

## Chapter 437

# Tests for the Ratio of Two Poisson Rates

## Introduction

The Poisson probability law gives the probability distribution of the number of events occurring in a specified interval of time or space. The Poisson distribution is often used to fit count data, such as the number of defects on an item, the number of accidents at an intersection during a year, the number of calls to a call center during an hour, or the number of meteors seen in the evening sky during an hour.

The Poisson distribution is characterized by a single parameter which is the mean number of occurrences during the specified interval.

The procedure documented in this chapter calculates the power or sample size for testing whether the ratio of two Poisson means is different from a specified value (usually one). The test procedure is described in Gu et al. (2008).

## Test Procedure

Assume that all subjects in each group are observed for a fixed time period and the number of events,  $X$ , (outcomes or defects) is recorded. The following table presents the various terms that are used.

Group	1	2
Fixed time interval	$t_1$	$t_2$
Sample Size	$N_1$	$N_2$
Number of events	$X_1$	$X_2$
Individual event rates	$\lambda_1$	$\lambda_2$
Distribution of $X$	Poisson( $\lambda_1 t_1$ )	Poisson( $\lambda_2 t_2$ )

Define the ratio of event rates,  $RR$ , as

$$RR = \frac{\lambda_2}{\lambda_1}$$

Gu (2008) considered several test statistics that can be used to test hypotheses about the ratio. For example,

$$H_0 : \frac{\lambda_2}{\lambda_1} = RR_0 \text{ versus } H_a : \frac{\lambda_2}{\lambda_1} > RR_0 .$$

or equivalently,

$$H_0 : RR = RR_0 \text{ versus } H_a : RR > RR_0 .$$

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where  $RR_0$  is the ratio of event rates under the null hypothesis.

Two test statistics are available in this case. The first is based on unconstrained maximum likelihood estimates

$$W_1 = \frac{X_2 - X_1 \left( \frac{\sqrt{RR_0}}{d} \right)}{\sqrt{X_2 + X_1 \left( \frac{RR_0}{d} \right)^2}}$$

with

$$d = t_1 N_1 / t_2 N_2.$$

The second test is based on constrained maximum likelihood estimates

$$W_2 = \frac{X_2 - X_1 \left( \frac{RR_0}{d} \right)}{\sqrt{(X_2 + X_1) \left( \frac{RR_0}{d} \right)}}$$

An equivalent pair of test statistics are available if logarithms are used. The statistical hypothesis is

$$H_0 : \ln\left(\frac{\lambda_2}{\lambda_1}\right) - \ln(RR_0) = 0 \text{ versus } H_a : \ln\left(\frac{\lambda_2}{\lambda_1}\right) - \ln(RR_0) > 0$$

or equivalently,

$$H_0 : \ln(RR) - \ln(RR_0) = 0 \text{ versus } H_a : \ln(RR) - \ln(RR_0) > 0$$

Two test statistics are available in this case as well. The first is based on unconstrained maximum likelihood estimates

$$W_3 = \frac{\ln\left(\frac{X_2}{X_1}\right) - \ln\left(\frac{RR_0}{d}\right)}{\sqrt{\frac{1}{X_2} + \frac{1}{X_1}}}$$

The second test is based on constrained maximum likelihood estimates

$$W_4 = \frac{\ln\left(\frac{X_2}{X_1}\right) - \ln\left(\frac{RR_0}{d}\right)}{\sqrt{\frac{\left(2 + \frac{d}{RR_0} + \frac{RR_0}{d}\right)}{X_1 + X_2}}}$$

## Tests for the Ratio of Two Poisson Rates

After extensive simulation, they recommend the following extension of the variance-stabilized test proposed by Huffman (1984) for the case when  $RR_0 / d > 1$ .

$$W_5 = \frac{2 \left[ \sqrt{X_2 + 3/8} - \sqrt{\frac{RR_0}{d}} (X_1 + 3/8) \right]}{\sqrt{1 + \frac{RR_0}{d}}}$$

Gu et al. (2008) show that all of these test statistics are approximately distributed as a standard normal and thus use the normal distribution as the basis of significance testing and power analysis.

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## Assumptions

The assumptions of the two-sample *Poisson* test are:

1. The data in each group are counts (discrete) that follow the Poisson distribution.
2. Each sample is a simple random sample from its population. Unlike most designs, in this design the sample size involves a fixed time parameter. That is, instead of specifying the number of people in a study, the number of man-hours is what is important. Hence, a sample size of 10 hours could be achieved by ten people being observed for one hour or two people being observed for five hours.

