Chapter 805

Tests for Two Correlations

Introduction

The correlation coefficient (or correlation), ρ, is a popular parameter for describing the strength of the association between two variables. The correlation coefficient is the slope of the regression line between two variables when both variables have been standardized. It ranges between plus and minus one. This chapter covers the case in which you want to test the difference between two correlations, each coming from a separate sample.

Since the correlation is the standardized slope between two variables, you could also apply this procedure to the case in which you want to test whether the slopes in two groups are equal.

Test Procedure

In the following discussion, ρ is the population correlation coefficient and *r* is the value calculated from a sample. The testing procedure is as follows. H_0 is the null hypothesis that $\rho_1 = \rho_2$. H_A represents the alternative hypothesis that $\rho_1 \neq \rho_2$ (one-tailed hypotheses are also available). To construct the hypothesis test, transform the correlations using the Fisher-*z* transformation.

$$z_i = \frac{1}{2}\log\left(\frac{1 + r_i}{1 - r_i}\right)$$

$$Z_i = \frac{1}{2}\log\left(\frac{1 + \rho_i}{1 - \rho_i}\right)$$

This transformation is used because the combined distribution of r_1 and r_2 is too difficult to work with, but the distributions of z_1 and z_2 are approximately normal.

Note that the reverse transformation is

$$r_i = \frac{e^{z_i} - e^{-z_i}}{e^{z_i} + e^{-z_i}}$$

Once the correlations have been converted into *z* values, the normal distribution may be used to conduct the test of $Z_1 - Z_2$. The standard deviation of the difference is given by

$$\sigma_{z_1 - z_2} = \sqrt{\frac{1}{N_1 - 3} + \frac{1}{N_2 - 3}}$$

The test statistic is given by

$$z = \frac{(z_1 - z_2) - (Z_1 - Z_2)}{\sigma_{z_1 - z_2}}$$

Note that the lower-case *z's* represent the values calculated from the two samples and the upper-case *Z's* represent the hypothesized population values.

Calculating the Power

The steps to calculate power are as follows:

1. Find z_α such that $1 - \Phi(z_\alpha) = \alpha$, where $\Phi(x)$ is the area under the standardized normal curve to the left of *x*.
2. Calculate: $Z_1 = \frac{1}{2}\log\left(\frac{1+\rho_1}{1-\rho_1}\right)$
3. Calculate: $Z_2 = \frac{1}{2}\log\left(\frac{1+\rho_2}{1-\rho_2}\right)$
4. Calculate: $\sigma_{z_1 - z_2} = \sqrt{\frac{1}{N_1 - 3} + \frac{1}{N_2 - 3}}$
5. Calculate: $x_\alpha = Z_1 - Z_2 + z_\alpha \sigma_{z_1 - z_2}$
6. Calculate: $z_\alpha = \frac{x_\alpha}{\sigma_{z_1 - z_2}}$
7. Calculate: $Power = 1 - \Phi(z_\alpha)$

Example 1 – Finding the Power

A researcher wants to compare the relationship between weight and heart rate in males and females. If the correlation between weight and heart rate is 0.3 in a sample of 100 males and 0.5 in a sample of 100 females, what is the power of a two-sided test for the difference between correlations at the 0.01 and 0.05 significance levels? Also compute the power for samples of 20, 200, 300, 400, and 600.

Setup

If the procedure window is not already open, use the PASS Home window to open it. The parameters for this example are listed below and are stored in the **Example 1** settings file. To load these settings to the procedure window, click **Open Example Settings File** in the Help Center or File menu.

Design Tab

Solve For .... **Power**
Alternative Hypothesis .... **Ha: ρ1 ≠ ρ2**
Alpha... **0.01 0.05**
Group Allocation .... **Equal (N1 = N2)**
Sample Size Per Group **20 100 200 300 400 600**
ρ1 (Correlation Group 1)............................... **0.3**
ρ2 (Correlation Group 2)............................... **0.5**

Output

Click the Calculate button to perform the calculations and generate the following output.

Numeric Reports

Numeric Results

Solve For: Power
Alternative Hypothesis: Ha: ρ1 ≠ ρ2

	Sample Size			Correlation			
Power	**N1**	**N2**	**N**	**ρ1**	**ρ2**	**ρ1 - ρ2**	**Alpha**
0.03081	20	20	40	0.3	0.5	-0.2	0.01
0.18250	100	100	200	0.3	0.5	-0.2	0.01
0.42230	200	200	400	0.3	0.5	-0.2	0.01
0.63541	300	300	600	0.3	0.5	-0.2	0.01
0.78888	400	400	800	0.3	0.5	-0.2	0.01
0.94144	600	600	1200	0.3	0.5	-0.2	0.01
0.10760	20	20	40	0.3	0.5	-0.2	0.05
0.38603	100	100	200	0.3	0.5	-0.2	0.05
0.66271	200	200	400	0.3	0.5	-0.2	0.05
0.83200	300	300	600	0.3	0.5	-0.2	0.05
0.92196	400	400	800	0.3	0.5	-0.2	0.05
0.98548	600	600	1200	0.3	0.5	-0.2	0.05

Power	The probability of rejecting a false null hypothesis when the alternative hypothesis is true.
N1 and N2	The number of items sampled from each population.
N	The total sample size. N = N1 + N2.
ρ1	The correlation in group 1.
ρ2	The correlation in group 2 assumed by the alternative hypothesis.
ρ1 - ρ2	The difference between correlations at which power and sample size calculations are made.
Alpha	The probability of rejecting a true null hypothesis.

