Chapter 549

Tests for Multiple Poisson Rates in a One-Way Design

Introduction

This module calculates the power and sample size for count data from a one-way design for testing for differences among means of those groups. This is analogous to the classical one-way ANOVA design that is used to analyze means of normal outcomes. In the case of only two groups, this procedure produces power and sample size estimates that match those from the two independent-group Poisson means procedure.

The methods used in this chapter come from Mai and Zhang (2017). They proposed using a likelihood ratio test that has similar power characteristics to Poisson regression.

Technical Details

Test Statistic

Technical details are given in Mai and Zhang (2017).

Suppose we have N_g (g = 1, …, G) subjects in each of G groups for a total of N subjects. Let Y_{gj} be the count response (0, 1, 2, …) of subject j in group g. The response is modeled by the model $Y_{gj} \sim Poisson(\mu_g)$.

The log-likelihood ratio test statistic

$$\widehat{D} = -2 \sum_{g=1}^{G} N_g \{ \bar{y}_g [\ln(\bar{y}) - \ln(\bar{y}_g) + (\bar{y}_g - \bar{y})] \}$$

where $\bar{y}$ is the overall mean of Y and $\bar{y}_g$ is the mean of Y in group g.

Under the null hypothesis of equal means, this test statistic follows the chi-squared distribution with G – 1 degrees of freedom.

Effect Size

Cramer's V is used as a standardized effect size measure. It gives the association between the count outcome variable and the group variable as a proportion of the maximum possible variation.

The definition of the effect size is

$$V^2 = -2 \sum_{g=1}^{G} \frac{N_g}{N(G-1)} \left\{ \mu_g \left[\ln(\mu_0) - \ln(\mu_g) \right] + (\mu_g - \mu_0) \right\}$$

where μ_g is the mean for group g, and μ_0 is the overall mean of all groups.

The sample effect size is estimated by

$$\hat{V} = \sqrt{\frac{\hat{D}}{N(G-1)}}$$

Power

The power of this test is given by

$$\text{power} = Pr\{D \geq C | H_1\}$$

$$= Pr\{\chi^2(df, \delta) \geq C\}$$

$$= 1 - \Psi_{df,\delta}(C)$$

$$= 1 - \Psi_{G-1,N(G-1)V^2}\left[\chi^2_{1-\alpha}(G-1)\right]$$

where $\Psi_{df,\delta}$ is a cumulative distribution function of a noncentral chi-squared random variable with df degrees of freedom and noncentrality δ, and C is the critical value from a central chi-squared distribution with degrees of freedom $G - 1$ and significance level $1 - \alpha$.

Sample Size

The above power formula can be used in a binary search to obtain an appropriate value for the sample size.

Example 1 – Determining Power

Researchers are planning a study comparing three medications: a standard drug and two experimental drugs. The experimental drugs are expected to have about the same impact on response rate. The outcome is a count—a non-negative integer value.

Each subject will receive an application of only one drug. The response is a count variable. With the standard drug, this response has been about 3.4. The researchers want a sample size large enough to detect a difference of 0.6 in the response rate. They will use the means of 3.4, 2.8, and 2.8 to represent this difference.

The test will be conducted at the 0.05 significance level. The subjects will be randomly split equally among the three groups.

What powers are achieved by group sample sizes between 100 and 200?

Setup

If the procedure window is not already open, use the PASS Home window to open it. The parameters for this example are listed below and are stored in the **Example 1** settings file. To load these settings to the procedure window, click **Open Example Settings File** in the Help Center or File menu.

Design Tab

Solve For ....**Power**
Alpha..**0.05**
G (Number of Groups) **3**
Group Allocation Input Type ....**Equal (N1 = N2 = ⋯ = NG)**
Ni (Subjects Per Group)................................**100 120 140 160 180 200**
μi's Input Type ....**μ1, μ2, ..., μG**
μ1, μ2, ..., μG..**3.4 2.8 2.8**

Output

Click the Calculate button to perform the calculations and generate the following output.

Numeric Reports

Numeric Results

Solve For: Power
Number of Groups: 3

Power	Total Number of Subjects N	Subjects per Group Ni	Group Means Set μi	Effect Size Cramer's V	Alpha
0.7082	300	100	μi(1)	0.1143	0.05
0.7898	360	120	μi(1)	0.1143	0.05
0.8517	420	140	μi(1)	0.1143	0.05
0.8973	480	160	μi(1)	0.1143	0.05
0.9300	540	180	μi(1)	0.1143	0.05
0.9529	600	200	μi(1)	0.1143	0.05

Item	Values
μi(1)	3.4, 2.8, 2.8

Power	The probability of rejecting a false null hypothesis when the alternative hypothesis is true.
N	The total number of subjects in the study.
Ni	The number of subjects per group.
μi	Group Means Set. Gives the name and number of the set containing the mean responses for each group.
Cramer's V	A measure of the effect size. It is the association between the outcome variable and the grouping variable. It is the proportion of the maximum variation in the outcome variable accounted for by the grouping variable.
Alpha	The probability of rejecting a true null hypothesis.