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## Technical Details

### Computing Power

If we define  $RR_a$  as the ratio of event rates under the alternative hypothesis at which the power is calculated, the power analysis for testing the hypothesis

$$H_0 : RR = RR_0 \text{ versus } H_a : RR > RR_0 .$$

using the test statistics defined above is completed as follows.

1. **Find the critical value.** Choose the critical value  $z_{1-\alpha}$  using the standard normal distribution so that the probability of rejecting  $H_0$  when it is true is  $\alpha$ .
2. **Compute the power.** Compute the power for each test as follows.

For  $W_1$ ,  $W_3$ , and  $W_4$ , the power is given by

$$Power(W_i) = 1 - \Phi \left( \frac{z_{1-\alpha} \sigma_i - \mu_i}{\sigma_i} \right)$$

where

$$\Phi(z) = \int_{-\infty}^z Normal(0,1)$$

$$\mu_1 = \left( \frac{RR_a}{d} - \frac{RR_0}{d} \right) t_1 N_1 \lambda_1, \quad \sigma_1^2 = \left( \frac{dRR_a + RR_0^2}{d^2} \right) t_1 N_1 \lambda_1$$

## Tests for the Ratio of Two Poisson Rates

$$\mu_3 = \ln\left(\frac{RR_a}{RR_0}\right), \quad \sigma_3^2 = \frac{d + RR_a}{t_1 N_1 \lambda_1 RR_a}$$

$$\mu_4 = \ln\left(\frac{RR_a}{RR_0}\right), \quad \sigma_4^2 = \frac{\left(2 + \frac{d}{RR_0} + \frac{RR_0}{d}\right)}{t_1 N_1 \lambda_1 \left(1 + \frac{RR_a}{d}\right)}$$

For  $W_2$  the power is computed using

$$Power(W_2) = 1 - \Phi\left(\frac{Ez_{1-\alpha} - F}{G}\right)$$

where

$$E = \sqrt{\left(\frac{RR_0}{RR_a}\right)^2 + \frac{RR_0^2}{RR_a d}}, \quad F = \left(1 - \frac{RR_0}{RR_a}\right) \sqrt{\frac{\lambda_1 t_1 N_1 RR_0}{d}},$$

$$G = \sqrt{\frac{RR_0}{RR_a} \left(1 + \frac{RR_0^2}{d RR_a}\right)}$$

For  $W_5$  the power is computed using

$$Power(W_5) = \Phi\left(\frac{|A|\sqrt{B} - z_{1-\alpha}C}{D}\right)$$

where

$$A = 2\left(1 - \sqrt{\frac{RR_0}{RR_a}}\right), \quad B = \lambda_1 t_1 N_1 + 3/8,$$

$$C = \sqrt{\frac{RR_0 + d}{RR_a}}, \quad D = \sqrt{\frac{RR_a + d}{RR_a}}$$

## Computing Sample Size

The sample size is found using the formula

$$N_1 = \frac{\left(\frac{z_{1-\alpha}C + z_{Power}D}{A}\right)^2 - 3/8}{\lambda_1 t_1}.$$

## Procedure Options

This section describes the options that are specific to this procedure. These are located on the Design tab. For more information about the options of other tabs, go to the Procedure Window chapter.

### Design Tab

The Design tab contains most of the parameters and options that you will be concerned with.

#### Solve For

##### Solve For

This option specifies the parameter to be solved for from the other parameters.

Select *Power* when you want to calculate the power of an experiment.

Select *Sample Size* when you want to calculate the sample size needed to achieve a given power and alpha level.

Select *t1 and t2 (Exposure Time with t1 = t2)* to calculate the required exposure time for a set number of subjects to achieve a desired power. The solution is found for the case where t1 = t2.

Select *RRa (Ratio of Event Rates)* to find the detectable ratio for a given power and sample size.

#### Test

##### Alternative Hypothesis

Specify whether the alternative hypothesis of the test is one-sided or two-sided. If a one-sided test is chosen, the hypothesis test direction is chosen based on whether the assumed event rate ratio (*RRa*) is greater than or less than the null hypothesized event ratio (*RR0*).

The options are

- **One-Sided**

The upper and lower null and alternative hypotheses are

Upper:  $H_0 : RR \leq RR_0$  versus  $H_a : RR > RR_0$

Lower:  $H_0 : RR \geq RR_0$  versus  $H_a : RR < RR_0$

- **Two-Sided**

The null and alternative hypotheses are

$H_0 : RR = RR_0$  versus  $H_a : RR \neq RR_0$

##### Test Statistic

This option allows you to select which test statistic will be used. Possible choices are

- **W1 = MLE**

This is the unconstrained maximum likelihood test in which  $\lambda_1$  and  $\lambda_2$  are estimated separately.

