

Chapter 650

Tests for One Variance

Introduction

Occasionally, researchers are interested in the estimation of the variance (or standard deviation) rather than the mean. This module calculates the sample size and performs power analysis for hypothesis tests concerning a single variance.

Technical Details

If a variable X is normally distributed with mean μ and variance σ^2 , the sample variance is distributed as a Chi-square random variable with $N - 1$ degrees of freedom, where N is the sample size. That is,

$$\chi^2 = \frac{(N - 1)s^2}{\sigma^2}$$

is distributed as a Chi-square random variable. The sample statistic, s^2 , is calculated as

$$s^2 = \frac{\sum_{i=1}^N (X_i - \bar{X})^2}{N - 1}.$$

If σ_1^2 is the assumed actual value of the variance under the alternative hypothesis, then the power or sample size of a hypothesis test about the variance can be calculated using the appropriate one of the following three formulas from Ostle and Malone (1988) page 130.

Case 1: $H_0: \sigma^2 = \sigma_0^2$ versus $H_a: \sigma^2 \neq \sigma_0^2$

$$\beta = P\left(\frac{\sigma_0^2}{\sigma_1^2} \chi_{\alpha/2, N-1}^2 < \chi^2 < \frac{\sigma_0^2}{\sigma_1^2} \chi_{1-\alpha/2, N-1}^2\right)$$

Case 2: $H_0: \sigma^2 \leq \sigma_0^2$ versus $H_a: \sigma^2 > \sigma_0^2$

$$\beta = P\left(\chi^2 < \frac{\sigma_0^2}{\sigma_1^2} \chi_{1-\alpha, N-1}^2\right)$$

Case 3: $H_0: \sigma^2 \geq \sigma_0^2$ versus $H_a: \sigma^2 < \sigma_0^2$

$$\beta = P\left(\chi^2 > \frac{\sigma_0^2}{\sigma_1^2} \chi_{\alpha, N-1}^2\right)$$

Procedure Options

This section describes the options that are specific to this procedure. These are located on the Design tab. For more information about the options of other tabs, go to the Procedure Window chapter.

Design Tab

The Design tab contains most of the parameters and options that you will be concerned with.

Solve For

Solve For

This option specifies the parameter to be solved for from the other parameters.

Test

Alternative Hypothesis

Specify the alternative hypothesis of the test. Since the null hypothesis is the opposite of the alternative, specifying the alternative is all that is needed. Usually, the two-tailed (\neq) option is selected.

Note that the alternative hypothesis enters into power calculations by specifying the rejection region of the hypothesis test. Its accuracy is critical.

Possible selections are:

- **Ha: $\sigma^2 \neq \sigma_0^2$**
This selection yields a *two-tailed* test. Use this option when you are testing whether the variance is different from σ_0^2 but don't want to specify a direction for the difference.
- **Ha: $\sigma^2 < \sigma_0^2$**
This option yields a *one-tailed* test. Use it when you are only interested in the case in which σ^2 is less than σ_0^2 .
- **Ha: $\sigma^2 > \sigma_0^2$**
This option yields a one-tailed test. Use it when you are only interested in the case in which σ^2 is greater than σ_0^2 .

Known Mean

Check this box if the mean is known.

The degrees of freedom of the Chi-square test is N-1 if the mean is calculated from the data (this is usually the case) or N if the mean is known.

Power and Alpha

Power

This option specifies one or more values for power. Power is the probability of rejecting a false null hypothesis, and is equal to one minus Beta. Beta is the probability of a type-II error, which occurs when a false null hypothesis is not rejected.

Values must be between zero and one. Historically, the value of 0.80 (Beta = 0.20) was used for power. Now, 0.90 (Beta = 0.10) is also commonly used.

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A single value may be entered here or a range of values such as *0.8 to 0.95 by 0.05* may be entered.

If your only interest is in determining the appropriate sample size for a confidence interval, set power or beta to 0.5.

Alpha

This option specifies one or more values for the probability of a type-I error. A type-I error occurs when a true null hypothesis is rejected.

Values must be between zero and one. Historically, the value of 0.05 has been used for alpha. This means that about one test in twenty will falsely reject the null hypothesis. You should pick a value for alpha that represents the risk of a type-I error you are willing to take in your experimental situation.