Summary Statements

A one-way design with 3 groups will be used to test whether there is a difference among the Poisson event rates (means) of the 3 groups. The comparison will be made using a likelihood ratio chi-square test with 2 degrees of freedom, with a Type I error rate (α) of 0.05. To detect the event rates 3.4, 2.8, 2.8 (effect size = 0.1143), with group sample sizes of 100, 100, 100 (for a total of 300 subjects), the power is 0.7082.

Dropout-Inflated Sample Size

Dropout Rate	Sample Size N	Dropout-Inflated Enrollment Sample Size N'	Expected Number of Dropouts D
20%	300	375	75
20%	360	450	90
20%	420	525	105
20%	480	600	120
20%	540	675	135
20%	600	750	150

Dropout Rate The percentage of subjects (or items) that are expected to be lost at random during the course of the study and for whom no response data will be collected (i.e., will be treated as "missing"). Abbreviated as DR.
N The evaluable sample size at which power is computed (as entered by the user). If N subjects are evaluated out of the N' subjects that are enrolled in the study, the design will achieve the stated power.
N' The total number of subjects that should be enrolled in the study in order to obtain N evaluable subjects, based on the assumed dropout rate. N' is calculated by inflating N using the formula N' = N / (1 - DR), with N' always rounded up. (See Julious, S.A. (2010) pages 52-53, or Chow, S.C., Shao, J., Wang, H., and Lokhnygina, Y. (2018) pages 32-33.)
D The expected number of dropouts. D = N' - N.

Dropout Summary Statements

Anticipating a 20% dropout rate, 375 subjects should be enrolled to obtain a final sample size of 300 subjects.

References

Y. Mai and Z.Zhang. 2017. Statistical power analysis for comparing means with binary or count data based on analogous ANOVA. In L. A. van der Ark, M. Wiberg, S. A. Culpepper, J. A. Douglas, & W.-C. Wang (Eds.), Quantitative Psychology - The 81st Annual Meeting of the Psychometric Society, Asheville, North Carolina, 2016: Springer.
Z. Zhang and K.H. Yuan. 2018. Practical Statistical Power Analysis: Using WebPower and R. ISDSA Press. Granger, IN.

This report gives the sample size for each value of the other parameters.

Plots Section

Plots
_____
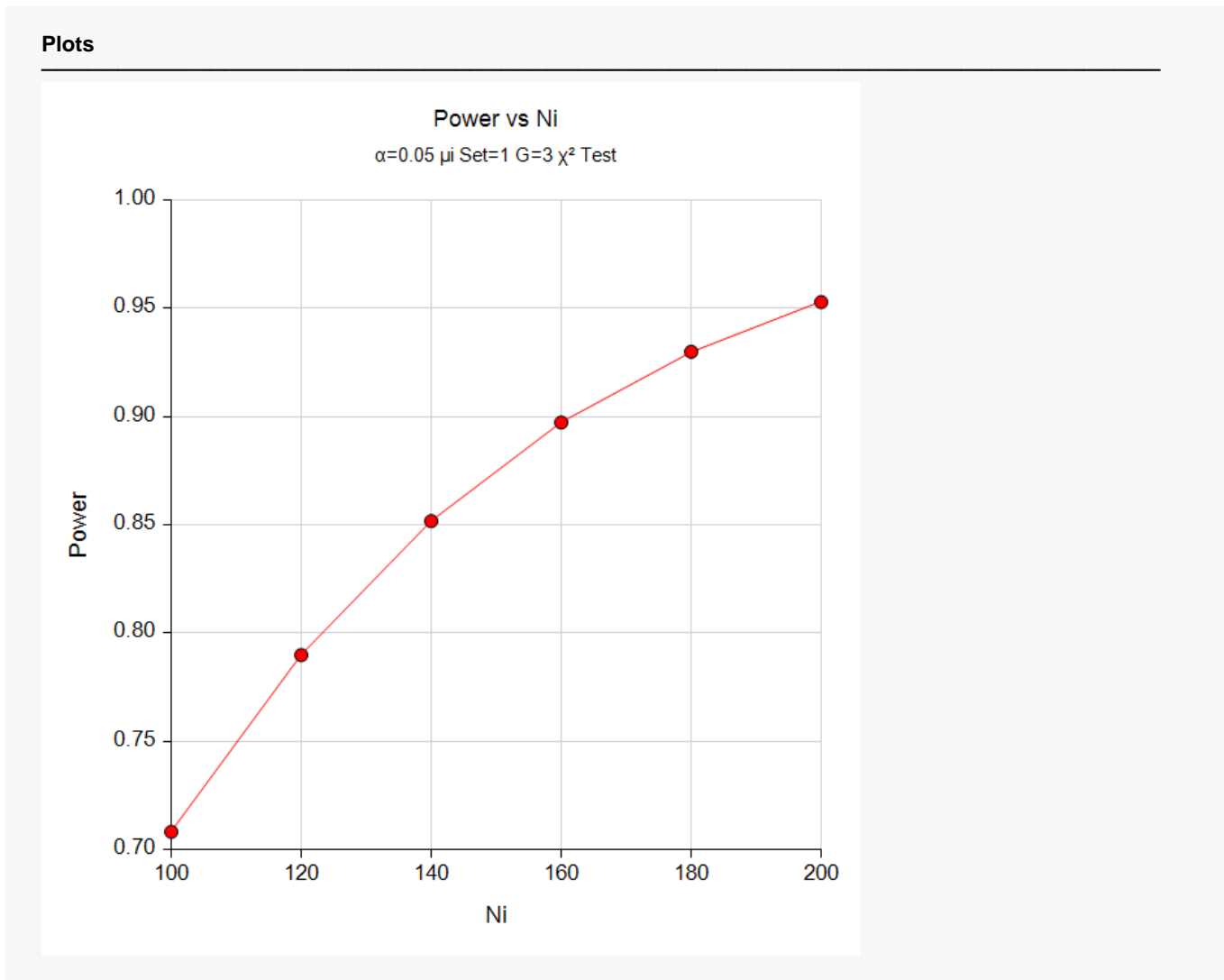
This shows the relationship among the design parameters.