- **W2 = CMLE**

This is the constrained maximum likelihood test in which  $\lambda_1$  and  $\lambda_2$  are estimated jointly.

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- **W3 = Ln(MLE)**  
This is the logarithmic version of the unconstrained maximum likelihood test in which  $\lambda_1$  and  $\lambda_2$  are estimated separately.
- **W4 = Ln(CMLE)**  
This is the logarithmic version of the constrained maximum likelihood test  $\lambda_1$  and  $\lambda_2$  are estimated jointly.
- **W5 = Variance Stabilized**  
This is the variance stabilized version of the unconstrained maximum likelihood test. This is the default value.

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## Power and Alpha

### Power

This option specifies one or more values for power. Power is the probability of rejecting a false null hypothesis, and is equal to one minus beta. Beta is the probability of a type-II error, which occurs when a false null hypothesis is not rejected.

Values must be between zero and one. Historically, the value of 0.80 (beta = 0.20) was used for power. Now, 0.90 (beta = 0.10) is also commonly used.

A single value may be entered here or a range of values such as *0.8 to 0.95 by 0.05* may be entered.

### Alpha

This option specifies one or more values for the probability of a type-I error. A type-I error occurs when a true null hypothesis is rejected.

Values must be between zero and one. For one-sided tests such as this, the value of 0.025 is recommended for alpha. You may enter a range of values such as *0.025 0.05 0.10* or *0.025 to 0.05 by 0.005*.

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## Sample Size

### t1 (Exposure Time of Group 1)

Enter a value (or range of values) for the fixed exposure (observation) time for group 1. Each subject in group 1 is observed for this amount of time. If the exposure times are variable, this is the average exposure time per subject in group 1. The value must be greater than 0.

A single value may be entered here or a range of values such as *0.8 to 1.0 by 0.05* may be entered.

### t2 (Exposure Time of Group 2)

Enter a value (or range of values) for the fixed exposure (observation) time for group 2. Each subject in group 2 is observed for this amount of time. If the exposure times are variable, this is the average exposure time per subject in group 2. The value must be greater than 0.

A single value may be entered here or a range of values such as *0.8 to 1.0 by 0.05* may be entered.

## Tests for the Ratio of Two Poisson Rates

**Sample Size (When Solving for Sample Size)****Group Allocation**

Select the option that describes the constraints on  $N1$  or  $N2$  or both.

The options are

- **Equal ( $N1 = N2$ )**  
This selection is used when you wish to have equal sample sizes in each group. Since you are solving for both sample sizes at once, no additional sample size parameters need to be entered.
- **Enter  $N1$ , solve for  $N2$**   
Select this option when you wish to fix  $N1$  at some value (or values), and then solve only for  $N2$ . Please note that for some values of  $N1$ , there may not be a value of  $N2$  that is large enough to obtain the desired power.
- **Enter  $N2$ , solve for  $N1$**   
Select this option when you wish to fix  $N2$  at some value (or values), and then solve only for  $N1$ . Please note that for some values of  $N2$ , there may not be a value of  $N1$  that is large enough to obtain the desired power.
- **Enter  $R = N2/N1$ , solve for  $N1$  and  $N2$**   
For this choice, you set a value for the ratio of  $N2$  to  $N1$ , and then PASS determines the needed  $N1$  and  $N2$ , with this ratio, to obtain the desired power. An equivalent representation of the ratio,  $R$ , is
 
$$N2 = R * N1.$$
- **Enter percentage in Group 1, solve for  $N1$  and  $N2$**   
For this choice, you set a value for the percentage of the total sample size that is in Group 1, and then PASS determines the needed  $N1$  and  $N2$  with this percentage to obtain the desired power.

 **$N1$  (Sample Size, Group 1)**

*This option is displayed if Group Allocation = "Enter  $N1$ , solve for  $N2$ "*

$N1$  is the number of items or individuals sampled from the Group 1 population.

$N1$  must be  $\geq 2$ . You can enter a single value or a series of values.

 **$N2$  (Sample Size, Group 2)**

*This option is displayed if Group Allocation = "Enter  $N2$ , solve for  $N1$ "*

$N2$  is the number of items or individuals sampled from the Group 2 population.

