Chapter 815

Tests for One Coefficient Alpha

Introduction

Coefficient alpha, or *Cronbach's alpha*, is a measure of the reliability of a test consisting of *k* parts. The *k* parts usually represent *k* items on a questionnaire or *k* raters. This module calculates power and sample size for testing whether coefficient alpha, ρ, is different from a given value such as zero.

Technical Details

Feldt et al. (1987) has shown that if $\hat{\rho}$ is the estimated value of coefficient alpha computed from a sample of size *N* questionnaires with *k* items, the statistic *W* is distributed as an *F* ratio with degrees of freedom *N*-1 and (*k*-1)(*N*-1), where

$$W = \frac{1-\rho_0}{1-\hat{\rho}}$$

and ρ_0 is the value of ρ assumed by the null hypothesis, H0.

Calculating the Power

Using the above definition of *W*, the power of the significance test of $\rho > \rho_0$ is calculated as follows:

1. Find F_α such that $\text{Prob}\left(F_{1-\alpha,N-1,(k-1)(N-1)}\right) = 1-\alpha$
2. Compute $\rho_C = \frac{F_\alpha+\rho_0-1}{F_\alpha}$
3. Compute $W_1 = \frac{1-\rho_1}{1-\rho_c}$, where ρ_1 is the value of ρ at which the power is calculated.
4. Compute the power = $1 - \Pr\left(W_1 > F_{N-1,(k-1)(N-1)}\right)$

Example 1 – Finding the Power

Suppose a study is being designed to test whether the coefficient alpha is 0.6 against the two-sided alternative. Find the power when *K* = 20, alpha = 0.05, CA1 = 0.65 0.70 0.75, and *N* = 50 100 200 300 500 700 1000 and 1400.

Setup

If the procedure window is not already open, use the PASS Home window to open it. The parameters for this example are listed below and are stored in the **Example 1** settings file. To load these settings to the procedure window, click **Open Example Settings File** in the Help Center or File menu.

Design Tab	
Solve For	**Power**
Alternative Hypothesis	**H1: CA ≠ CA0**
Alpha	**0.05**
N (Sample Size)	**50 100 200 300 500 700 1000 1400**
K (Number of Items or Raters)	**20**
CA0 (Coefficient Alpha | H0)	**0.6**
CA1 (Actual Coefficient Alpha)	**0.65 0.70 0.75**

Output

Click the Calculate button to perform the calculations and generate the following output.

Numeric Reports

Numeric Results

Solve For: Power
Hypotheses: H0: CA = CA0 vs. H1: CA ≠ CA0

			Coefficient Alpha		
Power	Sample Size N	Number of Items K	Null CA0	Actual CA1	Alpha
0.11084	50	20	0.6	0.65	0.05
0.16444	100	20	0.6	0.65	0.05
0.27111	200	20	0.6	0.65	0.05
0.37314	300	20	0.6	0.65	0.05
0.55224	500	20	0.6	0.65	0.05
0.69191	700	20	0.6	0.65	0.05
0.83338	1000	20	0.6	0.65	0.05
0.93197	1400	20	0.6	0.65	0.05
0.31842	50	20	0.6	0.70	0.05
0.53499	100	20	0.6	0.70	0.05
0.81016	200	20	0.6	0.70	0.05
0.93111	300	20	0.6	0.70	0.05
0.99273	500	20	0.6	0.70	0.05
0.99937	700	20	0.6	0.70	0.05
0.99999	1000	20	0.6	0.70	0.05
1.00000	1400	20	0.6	0.70	0.05
0.65625	50	20	0.6	0.75	0.05
0.90261	100	20	0.6	0.75	0.05
0.99469	200	20	0.6	0.75	0.05
0.99978	300	20	0.6	0.75	0.05
1.00000	500	20	0.6	0.75	0.05
1.00000	700	20	0.6	0.75	0.05
1.00000	1000	20	0.6	0.75	0.05
1.00000	1400	20	0.6	0.75	0.05

Power The probability of rejecting a false null hypothesis when the alternative hypothesis is true.
N The total sample size.
K The number of items or raters.
CA0 The value of coefficient alpha under the null hypothesis.
CA1 The value of coefficient alpha at which the power is computed.
Alpha The probability of rejecting a true null hypothesis.