Example 2 – Finding the Sample Size

Continuing with Example 1, the researchers want to determine the sample size corresponding to powers of 0.8 and 0.9.

Setup

If the procedure window is not already open, use the PASS Home window to open it. The parameters for this example are listed below and are stored in the **Example 2** settings file. To load these settings to the procedure window, click **Open Example Settings File** in the Help Center or File menu.

Design Tab

Solve For ....**Sample Size**
Power..**0.8 0.9**
Alpha..**0.05**
G (Number of Groups) **3**
Group Allocation Input Type ....**Equal (N1 = N2 = ⋯ = NG)**
μi's Input Type ....**μ1, μ2, ..., μG**
μ1, μ2, ..., μG...**3.4 2.8 2.8**

Output

Click the Calculate button to perform the calculations and generate the following output.

Numeric Results
_____

Solve For: Sample Size
Number of Groups: 3
_____

Power	Total Number of Subjects N	Group Subject Allocation Set ri	Group Means Set μi	Effect Size Cramer's V	Alpha
0.8003	369	ri(1)	μi(1)	0.1143	0.05
0.9011	486	ri(1)	μi(1)	0.1143	0.05

Item	Values
ri(1)	0.333, 0.333, 0.333
μi(1)	3.4, 2.8, 2.8

The sample sizes are 369 for a power of 0.8 and 486 for a power of 0.9. Note that the search routine only searches designs that have equal group sizes (i.e. that are divisible by 3 since there are 3 groups).

Example 3 – Comparing Various Effect Sizes

Continuing with Examples 1 and 2, the researchers want to compare the sample size of various sets of group means. To do this, they will compare the four sets of means shown in the following table. The first row is the mean for the standard medication. The second and third rows give the anticipated means of the experimental medications.

The values in this table must be loaded into the spreadsheet.

Table of Four Sets of Response Probabilities

C1	C2	C3	C4
3.4	3.4	3.4	3.4
2.8	3.0	3.1	2.8
2.8	3.0	3.1	3.1

Setup

If the procedure window is not already open, use the PASS Home window to open it. The parameters for this example are listed below and are stored in the **Example 3** settings file. To load these settings to the procedure window, click **Open Example Settings File** in the Help Center or File menu.