$N2$  must be  $\geq 2$ . You can enter a single value or a series of values.

 **$R$  (Group Sample Size Ratio)**

*This option is displayed only if Group Allocation = "Enter  $R = N2/N1$ , solve for  $N1$  and  $N2$ ."*

$R$  is the ratio of  $N2$  to  $N1$ . That is,

$$R = N2 / N1.$$

Use this value to fix the ratio of  $N2$  to  $N1$  while solving for  $N1$  and  $N2$ . Only sample size combinations with this ratio are considered.

$N2$  is related to  $N1$  by the formula:

$$N2 = [R \times N1],$$

where the value  $[Y]$  is the next integer  $\geq Y$ .

## Tests for the Ratio of Two Poisson Rates

For example, setting  $R = 2.0$  results in a Group 2 sample size that is double the sample size in Group 1 (e.g.,  $N1 = 10$  and  $N2 = 20$ , or  $N1 = 50$  and  $N2 = 100$ ).

$R$  must be greater than 0. If  $R < 1$ , then  $N2$  will be less than  $N1$ ; if  $R > 1$ , then  $N2$  will be greater than  $N1$ . You can enter a single or a series of values.

### Percent in Group 1

*This option is displayed only if Group Allocation = "Enter percentage in Group 1, solve for  $N1$  and  $N2$ ."*

Use this value to fix the percentage of the total sample size allocated to Group 1 while solving for  $N1$  and  $N2$ . Only sample size combinations with this Group 1 percentage are considered. Small variations from the specified percentage may occur due to the discrete nature of sample sizes.

The Percent in Group 1 must be greater than 0 and less than 100. You can enter a single or a series of values.

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## Sample Size (When Not Solving for Sample Size)

### Group Allocation

Select the option that describes how individuals in the study will be allocated to Group 1 and to Group 2.

The options are

- **Equal ( $N1 = N2$ )**  
This selection is used when you wish to have equal sample sizes in each group. A single per group sample size will be entered.
- **Enter  $N1$  and  $N2$  individually**  
This choice permits you to enter different values for  $N1$  and  $N2$ .
- **Enter  $N1$  and  $R$ , where  $N2 = R * N1$**   
Choose this option to specify a value (or values) for  $N1$ , and obtain  $N2$  as a ratio (multiple) of  $N1$ .
- **Enter total sample size and percentage in Group 1**  
Choose this option to specify a value (or values) for the total sample size ( $N$ ), obtain  $N1$  as a percentage of  $N$ , and then  $N2$  as  $N - N1$ .

### Sample Size Per Group

*This option is displayed only if Group Allocation = "Equal ( $N1 = N2$ )."*

The Sample Size Per Group is the number of items or individuals sampled from each of the Group 1 and Group 2 populations. Since the sample sizes are the same in each group, this value is the value for  $N1$ , and also the value for  $N2$ .

The Sample Size Per Group must be  $\geq 2$ . You can enter a single value or a series of values.

### $N1$ (Sample Size, Group 1)

*This option is displayed if Group Allocation = "Enter  $N1$  and  $N2$  individually" or "Enter  $N1$  and  $R$ , where  $N2 = R * N1$ ."*

$N1$  is the number of items or individuals sampled from the Group 1 population.

$N1$  must be  $\geq 2$ . You can enter a single value or a series of values.



## Tests for the Ratio of Two Poisson Rates

### N2 (Sample Size, Group 2)

*This option is displayed only if Group Allocation = "Enter N1 and N2 individually."*

$N2$  is the number of items or individuals sampled from the Group 2 population.

$N2$  must be  $\geq 2$ . You can enter a single value or a series of values.

### R (Group Sample Size Ratio)

*This option is displayed only if Group Allocation = "Enter N1 and R, where  $N2 = R * N1$ ."*

$R$  is the ratio of  $N2$  to  $N1$ . That is,

$$R = N2/N1$$

Use this value to obtain  $N2$  as a multiple (or proportion) of  $N1$ .

$N2$  is calculated from  $N1$  using the formula:

$$N2 = [R \times N1],$$

where the value  $[Y]$  is the next integer  $\geq Y$ .

For example, setting  $R = 2.0$  results in a Group 2 sample size that is double the sample size in Group 1.

$R$  must be greater than 0. If  $R < 1$ , then  $N2$  will be less than  $N1$ ; if  $R > 1$ , then  $N2$  will be greater than  $N1$ . You can enter a single value or a series of values.

