Chapter 815

Tests for One Coefficient Alpha

Introduction

Coefficient alpha, or *Cronbach's alpha*, is a measure of the reliability of a test consisting of *k* parts. The *k* parts usually represent *k* items on a questionnaire or *k* raters. This module calculates power and sample size for testing whether coefficient alpha, ρ, is different from a given value such as zero.

Technical Details

Feldt et al. (1987) has shown that if $\hat{\rho}$ is the estimated value of coefficient alpha computed from a sample of size *N* questionnaires with *k* items, the statistic *W* is distributed as an *F* ratio with degrees of freedom *N*-1 and (*k*-1)(*N*-1), where

$$W = \frac{1-\rho_0}{1-\hat{\rho}}$$

and ρ_0 is the value of ρ assumed by the null hypothesis, H0.

Calculating the Power

Using the above definition of *W*, the power of the significance test of $\rho > \rho_0$ is calculated as follows:

1. Find F_α such that $\text{Prob}\left(F_{1-\alpha,N-1,(k-1)(N-1)}\right) = 1-\alpha$
2. Compute $\rho_C = \dfrac{F_\alpha + \rho_0 - 1}{F_\alpha}$
3. Compute $W_1 = \dfrac{1-\rho_1}{1-\rho_c}$, where ρ_1 is the value of ρ at which the power is calculated.
4. Compute the power = $1 - \Pr\left(W_1 > F_{N-1,(k-1)(N-1)}\right)$

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Procedure Options

This section describes the options that are specific to this procedure. These are located on the Design tab. For more information about the options of other tabs, go to the Procedure Window chapter.

Design Tab

The Design tab contains most of the parameters and options that you will be concerned with.

Solve For

Solve For

This option specifies the parameter to be calculated from the values of the other parameters. Under most conditions, you would either select *Power* or *Sample Size*.

Select *Sample Size* when you want to determine the sample size needed to achieve a given power and alpha error level.

Select *Power* when you want to calculate the power of an experiment.

Test

Alternative Hypothesis

This option specifies whether the alternative hypothesis is one-sided or two-sided. It also specifies the direction of the hypothesis test. The null hypothesis is $H_0: \rho_0 = \rho$.The alternative hypothesis enters into power calculations by specifying the rejection region of the hypothesis test. Its accuracy is critical.

Possible selections are:

- **H1: CA ≠ CA0**
  This is the most common selection. It yields the *two-tailed* test. Use this option when you are testing whether values are different, but you do not want to specify beforehand which is larger.

- **H1: CA < CA0**
  This option yields a *one-tailed* test.

- **H1: CA > CA0**
  This option yields a *one-tailed* test.

Power and Alpha

Power

This option specifies one or more values for power. Power is the probability of rejecting a false null hypothesis, and is equal to one minus Beta. Beta is the probability of a type-II error, which occurs when a false null hypothesis is not rejected.

Values must be between zero and one. Historically, the value of 0.80 (Beta = 0.20) was used for power. Now, 0.90 (Beta = 0.10) is also commonly used.

A single value may be entered here or a range of values such as *0.8 to 0.95 by 0.05* may be entered.

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Alpha

This option specifies one or more values for the probability of a type-I error. A type-I error occurs when you reject the null hypothesis when in fact it is true.

Values of alpha must be between zero and one. Historically, the value of 0.05 has been used for alpha. This means that about one test in twenty will falsely reject the null hypothesis. You should pick a value for alpha that represents the risk of a type-I error you are willing to take in your experimental situation.

You may enter a range of values such as *0.01 0.05 0.10* or *0.01 to 0.10 by 0.01*.

Sample Size

N (Sample Size)

Specify the number of observations in the sample. You may enter a range such as *10 to 100 by 10* or a list of values separated by commas or blanks such as *20 50 100*.

K (Number of Items or Raters)

K is the number of items or raters in the study. Since it is a count, it must be an integer greater than one. You may enter a list of values separated by blanks.

Effect Size

CA0 (Coefficient Alpha | H0)

Specify the value of ρ_0, the value of coefficient alpha under the null hypothesis. Usually, this value will be zero, but any value between -1 and 1 is valid as long as it is not equal to CA1.

You may enter a list of values separated by blanks such as *0 0.1 0.2*.

CA1 (Actual Coefficient Alpha)

Specify the value of ρ_1, the value of coefficient alpha at which the power is computed. Usually, this value is positive, but any value between -1 and 1 is valid as long as it is not equal to CA0.

You may enter a list of values separated by blanks such as *0.1 0.2 0.3*.

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Example 1 – Finding the Power

Suppose a study is being designed to test whether the coefficient alpha is 0.6 against the two-sided alternative. Find the power when K = 20, alpha = 0.05, CA1 = 0.65 0.70 0.75, and N = 50 100 200 300 500 700 1000 and 1400.

