Chapter 412

Tests for One Poisson Rate

Introduction

The Poisson probability law gives the probability distribution of the number of events occurring in a specified interval of time or space. The Poisson distribution is often used to fit count data, such as the number of defects on an item, the number of accidents at an intersection during a year, the number of calls to a call center during an hour, or the number of meteors seen in the evening sky during an hour.

The Poisson distribution is characterized by a single parameter, λ, which is the mean number of occurrences during the interval. This procedure calculates the power or sample size for testing whether λ is less than or greater than a specified value. This test is usually called the *test of the Poisson rate (or mean)*.

The test is described in Ostle and Malone (1988) and the power calculation is given in Guenther (1977).

Test Procedure

Assume that the mean rate is λ_0. To test $H_0 : \lambda \leq \lambda_0$ vs. $H_a : \lambda > \lambda_0$, you would take the following steps.

1. **Find the critical value**. Choose the critical value X^* so that the probability of rejecting H_0 when it is true is equal to α. This is done by solving the following inequality for X^*.

$$\sum_{x=X^*}^{\infty} e^{-n\lambda_0} \frac{(n\lambda_0)^x}{x!} \leq \alpha \, .$$

   Note that because *X* is an integer, equality will seldom occur. Therefore, the minimum value of X^* is found for which the inequality holds.

2. **Select a sample of *n* items compute the total number of events** $X = \sum_{i=1}^{n} x_i$. If $X > X^*$ reject H_0 in favor of H_a.

The test in the other direction ($H_0 : \lambda \geq \lambda_0$ vs. $H_a : \lambda < \lambda_0$) is computed similarly.

Assumptions

The assumptions of the one-sample *Poisson* test are:

1. The data are counts (discrete) that follow the Poisson distribution.
2. The sample is a simple random sample from its population. Each individual in the population has an equal probability of being selected in the sample.

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Limitations

There are few limitations when using these tests. As long as the assumption that the mean occurrence rate is constant is met, the test is valid.

Technical Details

Computing Power

The power is computing for a specific alternative value λ_1 using the following formula.

$$Power = 1 - \beta$$

$$= \sum_{x=X^*}^{\infty} e^{-n\lambda_1} \frac{(n\lambda_1)^x}{x!}$$

Computing Sample Size

Following Guenther (1977), the sample size, *n*, is found by increasing the value of *d* in the following expression until the left-hand endpoint is less than the right-hand endpoint and the interval contains at least one integer.

$$\frac{X_{2d;1-\beta}^2}{2\lambda_1} \le n \le \frac{X_{2d;\alpha}^2}{2\lambda_0}, \quad d = 1,2,3,\cdots$$

Here $X_{v;P}^2$ is a percentage point of the chi-square distribution with v degrees of freedom.

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Example 1 – Power after a Study

This example demonstrates how to calculate the power for specific values of the other parameters. Suppose that accidents have occurred at an intersection at an average rate of 1 per month for the last several years. Recently, a distraction has been constructed near the intersection that appears to have increased the accident rate. Suppose the sample sizes are 12 and 24 months and alpha is 0.025. What is the power to detect alternatives of 1.1, 1.5, 2.0, and 2.5?

Setup

If the procedure window is not already open, use the PASS Home window to open it. The parameters for this example are listed below and are stored in the **Example 1** settings file. To load these settings to the procedure window, click **Open Example Settings File** in the Help Center or File menu.

Design Tab

Solve For	**Power**
Alternative Hypothesis	**Ha: λ1 > λ0**
Alpha	**0.025**
n (Sample Size)	**12 24**
λ0 (Null or Baseline Rate)	**1.0**
λ1 (Alternative Rate)	**1.1 1.4 1.8 2.2 2.5**

Output

Click the Calculate button to perform the calculations and generate the following output.

Numeric Reports

Numeric Results

Solve For: Power
Null Hypothesis: λ1 ≤ λ0
Alternative Hypothesis: λ1 > λ0

		Poisson Mean Rate				Alpha		
Power	Sample Size n	Null λ0	Alternative λ1	Difference λ1 - λ0	Effect Size	Target	Actual	Beta
0.0484	12	1	1.1	0.1	0.0953	0.025	0.0213	0.9516
0.0623	24	1	1.1	0.1	0.0953	0.025	0.0206	0.9377
0.2476	12	1	1.4	0.4	0.3381	0.025	0.0213	0.7524
0.4273	24	1	1.4	0.4	0.3381	0.025	0.0206	0.5727
0.6638	12	1	1.8	0.8	0.5963	0.025	0.0213	0.3362
0.9108	24	1	1.8	0.8	0.5963	0.025	0.0206	0.0892
0.9154	12	1	2.2	1.2	0.8090	0.025	0.0213	0.0846
0.9962	24	1	2.2	1.2	0.8090	0.025	0.0206	0.0038
0.9781	12	1	2.5	1.5	0.9487	0.025	0.0213	0.0219
0.9998	24	1	2.5	1.5	0.9487	0.025	0.0206	0.0002

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Power	The probability of rejecting a false null hypothesis when the alternative hypothesis is true.
n	The size of the sample drawn from the population.
λ0	The value of the population mean rate under the null hypothesis.
λ1	The value of the population mean rate under the alternative hypothesis.
λ1 - λ0	The difference being tested.
Effect Size	The value of (λ1 - λ0) / √(λ1).
Alpha	The probability of rejecting a true null hypothesis.
Beta	The probability of failing to reject the null hypothesis when the alternative hypothesis is true.