You may enter a range of values such as *0.01 0.05 0.10* or *0.01 to 0.10 by 0.01*.

Sample Size

N (Sample Size)

This is the number of observations in the study.

Effect Size

Input Type

This option indicates whether you want the input to be in terms of variances or standard deviations.

Regardless of the option chosen, the calculations are made based on variances. If you choose to input standard deviations, then the standard deviations are each simply squared to obtain the corresponding variances to be used in all power and sample size calculations

σ_0^2 (Baseline Variance)

This option is displayed only if Input Type = "Variances."

Enter one or more value(s) of the null, standard, or baseline variance. This variance is used to construct the hypotheses.

Only the ratio of the two variances is used, so you can enter a "1" here and enter the desired ratio in the σ_1^2 box. You can enter values in the range $\sigma_0^2 > 0$.

σ_1^2 (Alternative Variance)

This option is displayed only if Input Type = "Variances."

This is the value (or values) of the variance at which power and sample size are calculated.

Only the ratio of the two variances is used, so you can enter a "1" for σ_0^2 and enter the desired ratio here. You can enter values in the range $\sigma_1^2 > 0$.

σ_0 (Baseline Standard Deviation)

This option is displayed only if Input Type = "Standard Deviations."

Enter one or more value(s) of the null, standard, or baseline standard deviation. The square of this standard deviation (i.e. the null variance) is used to construct the hypotheses. You can enter values in the range $\sigma_0 > 0$.

σ_1 (Alternative Standard Deviation)

This option is displayed only if Input Type = "Standard Deviations."

This is the value (or values) of the standard deviation at which power and sample size are calculated. The square of this standard deviation (i.e. the alternative variance) is used in calculations. You can enter values in the range $\sigma_1 > 0$.

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Example 1 – Calculating the Power

A machine used to perform a particular analysis is to be replaced with a new type of machine if the new machine reduces the variation in the output. The current machine has been tested repeatedly and found to have an output variance of 42.5. The new machine will be cost effective if it can reduce the variance by 30% to 29.75. If the significance level is set to 0.05, calculate the power for sample sizes of 10, 50, 90, 130, 170, 210, and 250.

Setup

This section presents the values of each of the parameters needed to run this example. First, from the PASS Home window, load the **Tests for One Variance** procedure window by expanding **Variations**, then clicking on **One Variance**, and then clicking on **Tests for One Variance**. You may then make the appropriate entries as listed below, or open **Example 1** by going to the **File** menu and choosing **Open Example Template**.

<u>Option</u>	<u>Value</u>
Design Tab	
Solve For	Power
Alternative Hypothesis	Ha: $\sigma^2 < \sigma_0^2$
Known Mean	Unchecked
Alpha	0.05
N (Sample Size)	10 50 90 130 170 210 250
Input Type	Variations
σ_0^2 (Baseline Variance)	42.5
σ_1^2 (Alternative Variance)	29.75

Annotated Output

Click the Calculate button to perform the calculations and generate the following output.

Numeric Results

Numeric Results

Hypotheses: $H_0: \sigma^2 \geq \sigma_0^2$ vs. $H_a: \sigma^2 < \sigma_0^2$
 Mean Not Known, Chi-Square df = N - 1

Power	N	σ_0^2	σ_1^2	Alpha	Beta
0.14448	10	42.500	29.750	0.050	0.85552
0.50556	50	42.500	29.750	0.050	0.49444
0.74775	90	42.500	29.750	0.050	0.25225
0.88174	130	42.500	29.750	0.050	0.11826
0.94785	170	42.500	29.750	0.050	0.05215
0.97806	210	42.500	29.750	0.050	0.02194
0.99111	250	42.500	29.750	0.050	0.00889

References

Davies, Owen L. 1971. The Design and Analysis of Industrial Experiments. Hafner Publishing Company, New York.
 Ostle, B. 1988. Statistics in Research. Fourth Edition. Iowa State Press. Ames, Iowa.
 Zar, Jerrold H. 1984. Biostatistical Analysis (Second Edition). Prentice-Hall. Englewood Cliffs, New Jersey.