Setup

This section presents the values of each of the parameters needed to run this example. First, from the PASS Home window, load the **Tests for One Coefficient Alpha** procedure window by expanding **Correlation**, then clicking on **Coefficient (Cronbach's) Alpha**, and then clicking on **Tests for One Coefficient Alpha**. You may then make the appropriate entries as listed below, or open **Example 1** by going to the **File** menu and choosing **Open Example Template**.

Option	Value
Design Tab	
Solve For	**Power**
Alternative Hypothesis	**H1: CA ≠ CA0**
Alpha	**0.05**
N (Sample Size)	**50 100 200 300 500 700 1000 1400**
K (Number of Items or Raters)	**20**
CA0 (Coefficient Alpha | H0)	**0.6**
CA1 (Actual Coefficient Alpha)	**0.65 0.70 0.75**

Annotated Output

Click the Calculate button to perform the calculations and generate the following output.

Numeric Results

Numeric Results
Hypotheses: H0: CA = CA0 vs. H1: CA ≠ CA0

Power	Sample Size N	Number of Items K	Coefficient Alpha|H0 CA0	Actual Coefficient Alpha CA1	Alpha	Beta
0.11084	50	20	0.600	0.650	0.050	0.88916
0.16444	100	20	0.600	0.650	0.050	0.83556
0.27111	200	20	0.600	0.650	0.050	0.72889
0.37314	300	20	0.600	0.650	0.050	0.62686
0.55224	500	20	0.600	0.650	0.050	0.44776

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Report Definitions
Power is the probability of rejecting a false null hypothesis.
N is the total sample size.
K is the number of items or raters.
CA0 is the value of coefficient alpha under the null hypothesis.
CA1 is the value of coefficient alpha at which the power is computed.
Alpha is the probability of rejecting a true null hypothesis. It should be small.
Beta is the probability of accepting a false null hypothesis. It should be small.

Summary Statements
A sample of 50 subjects each responding to 20 items achieves 11% power to detect the difference between the actual coefficient alpha of 0.650 and the coefficient alpha under the null hypothesis of 0.600 using a two-sided F-test with a significance level of 0.050.

This report shows the values of each of the parameters, one scenario per row. The values from this table are plotted in the chart below.

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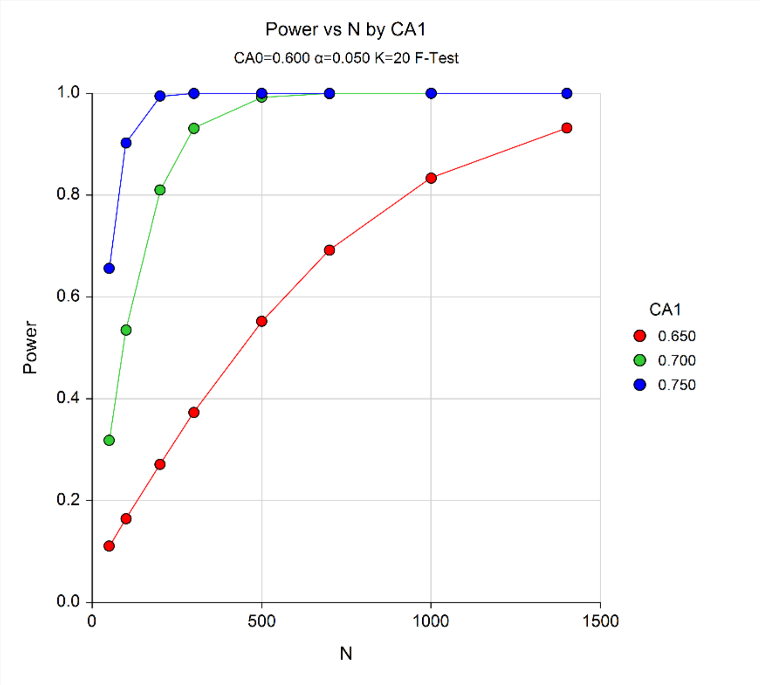
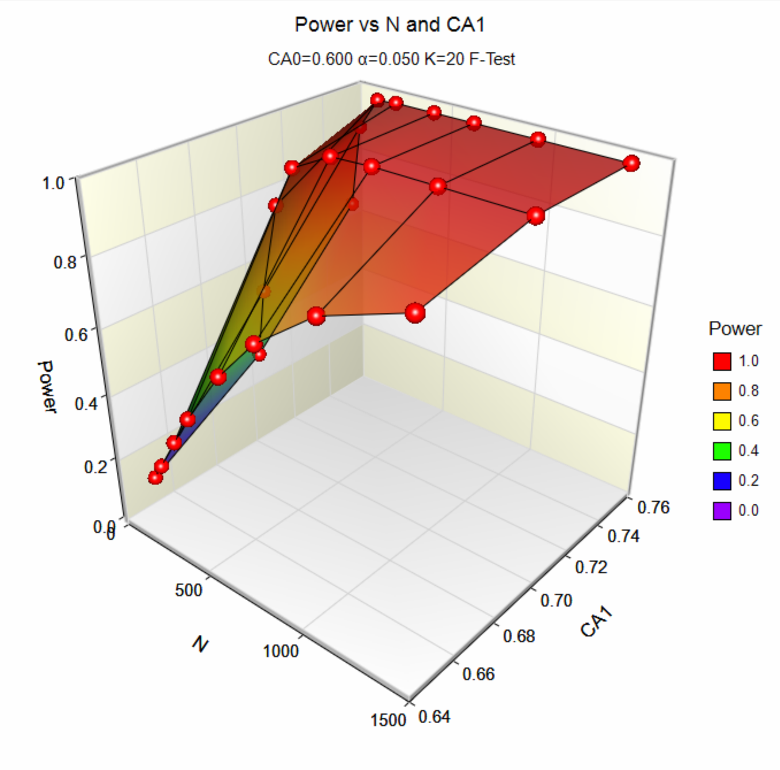
Plots Section