Summary Statements

A single-group coefficient alpha (or Cronbach's alpha) reliability design with 20 items (or raters) will be used to test whether the coefficient alpha (CA) is different from 0.6 (H0: CA = 0.6 versus H1: CA ≠ 0.6). The comparison will be made using a two-sided, one-sample coefficient alpha F-test, with a Type I error rate (α) of 0.05. To detect a coefficient alpha of 0.65 with a sample size of 50, the power is 0.11084.

Dropout-Inflated Sample Size

Dropout Rate	Sample Size N	Dropout-Inflated Enrollment Sample Size N'	Expected Number of Dropouts D
20%	50	63	13
20%	100	125	25
20%	200	250	50
20%	300	375	75
20%	500	625	125
20%	700	875	175
20%	1000	1250	250
20%	1400	1750	350

Dropout Rate	The percentage of subjects (or items) that are expected to be lost at random during the course of the study and for whom no response data will be collected (i.e., will be treated as "missing"). Abbreviated as DR.
N	The evaluable sample size at which power is computed (as entered by the user). If N subjects are evaluated out of the N' subjects that are enrolled in the study, the design will achieve the stated power.
N'	The total number of subjects that should be enrolled in the study in order to obtain N evaluable subjects, based on the assumed dropout rate. N' is calculated by inflating N using the formula N' = N / (1 - DR), with N' always rounded up. (See Julious, S.A. (2010) pages 52-53, or Chow, S.C., Shao, J., Wang, H., and Lokhnygina, Y. (2018) pages 32-33.)
D	The expected number of dropouts. D = N' - N.

Dropout Summary Statements

Anticipating a 20% dropout rate, 63 subjects should be enrolled to obtain a final sample size of 50 subjects.

References

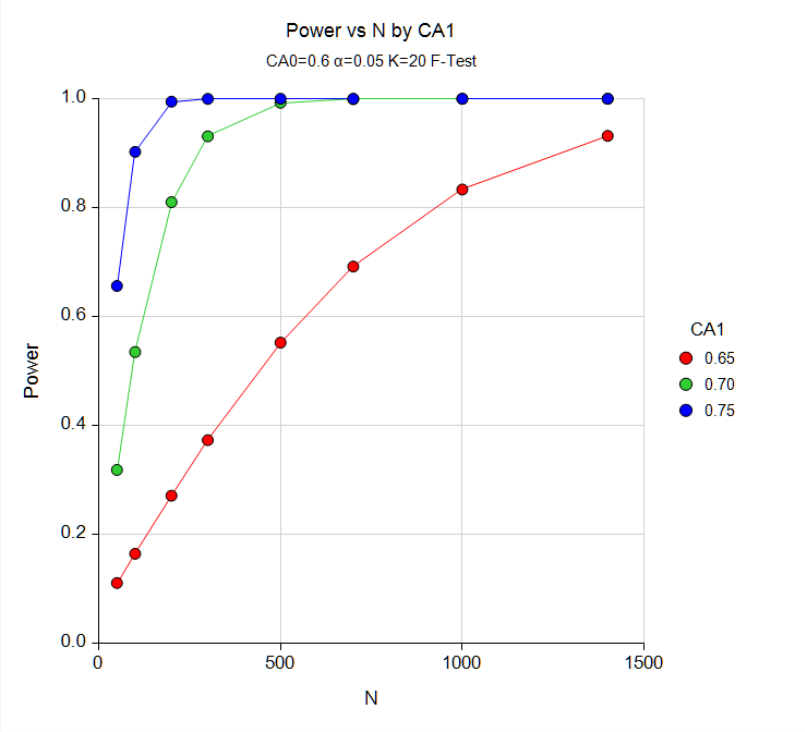
Bonett, Douglas. 2002. 'Sample Size Requirements for Testing and Estimating Coefficient Alpha.' Journal of Educational and Behavioral Statistics, Vol. 27, pages 335-340.