Summary Statements

A two-group correlation (Y versus X) design will be used to test whether the Group 1 correlation is different from the Group 2 correlation (H0: ρ1 = ρ2 versus Ha: ρ1 ≠ ρ2). The comparison will be made using a two-sided, two-sample Fisher-z-transformation Z-test, with a Type I error rate (α) of 0.01. To detect a Group 1 correlation of 0.3 and a Group 2 correlation of 0.5 (difference of -0.2), with sample sizes of 20 in Group 1 and 20 in Group 2, the power is 0.03081.

Dropout-Inflated Sample Size

	Sample Size			Dropout-Inflated Enrollment Sample Size			Expected Number of Dropouts		
Dropout Rate	N1	N2	N	N1'	N2'	N'	D1	D2	D
20%	20	20	40	25	25	50	5	5	10
20%	100	100	200	125	125	250	25	25	50
20%	200	200	400	250	250	500	50	50	100
20%	300	300	600	375	375	750	75	75	150
20%	400	400	800	500	500	1000	100	100	200
20%	600	600	1200	750	750	1500	150	150	300

Dropout Rate	The percentage of subjects (or items) that are expected to be lost at random during the course of the study and for whom no response data will be collected (i.e., will be treated as "missing"). Abbreviated as DR.
N1, N2, and N	The evaluable sample sizes at which power is computed (as entered by the user). If N1 and N2 subjects are evaluated out of the N1' and N2' subjects that are enrolled in the study, the design will achieve the stated power.
N1', N2', and N'	The number of subjects that should be enrolled in the study in order to obtain N1, N2, and N evaluable subjects, based on the assumed dropout rate. N1' and N2' are calculated by inflating N1 and N2 using the formulas N1' = N1 / (1 - DR) and N2' = N2 / (1 - DR), with N1' and N2' always rounded up. (See Julious, S.A. (2010) pages 52-53, or Chow, S.C., Shao, J., Wang, H., and Lokhnygina, Y. (2018) pages 32-33.)
D1, D2, and D	The expected number of dropouts. D1 = N1' - N1, D2 = N2' - N2, and D = D1 + D2.

Dropout Summary Statements

Anticipating a 20% dropout rate, 25 subjects should be enrolled in Group 1, and 25 in Group 2, to obtain final group sample sizes of 20 and 20, respectively.