### Total Sample Size (N)

*This option is displayed only if Group Allocation = "Enter total sample size and percentage in Group 1."*

This is the total sample size, or the sum of the two group sample sizes. This value, along with the percentage of the total sample size in Group 1, implicitly defines  $N1$  and  $N2$ .

The total sample size must be greater than one, but practically, must be greater than 3, since each group sample size needs to be at least 2.

You can enter a single value or a series of values.

### Percent in Group 1

*This option is displayed only if Group Allocation = "Enter total sample size and percentage in Group 1."*

This value fixes the percentage of the total sample size allocated to Group 1. Small variations from the specified percentage may occur due to the discrete nature of sample sizes.

The Percent in Group 1 must be greater than 0 and less than 100. You can enter a single value or a series of values.

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## Effect Size

### $\lambda 1$ (Event Rate of Group 1)

Enter a value (or range of values) for the mean event rate per time unit in group 1 (control).

#### Example of Estimating $\lambda 1$

If 200 patients were exposed for 1 year (i.e.  $t1 = 1$  year) and 40 experienced the event of interest, then the mean event rate would be

$$\lambda 1 = 40/(200*1) = 0.2 \text{ per patient-year}$$

If 200 patients were exposed for 2 years (i.e.  $t1 = 2$  years) and 40 experienced the event of interest, then the mean event rate would be

$$\lambda 1 = 40/(200*2) = 0.1 \text{ per patient-year}$$

## Tests for the Ratio of Two Poisson Rates

### Event Rate Ratio

$\lambda_1$  is used with  $\lambda_2a$  to calculate the event rate ratio as

$$RRa = \lambda_2a/\lambda_1$$

such that

$$\lambda_1 = \lambda_2a/RRa$$

The range of acceptable values is  $\lambda_1 > 0$ . You can enter a single value such as  $1$  or a series of values such as  $1$  to  $2$  by  $0.5$ .

### RR0 (Ratio of Event Rates under H0)

Enter a value (or range of values) for the ratio of the two mean event rates assumed by the null hypothesis, H0. Usually,  $RR0 = 1.0$  which implies that the two rates are equal (i.e.  $\lambda_1 = \lambda_2$ ). However, you may test other values of RR0 as well.

- **One-Sided**

The upper and lower null and alternative hypotheses are

Upper:  $H_0 : RR \leq RR_0$  versus  $H_a : RR > RR_0$

Lower:  $H_0 : RR \geq RR_0$  versus  $H_a : RR < RR_0$

- **Two-Sided**

The null and alternative hypotheses are

$$H_0 : RR = RR_0 \text{ versus } H_a : RR \neq RR_0$$

### Enter $\lambda_2$ or Ratio for Group 2

Indicate whether to enter the group 2 event rate ( $\lambda_2a$ ) or the event rate ratio (RRa) to specify the effect size.

The event rate ratio is calculated from  $\lambda_2a$  and  $\lambda_1$  as

$$RRa = \lambda_2a/\lambda_1$$

### RRa (Ratio of Event Rates under Ha)

*This option is displayed only if Enter  $\lambda_2$  or Ratio for Group 2 = "RRa (Ratio of Event Rates under Ha)."*

This is the value of the ratio of the two event rates,  $\lambda_1$  and  $\lambda_2a$ , at which the power is to be calculated.

The event rate ratio is calculated from  $\lambda_1$  and  $\lambda_2a$  as

$$RRa = \lambda_2a/\lambda_1$$

The range of acceptable values is  $RRa > 0$  and  $RRa \neq RR0$ .

You can enter a single value such as  $1.15$  or a series of values such as  $1.05$  to  $1.20$  by  $0.05$ .

### RRa (Ratio of Event Rates under Ha) [when solving for RRa (Ratio of Event Rates)]

Specify whether to solve for event rate ratios that are less than one or greater than RR0.

## Tests for the Ratio of Two Poisson Rates

 **$\lambda_{2a}$  (Event Rate of Group 2 under  $H_a$ )**

*This option is displayed only if Enter  $\lambda_2$  or Ratio for Group 2 = " $\lambda_{2a}$  (Event Rate of Group 2 under  $H_a$ )."*

Enter a value (or range of values) for the mean event rate per time unit in group 2 (treatment) under  $H_a$ , the alternative hypothesis.

