Chapter 199

Tests for Two Proportions using Effect Size

Introduction

This procedure provides sample size and power calculations for one- or two-sided hypothesis tests of the difference between two independent proportions using the effect size. The details of this procedure are given in Cohen (1988). The design corresponding to this test procedure is sometimes referred to as a *parallel-groups* design. In this design, two proportions from independent populations are compared by considering their difference. The difference is formed between transformed values of the proportions, formed to create variables that are more normally distributed than the raw proportions and that have a variance not related to the values of the proportions.

Test Procedure

If we assume that P_1 and P_2 represent the two proportions. The effect size is represented by the difference h formed as follows

$$h = \varphi_1 - \varphi_2$$

where

$$\varphi_i = 2\,\text{arcsine}\left(\sqrt{P_i}\right)$$

This is referred to as the arcsine, the arcsine root, or the angular transformation.

The null hypothesis is H_0: h = 0 and the alternative hypothesis depends on the number of "sides" of the test:

Two-Sided:	$H_1: h \neq 0$	or $H_1: \varphi_1 - \varphi_2 \neq 0$
Upper One-Sided:	$H_1: h > 0$	or $H_1: \varphi_1 - \varphi_2 > 0$
Lower One-Sided:	$H_1: h < 0$	or $H_1: \varphi_1 - \varphi_2 < 0$

A suitable Type I error probability (α) is chosen for the test, the data is collected, and a z-statistic is generated using the formula

$$z = \frac{\hat{\varphi}_1 - \hat{\varphi}_2}{\sqrt{\frac{N_1 + N_2}{2N_1N_2}}}$$

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This *z*-statistic follows a standard normal distribution. The null hypothesis is rejected in favor of the alternative if,

$$\text{for} \quad H_1: h \neq 0, \quad z < z_{\alpha/2} \quad \text{or} \quad z > z_{1-\alpha/2}$$

$$\text{for} \quad H_1: h > 0, \quad z > z_{1-\alpha}$$

$$\text{for} \quad H_1: h < 0, \quad z < z_{\alpha}$$

Comparing the z-statistic to the cut-off *z*-value is equivalent to comparing the *p*-value to α.

Power Calculation

The power of a one-sided test is calculated using the formulation of Cohen (1988):

$$z_{1-\beta} = h\sqrt{\frac{N'}{2}} - z_{1-\alpha}$$

where

$$N' = \frac{2N_1N_2}{N_1+N_2}$$

The Effect Size

As stated above, the effect size *h* is given by $h = \varphi_1 - \varphi_2$. Cohen (1988) proposed the following interpretation of the *h* values. An *h* near 0.2 is a *small* effect, an *h* near 0.5 is a *medium* effect, and an *h* near 0.8 is a *large* effect. These values for small, medium, and large effects are popular in the social sciences.

Cohen (1988) remarks that the value of h does not match directly with the value of $P_1 - P_2$, so care must be taken when using it. For example, all of the following pairs of values of P_1 and P_2 result in an *h* of about 0.30, even though the actual differences $P_1 - P_2$ are quite different.

P_1	P_2	$P_1 - P_2$	h
0.21	0.10	0.11	0.3
0.39	0.25	0.14	0.3
0.55	0.40	0.15	0.3
0.65	0.50	0.15	0.3
0.78	0.60	0.18	0.3
0.87	0.75	0.13	0.3
0.97	0.90	0.07	0.3

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Example 1 – Finding the Sample Size

Researchers wish to compare two types of local anesthesia using a balanced, parallel-group design. Subjects in pain will be randomized to one of two treatment groups, the treatment will be administered, and the subject's evaluation of pain intensity will be measured on a binary scale (acceptable, unacceptable).

The researchers would like to determine the sample sizes required to detect a small, medium, and large effect size with a two-sided t-test when the power is 80% or 90% and the significance level is 0.05.

Setup

If the procedure window is not already open, use the PASS Home window to open it. The parameters for this example are listed below and are stored in the **Example 1** settings file. To load these settings to the procedure window, click **Open Example Settings File** in the Help Center or File menu.

Design Tab

Solve For	**Sample Size**
Alternative Hypothesis	**Two-Sided**
Power	**0.80 0.90**
Alpha	**0.05**
Group Allocation	**Equal (N1 = N2)**
h	**0.2 0.5 0.8**

Output

Click the Calculate button to perform the calculations and generate the following output.