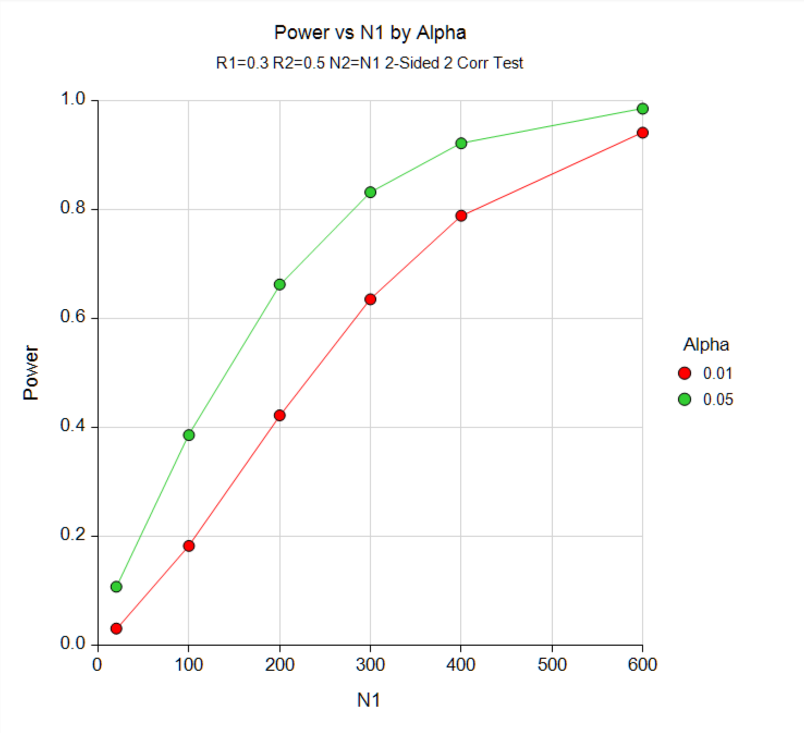
References

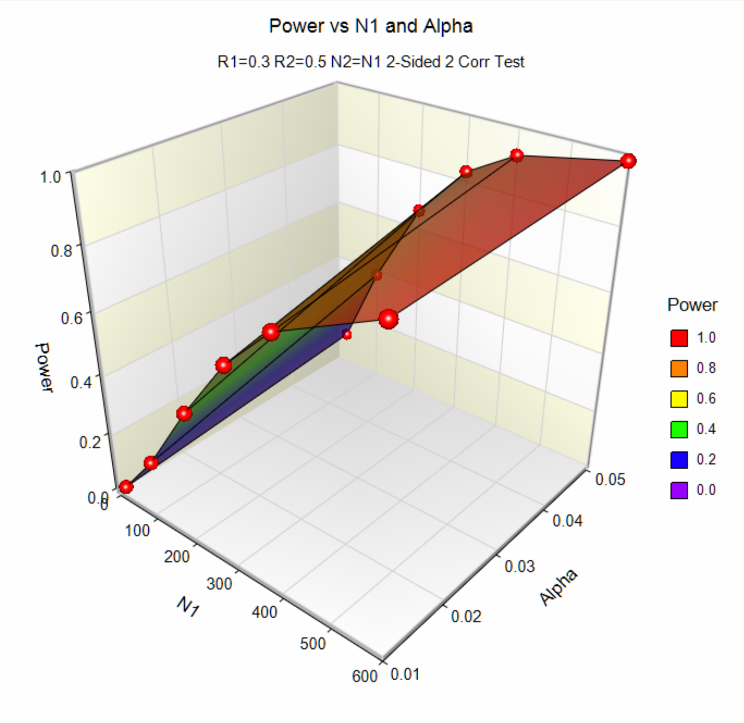
Zar, Jerrold H. 1984. Biostatistical Analysis. Second Edition. Prentice-Hall. Englewood Cliffs, New Jersey.

This report shows the values of each of the parameters, one scenario per row. The values from this table are displayed in the plots below.

Plots Section

Plots





These plots show the relationship between alpha, power, and sample size in this example.

Example 2 – Finding the Sample Size

Continuing with the previous example, suppose the researchers want to determine the exact sample size necessary to achieve 90% power at a 0.05 significance level.

Setup

If the procedure window is not already open, use the PASS Home window to open it. The parameters for this example are listed below and are stored in the **Example 2** settings file. To load these settings to the procedure window, click **Open Example Settings File** in the Help Center or File menu.

Design Tab

Solve For .... **Sample Size**
Alternative Hypothesis .... **Ha: ρ1 ≠ ρ2**
Power.. **0.90**
Alpha.. **0.05**
Group Allocation .... **Equal (N1 = N2)**
ρ1 (Correlation Group 1) **0.3**
ρ2 (Correlation Group 2) **0.5**

Output

Click the Calculate button to perform the calculations and generate the following output.

Numeric Results

Solve For: Sample Size
Alternative Hypothesis: Ha: ρ1 ≠ ρ2

Power		Sample Size			Correlation			
Target	**Actual**	**N1**	**N2**	**N**	**ρ1**	**ρ2**	**ρ1 - ρ2**	**Alpha**
0.9	0.9004	369	369	738	0.3	0.5	-0.2	0.05

PASS has calculated the sample size as 369 per group.

Example 3 – Validation using Zar (1984)

Zar (1984) page 314 presents an example of calculating the power for a test of two correlations. In his example, when *N1* = 95, *N2* = 98, *ρ1* = 0.84, *ρ2* = 0.78, and *alpha* = 0.05, the power is 22% for a two-sided test.

Setup

If the procedure window is not already open, use the PASS Home window to open it. The parameters for this example are listed below and are stored in the **Example 3** settings file. To load these settings to the procedure window, click **Open Example Settings File** in the Help Center or File menu.

Design Tab

Solve For .... **Power**
Alternative Hypothesis .... **Ha: ρ1 ≠ ρ2**
Alpha... **0.05**
Group Allocation .... **Enter N1 and N2 individually**
N1 .... **95**
N2 .... **98**
ρ1 (Correlation Group 1)............................... **0.84**
ρ2 (Correlation Group 2)............................... **0.78**

Output

Click the Calculate button to perform the calculations and generate the following output.

Numeric Results

Solve For: Power
Alternative Hypothesis: Ha: ρ1 ≠ ρ2

	Sample Size			Correlation			
Power	**N1**	**N2**	**N**	**ρ1**	**ρ2**	**ρ1 - ρ2**	**Alpha**
0.22498	95	98	193	0.84	0.78	0.06	0.05

PASS also calculates the power to be 22%.