Report Definitions

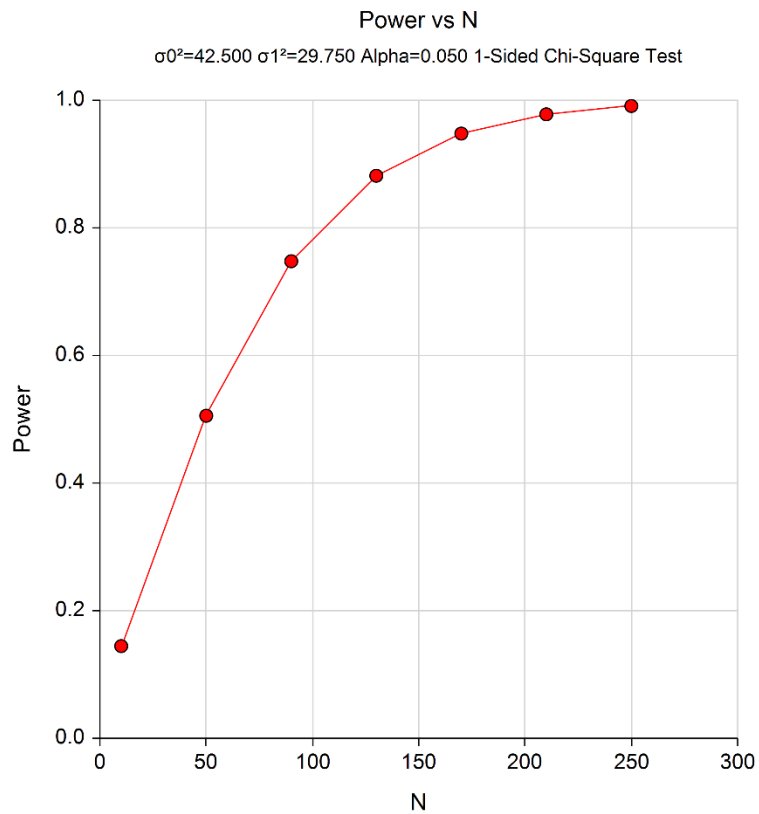
Power is the probability of rejecting a false null hypothesis. It should be close to one.
 N is the size of the sample drawn from the population.
 σ_0^2 is the value of the population variance under the null hypothesis.
 σ_1^2 is the value of the population variance under the alternative hypothesis.
 Alpha is the probability of rejecting a true null hypothesis. It should be small.
 Beta is the probability of accepting a false null hypothesis. It should be small.

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Summary Statements

A sample size of 10 achieves 14% power to detect a difference of 12.750 between the null hypothesis variance of 42.500 and the alternative hypothesis variance of 29.750 using a one-sided, Chi-Square hypothesis test with a significance level (alpha) of 0.050, assuming the mean is not known.

This report shows the calculated power for each scenario.

Plots Section

This plot shows the power versus the sample size. We see that a sample size of about 150 is necessary to achieve a power of 0.90.

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Example 2 – Calculating Sample Size

Continuing with the previous example, the analyst wants to find the necessary sample sizes to achieve a power of 0.9, for two significance levels, 0.01 and 0.05, and for several variance values. To make interpreting the output easier, the analyst decides to switch from the absolute scale to a ratio scale. To accomplish this, the baseline variance is set at 1.0 and the alternative variances of 0.2, 0.3, 0.4, 0.5, 0.6, and 0.7 are tried.

Setup

This section presents the values of each of the parameters needed to run this example. First, from the PASS Home window, load the **Tests for One Variance** procedure window by expanding **Variations**, then clicking on **One Variance**, and then clicking on **Tests for One Variance**. You may then make the appropriate entries as listed below, or open **Example 2** by going to the **File** menu and choosing **Open Example Template**.

<u>Option</u>	<u>Value</u>
Design Tab	
Solve For	Sample Size
Alternative Hypothesis	Ha: $\sigma^2 < \sigma_0^2$
Known Mean	Unchecked
Power	0.90
Alpha	0.01 0.05
Input Type	Variations
σ_0^2 (Baseline Variance)	1.0
σ_1^2 (Alternative Variance)	0.2 to 0.7 by 0.1

Output

Click the Calculate button to perform the calculations and generate the following output.