Numeric Reports

Numeric Results

Solve For:	Sample Size
Test Type:	Two-Sample Z-Test
Alternative Hypothesis:	H1: h ≠ 0

Power		Sample Size			Effect Size	
Target	Actual	N1	N2	N	h	Alpha
0.8	0.8006	393	393	786	0.2	0.05
0.9	0.9003	526	526	1052	0.2	0.05
0.8	0.8013	63	63	126	0.5	0.05
0.9	0.9031	85	85	170	0.5	0.05
0.8	0.8074	25	25	50	0.8	0.05
0.9	0.9014	33	33	66	0.8	0.05

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Target Power	The desired power. May not be achieved because of integer N1 and N2.
Actual Power	The achieved power. Because N1 and N2 are integers, this value is often (slightly) larger than the target power.
N1 and N2	The number of items sampled from each population.
N	The total sample size. N = N1 + N2.
h	Effect Size. Cohen recommended Low = 0.2, Medium = 0.5, and High = 0.8. h = φ1 - φ2, where $\varphi_i = 2 \times \text{ArcSine}(\sqrt{P_i})$.
Alpha	The probability of rejecting a true null hypothesis.

Summary Statements

A parallel two-group design will be used to test whether the proportion effect size (h = φ1 - φ2, where $\varphi_i = 2 \times \text{ArcSine}(\sqrt{P_i})$) is different from 0 (H0: h = 0 versus H1: h ≠ 0). The comparison will be made using a two-sided, two-sample Z-test, with a Type I error rate (α) of 0.05. To detect an effect size of 0.2 with 80% power, the number of subjects needed will be 393 in Group 1 and 393 in Group 2.

Dropout-Inflated Sample Size

	Sample Size			Dropout-Inflated Enrollment Sample Size			Expected Number of Dropouts		
Dropout Rate	**N1**	**N2**	**N**	**N1'**	**N2'**	**N'**	**D1**	**D2**	**D**
20%	393	393	786	492	492	984	99	99	198
20%	526	526	1052	658	658	1316	132	132	264
20%	63	63	126	79	79	158	16	16	32
20%	85	85	170	107	107	214	22	22	44
20%	25	25	50	32	32	64	7	7	14
20%	33	33	66	42	42	84	9	9	18

Dropout Rate	The percentage of subjects (or items) that are expected to be lost at random during the course of the study and for whom no response data will be collected (i.e., will be treated as "missing"). Abbreviated as DR.
N1, N2, and N	The evaluable sample sizes at which power is computed. If N1 and N2 subjects are evaluated out of the N1' and N2' subjects that are enrolled in the study, the design will achieve the stated power.
N1', N2', and N'	The number of subjects that should be enrolled in the study in order to obtain N1, N2, and N evaluable subjects, based on the assumed dropout rate. After solving for N1 and N2, N1' and N2' are calculated by inflating N1 and N2 using the formulas N1' = N1 / (1 - DR) and N2' = N2 / (1 - DR), with N1' and N2' always rounded up. (See Julious, S.A. (2010) pages 52-53, or Chow, S.C., Shao, J., Wang, H., and Lokhnygina, Y. (2018) pages 32-33.)
D1, D2, and D	The expected number of dropouts. D1 = N1' - N1, D2 = N2' - N2, and D = D1 + D2.

Dropout Summary Statements

Anticipating a 20% dropout rate, 492 subjects should be enrolled in Group 1, and 492 in Group 2, to obtain final group sample sizes of 393 and 393, respectively.

References

Cohen, Jacob. 1988. Statistical Power Analysis for the Behavioral Sciences. Lawrence Erlbaum Associates. Hillsdale, New Jersey

These reports show the values of each of the parameters, one scenario per row.