Summary Statements

A single-group design will be used to test whether the (Poisson) mean rate is greater than 1 (H0: λ ≤ 1 versus H1: λ > 1). The comparison will be made using a one-sided, one-sample Poisson rate test, with a Type I error rate (α) of 0.025. To detect a mean rate of 1.1 (corresponding to a difference of 0.1) with a sample size of 12, the power is 0.0484.

Dropout-Inflated Sample Size

Dropout Rate	Sample Size N	Dropout-Inflated Enrollment Sample Size N'	Expected Number of Dropouts D
20%	12	15	3
20%	24	30	6

Dropout Rate	The percentage of subjects (or items) that are expected to be lost at random during the course of the study and for whom no response data will be collected (i.e., will be treated as "missing"). Abbreviated as DR.
N	The evaluable sample size at which power is computed (as entered by the user). If N subjects are evaluated out of the N' subjects that are enrolled in the study, the design will achieve the stated power.
N'	The total number of subjects that should be enrolled in the study in order to obtain N evaluable subjects, based on the assumed dropout rate. N' is calculated by inflating N using the formula N' = N / (1 - DR), with N' always rounded up. (See Julious, S.A. (2010) pages 52-53, or Chow, S.C., Shao, J., Wang, H., and Lokhnygina, Y. (2018) pages 32-33.)
D	The expected number of dropouts. D = N' - N.

Dropout Summary Statements

Anticipating a 20% dropout rate, 15 subjects should be enrolled to obtain a final sample size of 12 subjects.

References

Guenther, William C. 1977. Sampling Inspection in Statistical Quality Control. Griffin's Statistical Monographs. Macmillan, NY. Pages 25-29.
Ostle, B. and Malone, L. 1988. Statistics in Research, 4th Edition. Iowa State University Press. Iowa. Pages 116-118.

This report shows the values of each of the parameters, one scenario per row. The values of power and beta were calculated from the other parameters.

Note that the actual power achieved is greater than the target power. Similarly, the actual alpha is less than the target alpha. These differences occur because only integer values of the count variable occur.

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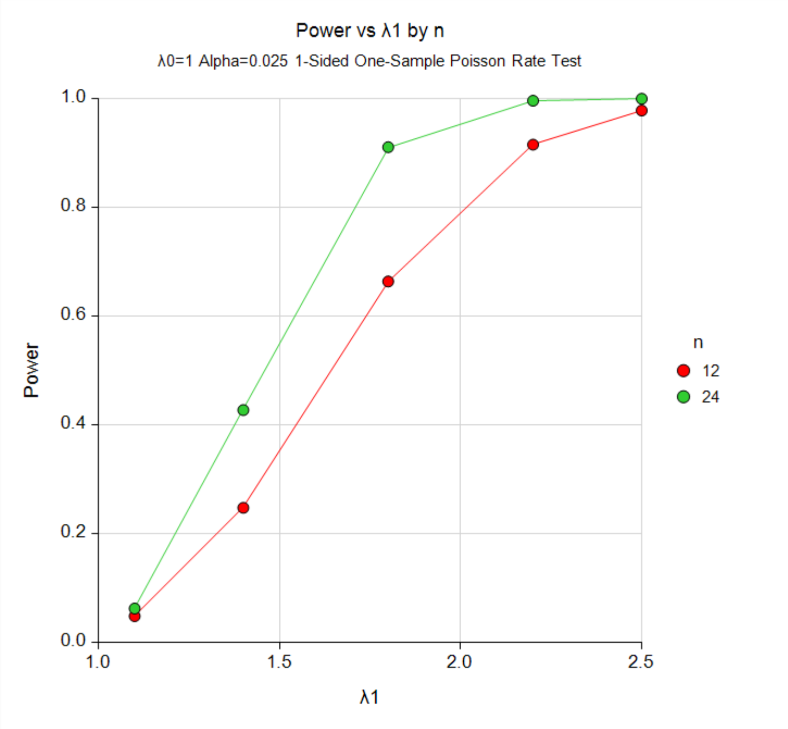
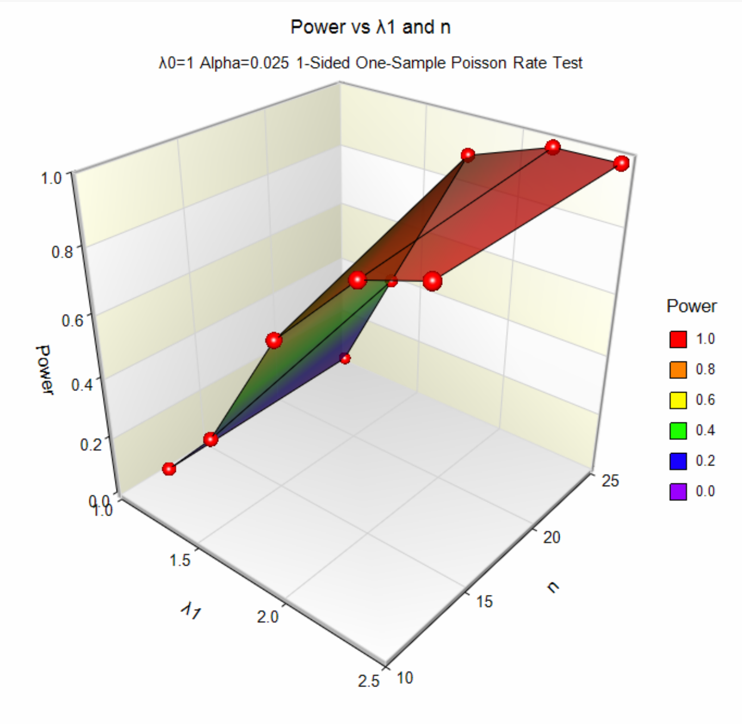
Plots Section