Numeric Results

Numeric Results

Hypotheses: $H_0: \sigma^2 \geq \sigma_0^2$ vs. $H_a: \sigma^2 < \sigma_0^2$

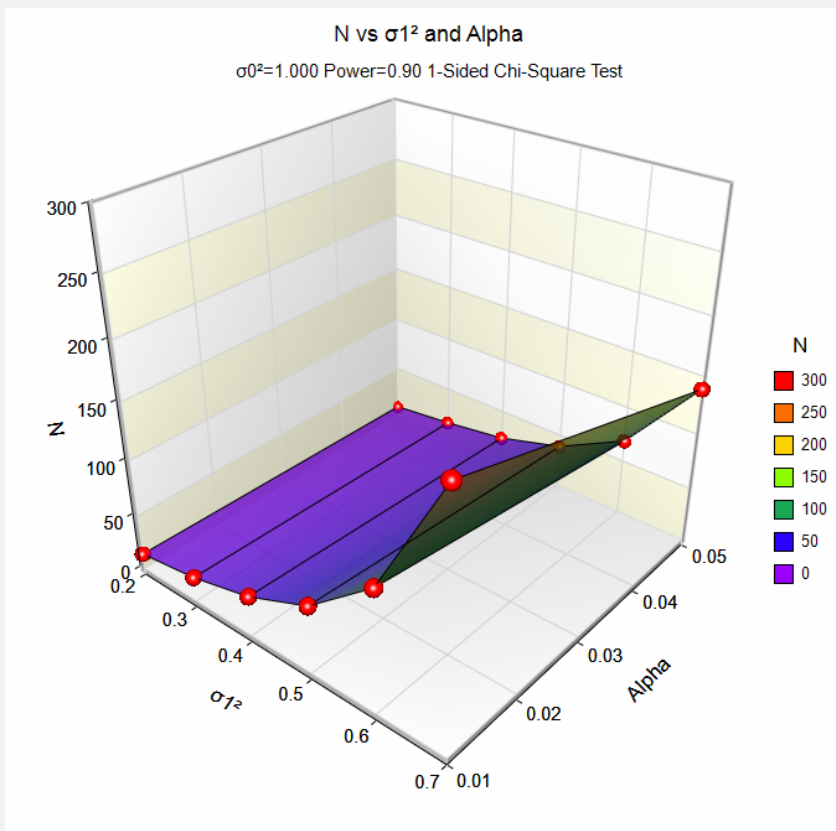
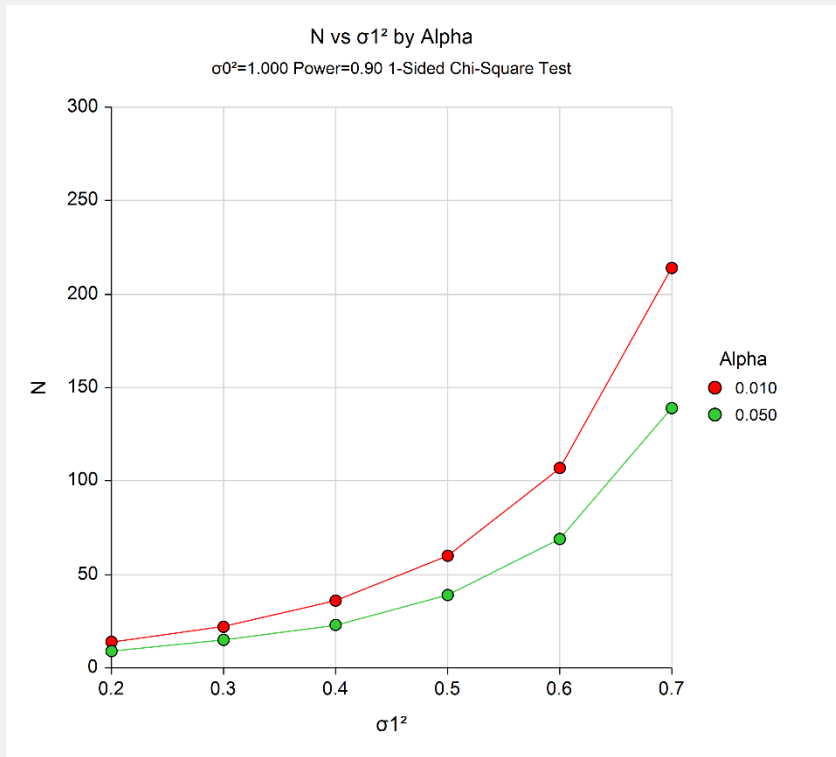
Mean Not Known, Chi-Square df = N - 1