Design Tab

Solve For ....**Sample Size**
Power..**0.9**
Alpha..**0.05**
G (Number of Groups) **3**
Group Allocation Input Type ....**Equal (N1 = N2 = ··· = NG)**
μi's Input Type ....**Columns containing sets of μi's**
Columns Containing Sets of μi's **C1-C4**

Input Spreadsheet Data

Row	C1	C2	C3	C4
1	3.4	3.4	3.4	3.4
2	2.8	3.0	3.1	2.8
3	2.8	3.0	3.1	3.1

Output

Click the Calculate button to perform the calculations and generate the following output.
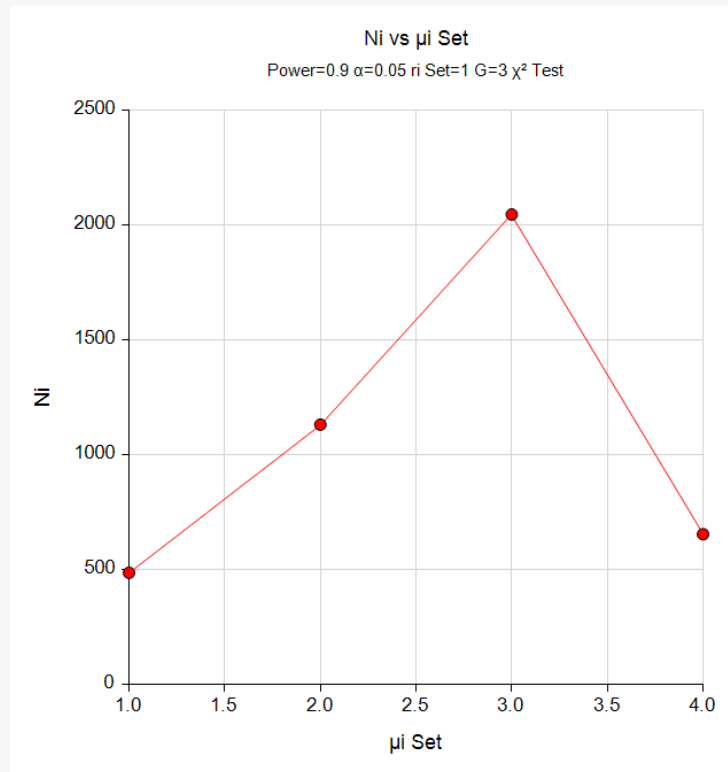
Numeric Results

Solve For: Sample Size
Number of Groups: 3

Power	Total Number of Subjects N	Group Subject Allocation Set ri	Group Means Set μi	Effect Size Cramer's V	Alpha
0.9011	486	ri(1)	C1(1)	0.1143	0.05
0.9002	1131	ri(1)	C2(2)	0.0748	0.05
0.9002	2046	ri(1)	C3(3)	0.0556	0.05
0.9006	654	ri(1)	C4(4)	0.0985	0.05

Item	Values
ri(1)	0.333, 0.333, 0.333
C1(1)	3.4, 2.8, 2.8
C2(2)	3.4, 3, 3
C3(3)	3.4, 3.1, 3.1
C4(4)	3.4, 2.8, 3.1

Plots

Note that the horizontal axis gives the index of the mean set footnote. Thus, '1.0' is column C1(1), '2.0' is column C2(2), and so on.

Example 4 – Validation of Power Calculation using Zhang and Yuan (2018)

Zhang and Yuan (2018) page 127-128 present an example in which G = 4, α = 0.05, and Ni = 25.

Their example provides the individual group means as μ1 = 3.48, μ2 = 4.24, μ3 = 3.12, and μ4 = 3.00. These result in a V of 0.148, so we will use these values in the validation.

Zhang and Yuan (2018) do not complete the power calculation, so we will do this using PASS's Probability Calculator. We find that the critical value of C = 7.814728 and that the noncentrality parameter is 100(3)(0.148)(0.148) = 6.5712. This results in a power of 0.5594 using the noncentral chi-squared calculator.

Setup

If the procedure window is not already open, use the PASS Home window to open it. The parameters for this example are listed below and are stored in the **Example 4** settings file. To load these settings to the procedure window, click **Open Example Settings File** in the Help Center or File menu.

Design Tab

Solve For ....**Power**
Alpha..**0.05**
G (Number of Groups) **4**
Group Allocation Input Type ....**Equal (N1 = N2 = ⋯ = NG)**
Ni (Subjects Per Group)................................**25**
μi's Input Type ....**μ1, μ2, ..., μG**
μ1, μ2, ..., μG...**3.48 4.24 3.12 3.00**

Output

Click the Calculate button to perform the calculations and generate the following output.

Numeric Results

Solve For: Power
Number of Groups: 4

Power	Total Number of Subjects N	Subjects per Group Ni	Group Means Set μi	Effect Size Cramer's V	Alpha
0.5594	100	25	μi(1)	0.1479522	0.05

Item	Values
μi(1)	3.48, 4.24, 3.12, 3

The power of 0.5594 matches the calculated power exactly, so the procedure is validated. The value of V of 0.148 matches the value from Zhang and Yuan (2018).