**Example of Estimating  $\lambda_{2a}$** 

If 200 patients were exposed for 1 year (i.e.  $t_2 = 1$  year) and 40 experienced the event of interest, then the mean event rate would be

$$\lambda_{2a} = 40/(200*1) = 0.2 \text{ per patient-year}$$

If 200 patients were exposed for 2 years (i.e.  $t_2 = 2$  years) and 40 experienced the event of interest, then the mean event rate would be

$$\lambda_{2a} = 40/(200*2) = 0.1 \text{ per patient-year}$$

**Event Rate Ratio**

$\lambda_{2a}$  is used with  $\lambda_1$  to calculate the event rate ratio as

$$RRa = \lambda_{2a}/\lambda_1$$

such that

$$\lambda_{2a} = RRa*\lambda_1$$

The range of acceptable values is  $\lambda_{2a} > 0$ . You can enter a single value such as 1 or a series of values such as 1 to 2 by 0.5.

## Example 1 – Finding the Sample Size

We will use the example of Gu (2008) in which epidemiologist wish to examine the relationship of post-menopausal hormone use and coronary heart disease (CHD). The incidence rate for those not using the hormone is 0.0005 ( $\lambda_1 = 0.0005$ ). How large of a sample is needed to detect a change in the incidence ratio from  $RR_0 = 1$  to  $RR_a = 2, 3, 4, 5, \text{ or } 6$ . Assume that 90% power is required and  $\alpha = 0.05$ . Assume that each subject will be observed for two years and that the design calls for an equal number of subjects in both groups.

### Setup

This section presents the values of each of the parameters needed to run this example. First, from the PASS Home window, load the procedure window. You may make the appropriate entries as listed below or open **Example 1** by going to the **File** menu and choosing **Open Example Template**.

<u>Option</u>	<u>Value</u>
<b>Design Tab</b>	
Solve For .....	<b>Sample Size</b>
Alternative Hypothesis .....	<b>One-Sided</b>
Test Statistic .....	<b>W5 = Variance Stabilized</b>
Power.....	<b>0.90</b>
Alpha.....	<b>0.05</b>
t1 (Exposure Time of Group 1) .....	<b>2</b>
t2 (Exposure Time of Group 2) .....	<b>2</b>
Group Allocation .....	<b>Equal (N1 = N2)</b>
$\lambda_1$ (Group 1 Event Rate).....	<b>0.0005</b>
RR0 (Ratio of Event Rates under H0) ....	<b>1</b>
Enter $\lambda_2$ or Ratio for Group 2 .....	<b>RRa (Ratio of Event Rates under H0)</b>
RRa (Ratio of Event Rates under Ha) ....	<b>2 3 4 5 6</b>
<b>Reports Tab</b>	
Decimals for Event Rates .....	<b>4</b>

## Tests for the Ratio of Two Poisson Rates

## Annotated Output

Click the Calculate button to perform the calculations and generate the following output.

## Numeric Results

## Numeric Results for Testing the Ratio of Two Poisson Rates

Alternative Hypothesis: One-Sided ( $H_0: RR \leq RR_0$  vs.  $H_a: RR > RR_0$ )

Test Statistic: W5 = Variance Stabilized

Power	N1	N2	N	Grp 1 Expos Time t1	Grp 2 Expos Time t2	Grp 1 Event Rate $\lambda_1$	Grp 2 Event Rate $\lambda_{2a}$	Alt Rate Ratio RRa	Null Rate Ratio RR0	Alpha
0.90001	29737	29737	59474	2.0	2.0	0.0005	0.0010	2.000	1.000	0.050
0.90000	10777	10777	21554	2.0	2.0	0.0005	0.0015	3.000	1.000	0.050
0.90001	6364	6364	12728	2.0	2.0	0.0005	0.0020	4.000	1.000	0.050
0.90002	4513	4513	9026	2.0	2.0	0.0005	0.0025	5.000	1.000	0.050
0.90001	3514	3514	7028	2.0	2.0	0.0005	0.0030	6.000	1.000	0.050

## References

Gu, K., Ng, H.K.T., Tang, M.L., and Schucany, W. 2008. 'Testing the Ratio of Two Poisson Rates.' Biometrical Journal, 50, 2, 283-298.

Huffman, Michael. 1984. 'An Improved Approximate Two-Sample Poisson Test.' Applied Statistics, 33, 2, 224-226.

## Report Definitions

Power is the probability of rejecting the null hypothesis when it is false. It should be close to one.

N1 and N2 are the number of subjects in groups 1 and 2, respectively.

N is the total sample size.  $N = N1 + N2$ .

t1 and t2 are the exposure (observation) times in groups 1 and 2, respectively.

$\lambda_1$  is the mean event rate per time unit in group 1 (control). This is often the baseline event rate.