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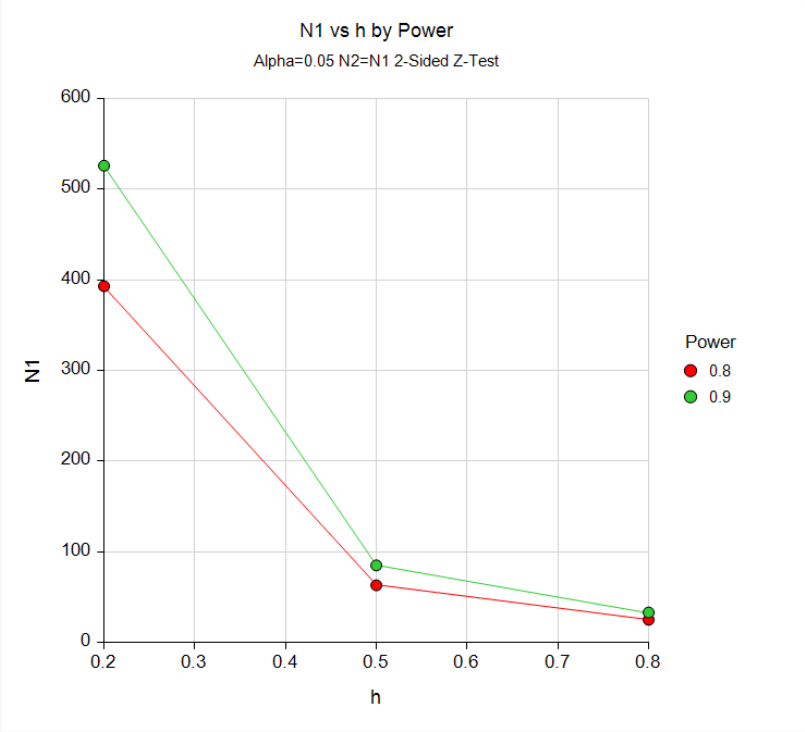
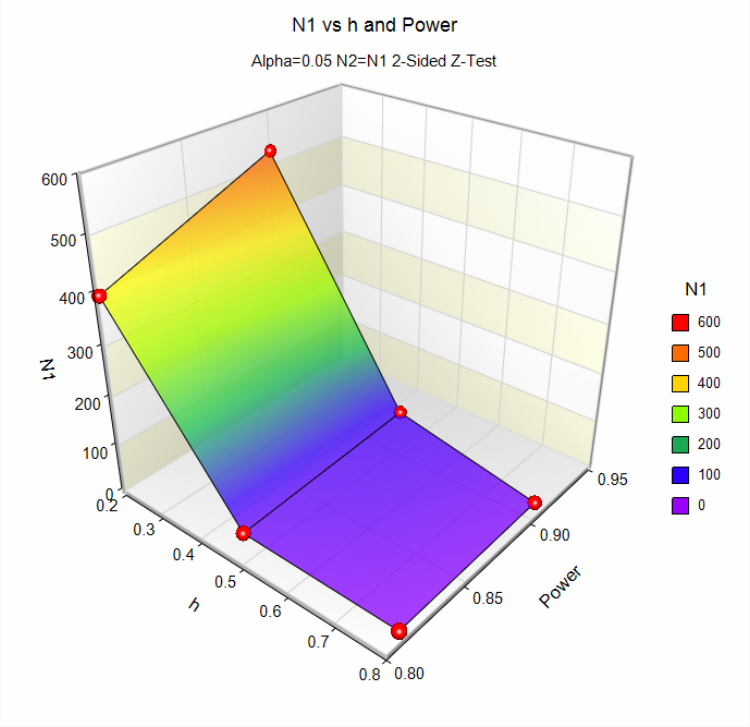
Plots Section

Plots





These plots show the relationship between effect size, power, and sample size.

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Example 2 – Validation using Cohen (1988)

Cohen (1988) gives an example on page 199 of a one-sided test in which alpha = 0.05, h = 0.3, and N1 = N2 = 80. He finds the power to be 0.60.

Setup

If the procedure window is not already open, use the PASS Home window to open it. The parameters for this example are listed below and are stored in the **Example 2** settings file. To load these settings to the procedure window, click **Open Example Settings File** in the Help Center or File menu.

Design Tab	
Solve For	**Power**
Alternative Hypothesis	**One-Sided**
Alpha	**0.05**
Group Allocation	**Equal (N1 = N2)**
Sample Size Per Group	**80**
h	**0.30**

Output

Click the Calculate button to perform the calculations and generate the following output.

Numeric Results

Solve For:	Power
Test Type:	Two-Sample Z-Test
Alternative Hypothesis:	H1: h > 0

	Sample Size			Effect Size	
Power	**N1**	**N2**	**N**	**h**	**Alpha**
0.5997	80	80	160	0.3	0.05

PASS also calculated the power as 0.60 which validates the procedure.

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