Chapter 820

Tests for Two Coefficient Alphas

Introduction

Coefficient alpha, or *Cronbach's alpha*, is a popular measure of the reliability of a scale consisting of *k* parts. The *k* parts often represent *k* items on a questionnaire (scale) or *k* raters. This module calculates power and sample size for testing whether two coefficient alphas are different when the two samples are either dependent or independent.

Technical Details

Feldt et al. (1999) presents methods for testing one-, or two-, sided hypotheses about two coefficient alphas, which we label ρ_1 and ρ_2. The results assume that N_1 observations for each of k_1 items are available for one scale and N_2 observations for each of k_2 items are available for another scale. These sets of observations may either be from two independent groups of subjects (independent case) or two sets of observations on each subject (dependent case). In the dependent case, $N_1 = N_2$ and the correlation coefficient between the overall scores of each scale is represented by φ. For the independent case $\varphi = 0$.

Suppose $\hat{\rho}_1$ and $\hat{\rho}_2$ are the sample estimates of ρ_1 and ρ_2, respectively. Hypothesis tests are based on the result that the test statistic,

$$W = \left(\frac{1-\hat{\rho}_2}{1-\hat{\rho}_1}\right)\left(\frac{1-\rho_1}{1-\rho_2}\right)$$

$$= \hat{\delta}\left(\frac{1-\rho_1}{1-\rho_2}\right)$$

is approximately distributed as a central *F* variable with degrees of freedom v_1 and v_2. The values of v_1 and v_2 depend on N_1, N_2, k_1, k_2, and φ.

Also define

$$c_i = (N_i - 1)(k_i - 1), \quad i = 1,2$$

Independent Case

When the two scales are independent, there are two situations that must be considered separately. If $c_i > 1000$ and $k_i > 25$, the values of v_1 and v_2 are computed using

$$v_1 = N_1 - 1$$

$$v_2 = N_2 - 1$$

otherwise, they are computed using

$$v_1 = \frac{2A^2}{2B - AB - A^2}$$

$$v_2 = \frac{2A}{A - 1}$$

where

$$A = \frac{c_1(N_2 - 1)}{(c_1 - 2)(N_2 - 3)}$$

$$B = \frac{(N_1 + 1)(N_2 - 1)^2(c_2 + 2)c_1^2}{(N_2 - 3)(N_2 - 5)(N_1 - 1)(c_1 - 2)(c_1 - 4)c_2}$$

Dependent Case

When the two scales are dependent, it follows that $N_1 = N_2 = N$. There are two situations that must be considered separately.

If $c_i > 1000$ and $k_i > 25$, the values of v_1 and v_2 are computed using

$$v_1 = v_2 = \frac{N - 1 - 7\varphi^2}{1 - \varphi^2}$$

otherwise, they are computed using

$$v_1 = \frac{2M^2}{V(2 - M) - M^2(M - 1)}$$

$$v_2 = \frac{2M}{M - 1}$$

where

$$M = A - \frac{2\varphi^2}{N-1}$$

$$V = B - A^2 - \frac{4\varphi^2}{N-1}$$

Calculating the Power

Let ρ_{20} be the value of coefficient alpha in the second set under H0, ρ_{21} be the value of coefficient alpha in the second set at which the power is calculated, and ρ_1 be the value of coefficient alpha in the first set. The power of the one-sided hypothesis that $H_0: \rho_{20} \leq \rho_1$ versus the alternative that $H_1: \rho_{20} > \rho_1$ is calculated as follows:

1. Find F_α such that $\text{Prob}\left(F < F_{\alpha,v1,v2}\right) = \alpha$
2. Compute $\delta' = \frac{1}{F_\alpha}\left(\frac{1-\rho_1}{1-\rho_{20}}\right)$
3. Compute $W_1 = \left(\frac{1-\rho_1}{1-\rho_{21}}\right)\delta'$
4. Compute the power = $1 - \Pr\left(W_1 > F_{v1,v2}\right)$

Example 1 – Finding the Power

Suppose a study is being designed to compare the coefficient alphas of two scales. The researchers are going to use a two-sided F-test at a significance level of 0.05. Past experience has shown that CA1 is approximately 0.4. The researchers will use different subjects in each dataset. Find the power when K1 = K2 = 10, CA2.0 = CA1,
N1 = 50, 100, 150, 200, 250, and 300, N2 = N1, and CA2.1 = 0.6 and 0.7.