Plots





These plots show the relationship between sample size and power for various values of the alternative rate.

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Example 2 – Finding the Sample Size

This example will extend Example 1 to the case in which we want to find the necessary sample size to achieve at least 90% power. This is done as follows.

Setup

If the procedure window is not already open, use the PASS Home window to open it. The parameters for this example are listed below and are stored in the **Example 2** settings file. To load these settings to the procedure window, click **Open Example Settings File** in the Help Center or File menu.

Design Tab

Solve For .... **Sample Size**
Alternative Hypothesis .... **Ha: λ1 > λ0**
Power.. **0.90**
Alpha.. **0.025**
λ0 (Null or Baseline Rate) **1.0**
λ1 (Alternative Rate) **1.1 1.4 1.8 2.2 2.5**

Output

Click the Calculate button to perform the calculations and generate the following output.

Numeric Results

Solve For: Sample Size
Null Hypothesis: λ1 ≤ λ0
Alternative Hypothesis: λ1 > λ0

		Poisson Mean Rate				Alpha		
Power	Sample Size n	Null λ0	Alternative λ1	Difference λ1 - λ0	Effect Size	Target	Actual	Beta
0.9002	1100	1	1.1	0.1	0.0953	0.025	0.0250	0.0998
0.9009	80	1	1.4	0.4	0.3381	0.025	0.0220	0.0991
0.9108	24	1	1.8	0.8	0.5963	0.025	0.0206	0.0892
0.9154	12	1	2.2	1.2	0.8090	0.025	0.0213	0.0846
0.9366	9	1	2.5	1.5	0.9487	0.025	0.0220	0.0634

This report shows the sample sizes that are necessary to achieve the required power.

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Example 3 – Finding the Minimum Detectable Difference

Continuing with the previous example, suppose only 10 months of data are available. What is the minimum detectable difference that can be detected by this design?

Setup

If the procedure window is not already open, use the PASS Home window to open it. The parameters for this example are listed below and are stored in the **Example 3** settings file. To load these settings to the procedure window, click **Open Example Settings File** in the Help Center or File menu.

Design Tab

Solve For **λ1**
Alternative Hypothesis .... **Ha: λ1 > λ0**
Power... **0.90**
Alpha... **0.025**
n (Sample Size) **10**
λ0 (Null or Baseline Rate) **1.0**

Output

Click the Calculate button to perform the calculations and generate the following output.

Numeric Results

Solve For: λ1
Null Hypothesis: λ1 ≤ λ0
Alternative Hypothesis: λ1 > λ0

		Poisson Mean Rate				Alpha		
Power	Sample Size n	Null λ0	Alternative λ1	Difference λ1 - λ0	Effect Size	Target	Actual	Beta
0.9	10	1	2.36	1.36	0.8856	0.025	0.0143	0.1

This report shows that the minimum detectable difference is 2.36 – 1.00 = 1.36.

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Example 4 – Validation using Guenther (1977)

Guenther (1977) page 27 gives an example in which λ0 = 0.05, λ1 = .2, α = 0.05, β = 0.10, and n = 47. We will now run this example.

Setup

If the procedure window is not already open, use the PASS Home window to open it. The parameters for this example are listed below and are stored in the **Example 4** settings file. To load these settings to the procedure window, click **Open Example Settings File** in the Help Center or File menu.

Design Tab

Solve For .... **Sample Size**
Alternative Hypothesis .... **Ha: λ1 > λ0**
Power... **0.90**
Alpha.. **0.05**
λ0 (Null or Baseline Rate) **0.05**
λ1 (Alternative Rate) **0.2**

Output

Click the Calculate button to perform the calculations and generate the following output.

Numeric Results

Solve For: Sample Size
Null Hypothesis: λ1 ≤ λ0
Alternative Hypothesis: λ1 > λ0

		Poisson Mean Rate				Alpha		
Power	Sample Size n	Null λ0	Alternative λ1	Difference λ1 - λ0	Effect Size	Target	Actual	Beta
0.9065	47	0.05	0.2	0.15	0.3354	0.05	0.0327	0.0935

Note that the value of *n* is indeed 47.

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