These plots show the relationship between CA1, N, and power.

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Example 2 – Finding the Sample Size

Continuing with Example 1, find the sample size necessary to achieve a power of 90% with a 0.05 significance level.

Setup

This section presents the values of each of the parameters needed to run this example. First, from the PASS Home window, load the **Tests for One Coefficient Alpha** procedure window by expanding **Correlation**, then clicking on **Coefficient (Cronbach's) Alpha**, and then clicking on **Tests for One Coefficient Alpha**. You may then make the appropriate entries as listed below, or open **Example 2** by going to the **File** menu and choosing **Open Example Template**.

Option	Value
Design Tab	
Solve For	**Sample Size**
Alternative Hypothesis	**H1: CA ≠ CA0**
Power	**0.90**
Alpha	**0.05**
K (Number of Items or Raters)	**20**
CA0 (Coefficient Alpha | H0)	**0.6**
CA1 (Actual Coefficient Alpha)	**0.65 0.70 0.75**

Output

Click the Calculate button to perform the calculations and generate the following output.

Numeric Results

Numeric Results
Hypotheses: H0: CA = CA0 vs. H1: CA ≠ CA0

Power	Sample Size N	Number of Items K	Coefficient Alpha|H0 CA0	Actual Coefficient Alpha CA1	Alpha	Beta
0.90022	1233	20	0.600	0.650	0.050	0.09978
0.90073	265	20	0.600	0.700	0.050	0.09927
0.90261	100	20	0.600	0.750	0.050	0.09739

This report shows the dramatic increase in sample size that is needed to achieve the desired sample power as CA1 gets closer to CA0.

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Example 3 – Validation using Bonett (2002)

Bonett (2002) page 337 presents a table in which the sample sizes were calculated for several parameter configurations. When CA0 = 0, CA1 = 0.50, alpha = 0.10, power = 0.95, and *k* = 2, 5, 10, and 100, he finds *N* to be 93, 59, 52, and 48, respectively.

Setup

This section presents the values of each of the parameters needed to run this example. First, from the PASS Home window, load the **Tests for One Coefficient Alpha** procedure window by expanding **Correlation**, then clicking on **Coefficient (Cronbach's) Alpha**, and then clicking on **Tests for One Coefficient Alpha**. You may then make the appropriate entries as listed below, or open **Example 3** by going to the **File** menu and choosing **Open Example Template**.

Option	Value
Design Tab	
Solve For	**Sample Size**
Alternative Hypothesis	**H1: CA ≠ CA0**
Power	**0.95**
Alpha	**0.1**
K (Number of Items or Raters)	**2 5 10 100**
CA0 (Coefficient Alpha | H0)	**0**
CA1 (Actual Coefficient Alpha)	**0.5**

Output

Click the Run button to perform the calculations and generate the following output.

Numeric Results

Numeric Results
Hypotheses: H0: CA = CA0 vs. H1: CA ≠ CA0

Power	Sample Size N	Number of Items K	Coefficient Alpha|H0 CA0	Actual Coefficient Alpha CA1	Alpha	Beta
0.95176	93	2	0.000	0.500	0.100	0.04824
0.95253	59	5	0.000	0.500	0.100	0.04747
0.95047	52	10	0.000	0.500	0.100	0.04953
0.95213	48	100	0.000	0.500	0.100	0.04787

The sample sizes match Bonett's results exactly.

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