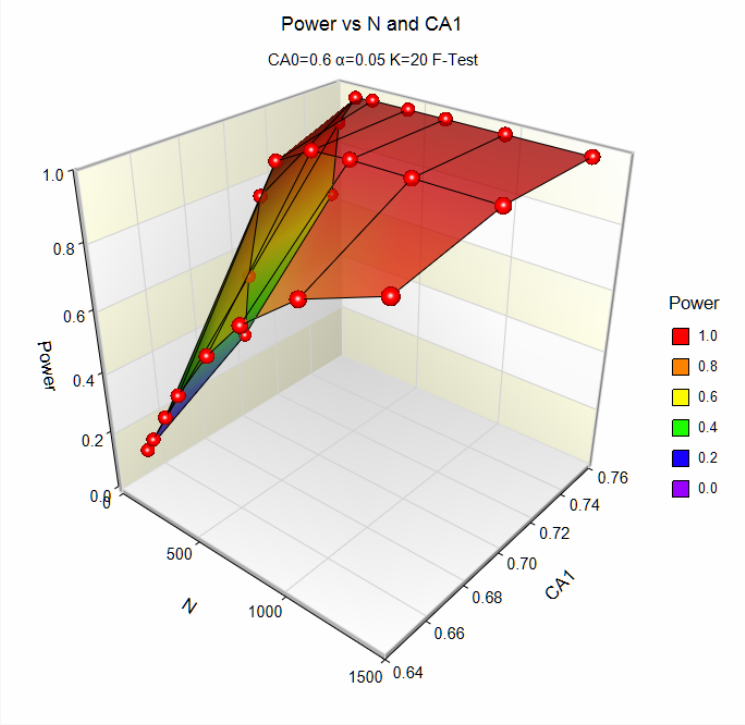
Feldt, L.S.; Woodruff, D.J.; & Salih, F.A. 1987. 'Statistical Inference for Coefficient Alpha.' Applied Psychological Measurement, Vol. 11, pages 93-103.

This report shows the values of each of the parameters, one scenario per row. The values from this table are plotted in the chart below.

Plots Section

Plots





These plots show the relationship between CA1, N, and power.

Example 2 – Finding the Sample Size

Continuing with Example 1, find the sample size necessary to achieve a power of 90% with a 0.05 significance level.

Setup

If the procedure window is not already open, use the PASS Home window to open it. The parameters for this example are listed below and are stored in the **Example 2** settings file. To load these settings to the procedure window, click **Open Example Settings File** in the Help Center or File menu.

Design Tab	
Solve For	**Sample Size**
Alternative Hypothesis	**H1: CA ≠ CA0**
Power	**0.90**
Alpha	**0.05**
K (Number of Items or Raters)	**20**
CA0 (Coefficient Alpha | H0)	**0.6**
CA1 (Actual Coefficient Alpha)	**0.65 0.70 0.75**

Output

Click the Calculate button to perform the calculations and generate the following output.

Numeric Results

Solve For: Sample Size
Hypotheses: H0: CA = CA0 vs. H1: CA ≠ CA0

			Coefficient Alpha		
Power	Sample Size N	Number of Items K	Null CA0	Actual CA1	Alpha
0.90022	1233	20	0.6	0.65	0.05
0.90073	265	20	0.6	0.70	0.05
0.90261	100	20	0.6	0.75	0.05

This report shows the dramatic increase in sample size that is needed to achieve the desired sample power as CA1 gets closer to CA0.

Example 3 – Validation using Bonett (2002)

Bonett (2002) page 337 presents a table in which the sample sizes were calculated for several parameter configurations. When CA0 = 0, CA1 = 0.50, alpha = 0.10, power = 0.95, and *k* = 2, 5, 10, and 100, he finds *N* to be 93, 59, 52, and 48, respectively.

Setup

If the procedure window is not already open, use the PASS Home window to open it. The parameters for this example are listed below and are stored in the **Example 3** settings file. To load these settings to the procedure window, click **Open Example Settings File** in the Help Center or File menu.

Design Tab	
Solve For	**Sample Size**
Alternative Hypothesis	**H1: CA ≠ CA0**
Power	**0.95**
Alpha	**0.1**
K (Number of Items or Raters)	**2 5 10 100**
CA0 (Coefficient Alpha | H0)	**0**
CA1 (Actual Coefficient Alpha)	**0.5**

Output

Click the Run button to perform the calculations and generate the following output.

Numeric Results

Solve For: Sample Size
Hypotheses: H0: CA = CA0 vs. H1: CA ≠ CA0

			Coefficient Alpha		
Power	Sample Size N	Number of Items K	Null CA0	Actual CA1	Alpha
0.95176	93	2	0	0.5	0.1
0.95253	59	5	0	0.5	0.1
0.95047	52	10	0	0.5	0.1
0.95213	48	100	0	0.5	0.1

The sample sizes match Bonett's results exactly.