Power	N	σ_0^2	σ_1^2	Alpha	Beta
0.91734	14	1.000	0.200	0.010	0.08266
0.90902	9	1.000	0.200	0.050	0.09098
0.90091	22	1.000	0.300	0.010	0.09909
0.91935	15	1.000	0.300	0.050	0.08065
0.90368	36	1.000	0.400	0.010	0.09632
0.90067	23	1.000	0.400	0.050	0.09933
0.90163	60	1.000	0.500	0.010	0.09837
0.90423	39	1.000	0.500	0.050	0.09577
0.90126	107	1.000	0.600	0.010	0.09874
0.90078	69	1.000	0.600	0.050	0.09922
0.90060	214	1.000	0.700	0.010	0.09940
0.90117	139	1.000	0.700	0.050	0.09883

This report shows the necessary sample size for each scenario.

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Plots Section



These plots show the necessary sample size for various values of σ_1^2 . Note that as σ_1^2 gets farther from zero, the required sample size increases.

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Example 3 – Validation using Zar (1984)

Zar (1984) page 117 presents an example with $\sigma_0^2 = 1.5$, $\sigma_1^2 = 2.6898$, $N = 40$, $Alpha = 0.05$, and $Power = 0.84$.

Setup

This section presents the values of each of the parameters needed to run this example. First, from the PASS Home window, load the **Tests for One Variance** procedure window by expanding **Variations**, then clicking on **One Variance**, and then clicking on **Tests for One Variance**. You may then make the appropriate entries as listed below, or open **Example 3** by going to the **File** menu and choosing **Open Example Template**.

<u>Option</u>	<u>Value</u>
Design Tab	
Solve For	Power
Alternative Hypothesis	Ha: $\sigma^2 > \sigma_0^2$
Known Mean	Unchecked
Alpha.....	0.05
N (Sample Size).....	40
Input Type.....	Variations
σ_0^2 (Baseline Variance).....	1.5
σ_1^2 (Alternative Variance).....	2.6898

Output

Click the Calculate button to perform the calculations and generate the following output.

Numeric Results

Numeric Results					
Hypotheses: $H_0: \sigma^2 \leq \sigma_0^2$ vs. $H_a: \sigma^2 > \sigma_0^2$					
Mean Not Known, Chi-Square df = N - 1					
Power	N	σ_0^2	σ_1^2	Alpha	Beta
0.83517	40	1.5000	2.6898	0.050	0.16483

PASS calculated the power to be 0.83517, which matches Zar's result of 0.84 within rounding.

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Example 4 – Validation using Davies (1971)

Davies (1971) page 40 presents an example of determining N when (in the standard deviation scale) $\sigma_0 = 0.04$, $\sigma_1 = 0.10$, Alpha = 0.05, and Power = 0.99. Davies calculates N to be 13.

Setup

This section presents the values of each of the parameters needed to run this example. First, from the PASS Home window, load the **Tests for One Variance** procedure window by expanding **Variations**, then clicking on **One Variance**, and then clicking on **Tests for One Variance**. You may then make the appropriate entries as listed below, or open **Example 4** by going to the **File** menu and choosing **Open Example Template**.

<u>Option</u>	<u>Value</u>
Design Tab	
Solve For	Sample Size
Alternative Hypothesis	Ha: $\sigma^2 > \sigma_0^2$
Known Mean	Unchecked
Power	0.99
Alpha	0.05
Input Type	Standard Deviations
σ_0 (Baseline Std. Dev.)	0.04
σ_1 (Alternative Std. Dev.)	0.10

Output

Click the Calculate button to perform the calculations and generate the following output.

Numeric Results

Numeric Results							
Hypotheses: $H_0: \sigma^2 \leq \sigma_0^2$ vs. $H_a: \sigma^2 > \sigma_0^2$							
Mean Not Known, Chi-Square df = N - 1							
Power	N	σ_0	σ_1	σ_0^2	σ_1^2	Alpha	Beta
0.99238	13	0.040	0.100	0.002	0.010	0.050	0.00762

PASS calculated an N of 13 which matches Davies' result.