Setup

If the procedure window is not already open, use the PASS Home window to open it. The parameters for this example are listed below and are stored in the **Example 1** settings file. To load these settings to the procedure window, click **Open Example Settings File** in the Help Center or File menu.

Design Tab

Solve For ..... **Power**
Alternative Hypothesis ..... **H1: CA1 ≠ CA2.0**
Alpha..... **0.05**
N1 (Sample Size in Set 1)..... **50 to 300 by 50**
K1 (Items/Scale in Set 1) **10**
N2 (Sample Size in Set 2)..... **N1**
K2 (Items/Scale in Set 2) **K1**
CA1 (Actual Coefficient Alpha in Set 1) **0.4**
CA2.0 (Coefficient Alpha in Set 2|H0)..... **CA1**
CA2.1 (Actual Coefficient Alpha in Set 2) **0.6 0.7**
φ (Correlation Between Sets) **0**

Output

Click the Calculate button to perform the calculations and generate the following output.

Numeric Reports

Numeric Results

Solve For: Power
Hypotheses: H0: CA1 = CA2.0 vs. H1: CA1 ≠ CA2.0

	Dataset Sample Size			Number of Items per Dataset		Coefficient Alpha				
						Dataset 1	Dataset 2			
Power	N1	N2	N	K1	K2	Actual CA1	Null CA2.0	Actual CA2.1	Correlation Between Datasets φ	Alpha
0.26423	50	50	100	10	10	0.4	0.4	0.6	0	0.05
0.47746	100	100	200	10	10	0.4	0.4	0.6	0	0.05
0.64813	150	150	300	10	10	0.4	0.4	0.6	0	0.05
0.77250	200	200	400	10	10	0.4	0.4	0.6	0	0.05
0.85759	250	250	500	10	10	0.4	0.4	0.6	0	0.05
0.91319	300	300	600	10	10	0.4	0.4	0.6	0	0.05
0.62531	50	50	100	10	10	0.4	0.4	0.7	0	0.05
0.90263	100	100	200	10	10	0.4	0.4	0.7	0	0.05
0.97926	150	150	300	10	10	0.4	0.4	0.7	0	0.05
0.99611	200	200	400	10	10	0.4	0.4	0.7	0	0.05
0.99934	250	250	500	10	10	0.4	0.4	0.7	0	0.05
0.99989	300	300	600	10	10	0.4	0.4	0.7	0	0.05

Power	The probability of rejecting a false null hypothesis when the alternative hypothesis is true.
N1, N2, and N	The sample sizes of datasets 1, 2, and both, respectively.
K1 and K2	The number of items in datasets 1 and 2, respectively.
CA1	The coefficient alpha in dataset 1.
CA2.0	The coefficient alpha in dataset 2 under H0.
CA2.1	The coefficient alpha in dataset 2 at which the power is calculated.
φ	Phi is the correlation between the average scores of each of the two datasets.
Alpha	The probability of rejecting a true null hypothesis.

Summary Statements

A two-group coefficient alpha (or Cronbach's alpha) reliability design with 10 items (or raters) for Group 1 and 10 items (or raters) for Group 2 will be used to test whether the Group 1 coefficient alpha (CA1) is different from the Group 2 coefficient alpha (CA2) (H0: CA1 = CA2 versus Ha: CA1 ≠ CA2). The comparison will be made using a two-sample coefficient alpha F-test, with a Type I error rate (α) of 0.05. Under the null hypothesis, the coefficient alphas in Groups 1 and 2 are assumed to be 0.4 and 0.4, respectively. The correlation between the two datasets is assumed to be 0. To detect a Group 2 coefficient alpha of 0.6 (and a Group 1 coefficient alpha of 0.4), with sample sizes of 50 in Group 1 and 50 in Group 2, the power is 0.26423.

Dropout-Inflated Sample Size

	Sample Size			Dropout-Inflated Enrollment Sample Size			Expected Number of Dropouts		
Dropout Rate	**N1**	**N2**	**N**	**N1'**	**N2'**	**N'**	**D1**	**D2**	**D**
20%	50	50	100	63	63	126	13	13	26
20%	100	100	200	125	125	250	25	25	50
20%	150	150	300	188	188	376	38	38	76
20%	200	200	400	250	250	500	50	50	100
20%	250	250	500	313	313	626	63	63	126
20%	300	300	600	375	375	750	75	75	150