$\lambda_{2a}$  is the mean event rate per time unit in group 2 (treatment) under the alternative hypothesis.

RRa is the ratio of the two event rates under the alternative hypothesis.  $RRa = \lambda_{2a}/\lambda_1$ .

RR0 is the ratio of the two event rates under the null hypothesis.

Alpha is the probability of rejecting the null hypothesis when it is true. It should be small.

## Summary Statements

For a one-sided test of the null hypothesis  $H_0: RR \leq 1.000$  vs. the alternative  $H_a: RR > 1.000$  using the W5 = Variance Stabilized test statistic, samples of 29737 subjects in group 1 with exposure time of 2.0 and 29737 subjects in group 2 with exposure time of 2.0 achieve 90.001% power to detect an event rate ratio (RRa) of 2.000 when the event rate in group 1 ( $\lambda_1$ ) is 0.0005 and the significance level (alpha) is 0.050.

This report shows the values of each of the parameters, one scenario per row. The values of power and beta were calculated from the other parameters.

## Power

Power is the probability of rejecting the null hypothesis when it is false. It should be close to one.

## N1 and N2

N1 and N2 are the number of subjects in groups 1 and 2, respectively. To calculate the sample size in terms of person-time, you would multiply the sample size by the exposure time,  $t_i$ . For example, the group sample size in the first row calculated in person-years is  $29737 \times 2.0 = 59474$  person-years.

## N

N is the total sample size.  $N = N1 + N2$ .

## t1 and t2

t1 and t2 are the exposure (observation) times in groups 1 and 2, respectively.

 $\lambda_1$ 

$\lambda_1$  is the mean event rate per time unit in group 1 (control). This is often the baseline event rate.

## Tests for the Ratio of Two Poisson Rates

 **$\lambda_2a$** 

$\lambda_2a$  is the mean event rate per time unit in group 2 (treatment) under the alternative hypothesis.

**RRa**

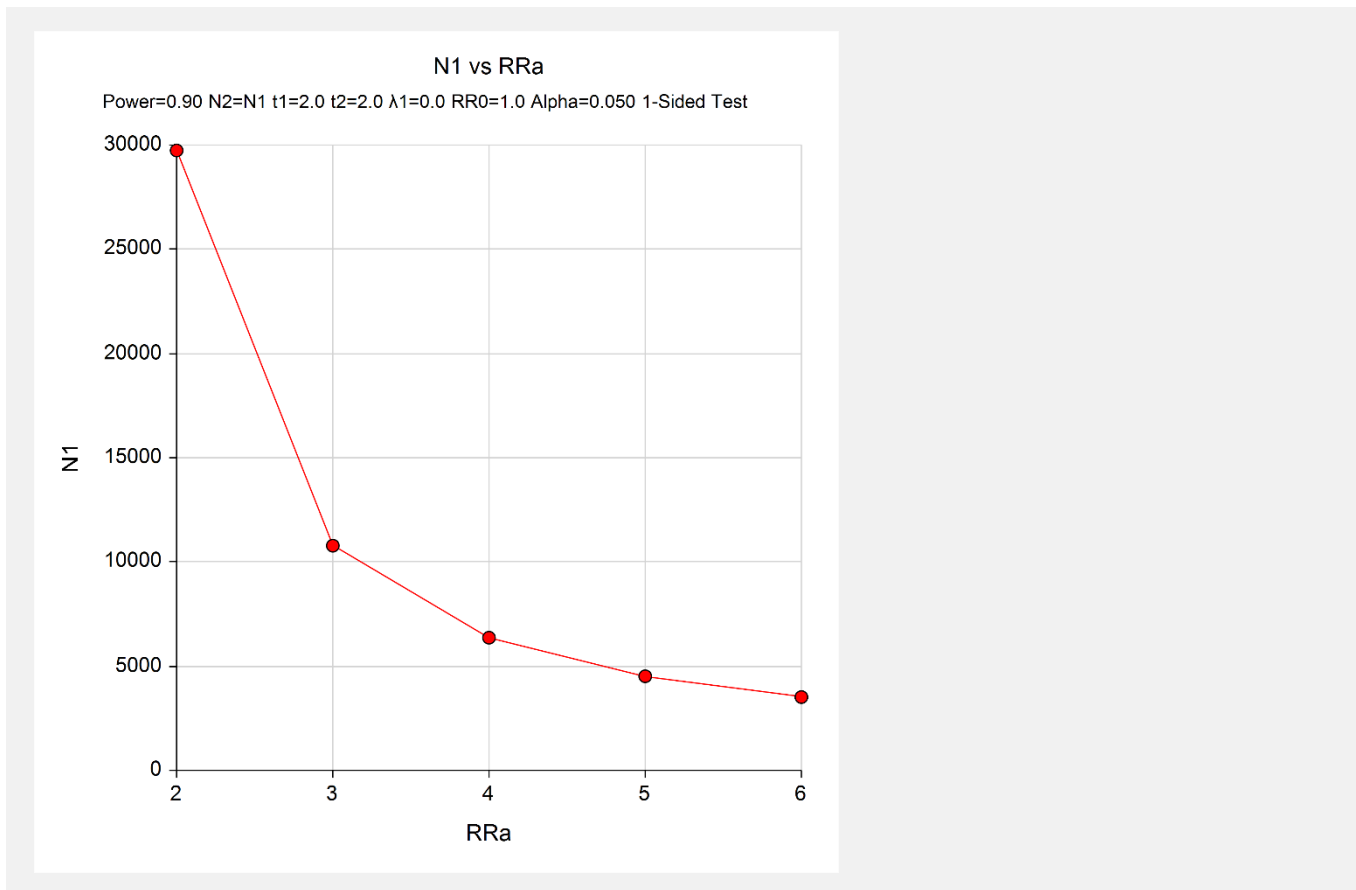
RRa is the ratio of the two event rates under the alternative hypothesis.  $RRa = \lambda_2a/\lambda_1$ .

