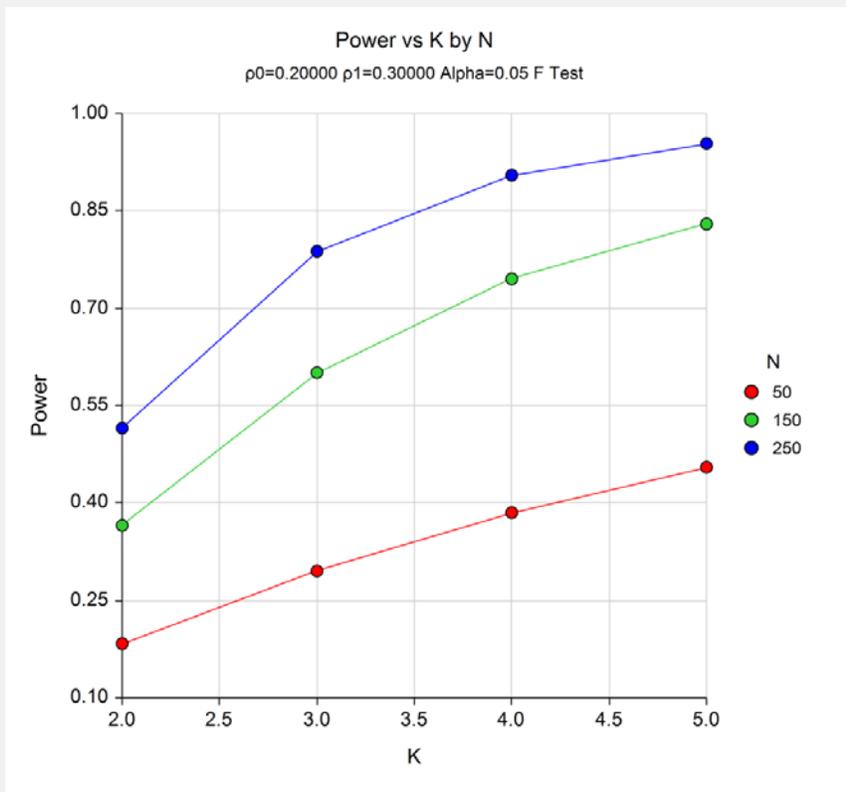
These plots show the relation between power, number of subjects, and observations per subject.

Example 2 – Validation using Walter

Walter *et al.* (1998) page 106 give a table of sample sizes. When ρ0 is 0.2, ρ1 is 0.3, N is 544, K is 2, and alpha is 0.05, the power is 0.80.

Setup

This section presents the values of each of the parameters needed to run this example. First, from the PASS Home window, load the **Tests for Intraclass Correlation** procedure window by expanding **Correlation**, then clicking on **Intraclass Correlation**, and then clicking on **Tests for Intraclass Correlation**. You may then make the appropriate entries as listed below, or open **Example 2** by going to the **File** menu and choosing **Open Example Template**.

Option	Value
Design Tab	
Solve For	**Power**
Alpha..	**0.05**
K (Number of Subjects)	**544**
N (Observations Per Subject).................	**2**
ρ0 (Intraclass Correlation 0)	**0.2**
ρ1 (Intraclass Correlation 1) > ρ0	**0.3**

Output

Click the Calculate button to perform the calculations and generate the following output.

Numeric Results

Power	N Number of Subjects	K Observations Per Subject	ρ0 Intraclass Correlation 0	ρ1 Intraclass Correlation 1	Alpha	Beta
0.80033	544	2	0.200	0.300	0.05000	0.19967

PASS has also calculated the power as 0.80.