Dropout Rate The percentage of subjects (or items) that are expected to be lost at random during the course of the study and for whom no response data will be collected (i.e., will be treated as "missing"). Abbreviated as DR.
N1, N2, and N The evaluable sample sizes at which power is computed (as entered by the user). If N1 and N2 subjects are evaluated out of the N1' and N2' subjects that are enrolled in the study, the design will achieve the stated power.
N1', N2', and N' The number of subjects that should be enrolled in the study in order to obtain N1, N2, and N evaluable subjects, based on the assumed dropout rate. N1' and N2' are calculated by inflating N1 and N2 using the formulas N1' = N1 / (1 - DR) and N2' = N2 / (1 - DR), with N1' and N2' always rounded up. (See Julious, S.A. (2010) pages 52-53, or Chow, S.C., Shao, J., Wang, H., and Lokhnygina, Y. (2018) pages 32-33.)
D1, D2, and D The expected number of dropouts. D1 = N1' - N1, D2 = N2' - N2, and D = D1 + D2.

Dropout Summary Statements

Anticipating a 20% dropout rate, 63 subjects should be enrolled in Group 1, and 63 in Group 2, to obtain final group sample sizes of 50 and 50, respectively.

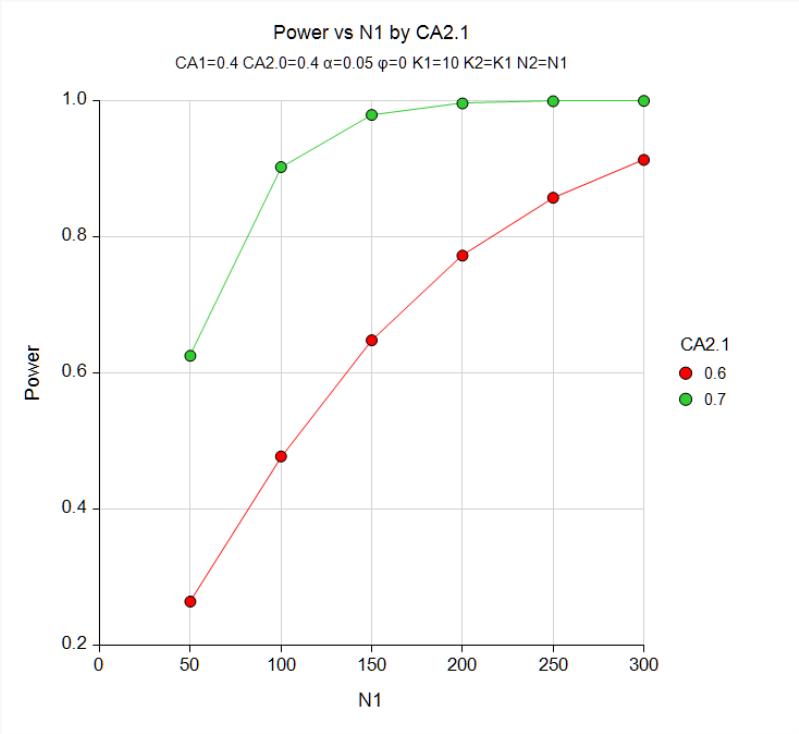
References

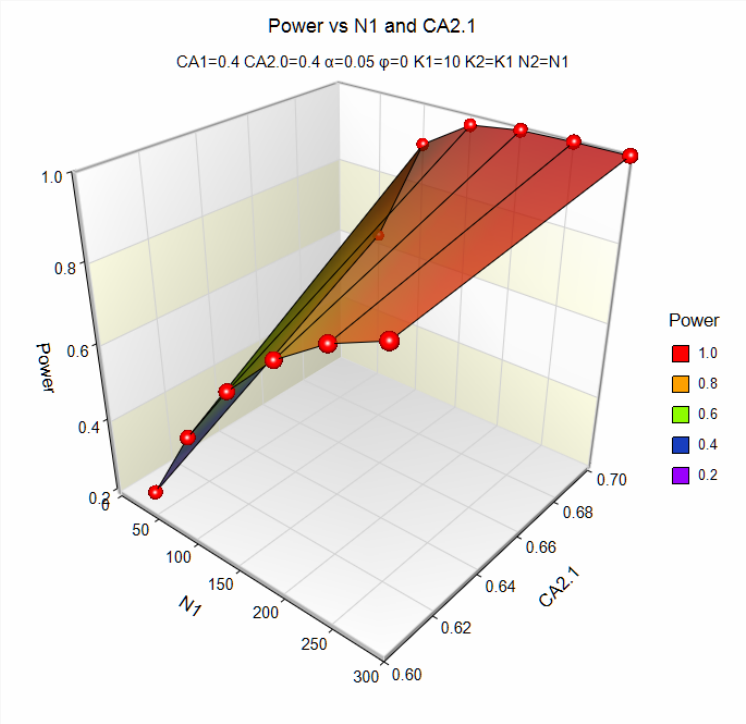
Feldt, L.S.; Ankenmann, R.D. 1999. 'Determining Sample Size for a Test of the Equality of Alpha Coefficients When the Number of Part-Tests is Small.' Psychological Methods, Vol. 4(4), pages 366-377.