**RR0**

RR0 is the ratio of the two event rates under the null hypothesis.

**Alpha**

Alpha is the probability of rejecting the null hypothesis when it is true. It should be small.

**Plots Section**

This plot shows the relationship between group sample size and RRa.

## Tests for the Ratio of Two Poisson Rates

**Example 2 – Validation using Gu et al. (2008)**

Gu et al. (2008) present an example that we will use to validate this procedure. Using the scenario cited in Example 1 above, they give a sample size calculation on page 295. In this example,  $\lambda_1 = 0.0005$ ,  $\rho_0 = 1$ ,  $\rho_a = 4$ ,  $t_1 = t_2 = 2$ ,  $\alpha = 0.05$ ,  $R = 0.5$ , and power = 0.9. In their Table 6, they list the sample size for  $p_5^{(A)}$  in this scenario as 8627. However, this number is inaccurate because of the two-decimal place rounding that was done during their calculation. In a private communication, they agreed that the more accurate number is 8590.

**Setup**

This section presents the values of each of the parameters needed to run this example. First, from the PASS Home window, load the procedure window. You may make the appropriate entries as listed below or open **Example 2** by going to the **File** menu and choosing **Open Example Template**.

<u>Option</u>	<u>Value</u>
<b>Design Tab</b>	
Solve For .....	<b>Sample Size</b>
Alternative Hypothesis .....	<b>One-Sided</b>
Test Statistic .....	<b>W5 = Variance Stabilized</b>
Power .....	<b>0.90</b>
Alpha .....	<b>0.05</b>
t1 (Exposure Time of Group 1) .....	<b>2</b>
t2 (Exposure Time of Group 2) .....	<b>2</b>
Group Allocation .....	<b>Enter R = N2/N1, solve for N1 and N2</b>
R .....	<b>0.5</b>
$\lambda_1$ (Group 1 Event Rate).....	<b>0.0005</b>
RR0 (Ratio of Event Rates under H0) ....	<b>1</b>
Enter $\lambda_2$ or Ratio for Group 2 .....	<b>RRa (Ratio of Event Rates under H0)</b>
RRa (Ratio of Event Rates under Ha) ....	<b>4</b>
<b>Reports Tab</b>	
Decimals for Event Rates .....	<b>4</b>

**Output**

Click the Calculate button to perform the calculations and generate the following output.

**Numeric Results**

<b>Numeric Results for Testing the Ratio of Two Poisson Rates</b>											
Alternative Hypothesis: One-Sided (H0: RR ≤ RR0 vs. Ha: RR > RR0)											
Test Statistic: W5 = Variance Stabilized											
					Grp 1	Grp 2	Grp 1	Grp 2	Alt	Null	
Power	N1	N2	N	R	Expos	Expos	Event	Event	Rate	Rate	Alpha
					t1	t2	Rate	Rate	RRa	RR0	
							$\lambda_1$	$\lambda_{2a}$			
0.90001	8590	4295	12885	0.50	2.0	2.0	0.0005	0.0020	4.000	1.000	0.050

These results match the more accurate value of 8590.