This report shows the values of each of the parameters, one scenario per row. The values from this table are displayed in the plots below.

Plots Section

Plots





These plots show the relationship between CA2.1, N1, and power.

Example 2 – Finding the Sample Size

Continuing with Example 1, find the sample size necessary to achieve a power of 90% at the 0.05 significance level.

Setup

If the procedure window is not already open, use the PASS Home window to open it. The parameters for this example are listed below and are stored in the **Example 2** settings file. To load these settings to the procedure window, click **Open Example Settings File** in the Help Center or File menu.

Design Tab

Solve For ..... **Sample Size (N1)**
Alternative Hypothesis ..... **H1: CA1 ≠ CA2.0**
Power..... **0.90**
Alpha..... **0.05**
K1 (Items/Scale in Set 1) **10**
N2 (Sample Size in Set 2)..... **N1**
K2 (Items/Scale in Set 2) **K1**
CA1 (Actual Coefficient Alpha in Set 1) **0.4**
CA2.0 (Coefficient Alpha in Set 2|H0)..... **CA1**
CA2.1 (Actual Coefficient Alpha in Set 2) **0.6 0.7**
φ (Correlation Between Sets) **0**

Output

Click the Run button to perform the calculations and generate the following output.

Numeric Results

Solve For: Sample Size (N1)
Hypotheses: H0: CA1 = CA2.0 vs. H1: CA1 ≠ CA2.0

	Dataset Sample Size			Number of Items per Dataset		Coefficient Alpha				
						Dataset 1	Dataset 2			
Power	N1	N2	N	K1	K2	Actual CA1	Null CA2.0	Actual CA2.1	Correlation Between Datasets φ	Alpha
0.90004	286	286	572	10	10	0.4	0.4	0.6	0	0.05
0.90263	100	100	200	10	10	0.4	0.4	0.7	0	0.05

This report shows that 286 subjects per dataset are needed when CA2.1 is 0.60 and 100 subjects per dataset are needed when CA2.1 is 0.70.

Example 3 – Validation using Feldt et al. (1999)

Feldt et al. (1999) presents an example in which CA1 = 0, CA2.0 = 0, CA2.1 = 0.5, alpha = 0.05, $\varphi = 0$, N1 = N2 = 60, and *k* = 5. They find the power of a one-sided test to be 0.761.

Setup

If the procedure window is not already open, use the PASS Home window to open it. The parameters for this example are listed below and are stored in the **Example 3** settings file. To load these settings to the procedure window, click **Open Example Settings File** in the Help Center or File menu.

Design Tab

Solve For ..... **Power**
Alternative Hypothesis ..... **H1: CA1 < CA2.0**
Alpha..... **0.05**
N1 (Sample Size in Set 1)..... **60**
K1 (Items/Scale in Set 1) **5**
N2 (Sample Size in Set 2)..... **N1**
K2 (Items/Scale in Set 2) **K1**
CA1 (Actual Coefficient Alpha in Set 1) **0**
CA2.0 (Coefficient Alpha in Set 2|H0)..... **CA1**
CA2.1 (Actual Coefficient Alpha in Set 2) **0.5**
φ (Correlation Between Sets) **0**

Output

Click the Calculate button to perform the calculations and generate the following output.

Numeric Results

Solve For: Power
Hypotheses: H0: CA1 ≥ CA2.0 vs. H1: CA1 < CA2.0

	Dataset Sample Size			Number of Items per Dataset		Coefficient Alpha				
						Dataset 1	Dataset 2			
Power	N1	N2	N	K1	K2	Actual CA1	Null CA2.0	Actual CA2.1	Correlation Between Datasets φ	Alpha
0.76548	60	60	120	5	5	0	0	0.5	0	0.05

Note that **PASS's** result is slightly different from Feldt's because **PASS** uses fractional degrees of freedom and Feldt rounds to the closest integer. Although the difference in power is small, allowing fractional degrees of freedom is more accurate.