

Chapter 411

Wilcoxon Signed-Rank Tests

Introduction

The one-sample t -test is used to test whether the mean of a population is greater than, less than, or not equal to a specific value. Because the t distribution is used to calculate critical values for the test, this test is often called the one-sample t -test. The t -test assumes that the population standard deviation is unknown and will be estimated by the data.

The nonparametric analog of the t -test is the *Wilcoxon Signed-Rank Test* and may be used when the one-sample t -test assumptions are violated.

Other PASS Procedures for Testing One Mean or Median

Procedures in **PASS** are primarily built upon the testing methods, test statistic, and test assumptions that will be used when the analysis of the data is performed. You should check to identify that the test procedure described below in the Test Procedure section matches your intended procedure. If your assumptions or testing method are different, you may wish to use one of the other one-sample procedures available in **PASS**—the One-Sample Z-Tests and the One-Sample T-Tests procedures. The methods, statistics, and assumptions for those procedures are described in the associated chapters.

If you wish to show that the mean of a population is larger (or smaller) than a reference value by a specified amount, you should use one of the clinical superiority procedures for comparing means. Non-inferiority, equivalence, and confidence interval procedures are also available.

Assumptions for One-Sample Tests

This section describes the assumptions that are made when you use one of the one-sample tests. The key assumption relates to normality or non-normality of the data. One of the reasons for the popularity of the t -test is its robustness in the face of assumption violation. However, if an assumption is not met even approximately, the significance levels and the power of the t -test are invalidated. Unfortunately, in practice it often happens that several assumptions are not met. This makes matters even worse! Hence, take the steps to check the assumptions before you make important decisions based on these tests.

One-Sample Z-Test Assumptions

The assumptions of the one-sample z-test are:

1. The data are continuous (not discrete).
2. The data follow the normal probability distribution.
3. The sample is a simple random sample from its population. Each individual in the population has an equal probability of being selected in the sample.
4. The population standard deviation is known.

One-Sample T-Test Assumptions

The assumptions of the one-sample t -test are:

1. The data are continuous (not discrete).
2. The data follow the normal probability distribution.
3. The sample is a simple random sample from its population. Each individual in the population has an equal probability of being selected in the sample.

Paired T-Test Assumptions

The assumptions of the paired t -test are:

1. The data are continuous (not discrete).
2. The data, i.e., the differences for the matched pairs, follow a normal probability distribution.
3. The sample of pairs is a simple random sample from its population. Each individual in the population has an equal probability of being selected in the sample.

Wilcoxon Signed-Rank Test Assumptions

The assumptions of the Wilcoxon signed-rank test are as follows (note that the difference is between a data value and the hypothesized median or between the two data values of a pair):

1. The differences are continuous (not discrete).
2. The distribution of each difference is symmetric.
3. The differences are mutually independent.
4. The differences all have the same median.
5. The measurement scale is at least interval.

Limitations

There are few limitations when using these tests. Sample sizes may range from a few to several hundred. If your data are discrete with at least five unique values, you can often ignore the continuous variable assumption. Perhaps the greatest restriction is that your data come from a random sample of the population. If you do not have a random sample, your significance levels will probably be incorrect.

Wilcoxon Signed-Rank Test Statistic

The Wilcoxon signed-rank test is a popular, nonparametric substitute for the t -test. It assumes that the data follow a symmetric distribution. The test is computed using the following steps.

1. Subtract the hypothesized mean, μ_0 , from each data value. Rank the values according to their absolute values.
2. Compute the sum of the positive ranks S_p and the sum of the negative ranks S_n . The test statistic, W_R , is the minimum of S_p and S_n .
3. Compute the mean and standard deviation of W_R using the formulas

$$\mu_{W_R} = \frac{n(n+1)}{4}$$

$$\sigma_{W_R} = \sqrt{\frac{n(n+1)(2n+1)}{24} - \frac{\sum t^3 - \sum t}{48}}$$

where t represents the number of times the i^{th} value occurs.

4. Compute the z-value using

$$z_W = \frac{W_R - \mu_{W_R}}{\sigma_{W_R}}$$

The significance of the test statistic is determined by computing the p-value using the standard normal distribution. If this p-value is less than a specified level (usually 0.05), the null hypothesis is rejected in favor of the alternative hypothesis. Otherwise, no conclusion can be reached.

Population Size

This is the number of subjects in the population. Usually, you assume that samples are drawn from a very large (infinite) population. Occasionally, however, situations arise in which the population of interest is of limited size. In these cases, appropriate adjustments must be made.

When a finite population size is specified, the standard deviation is reduced according to the formula:

$$\sigma_1^2 = \left(1 - \frac{n}{N}\right) \sigma^2$$

where n is the sample size, N is the population size, σ is the original standard deviation, and σ_1 is the new standard deviation.

The quantity n/N is often called the sampling fraction. The quantity $\left(1 - \frac{n}{N}\right)$ is called the *finite population correction factor*.

Power Calculation for the Wilcoxon Signed-Rank Test

The power calculation for the Wilcoxon signed-rank test is the same as that for the one-sample t -test except that an adjustment is made to the sample size based on an assumed data distribution as described in Al-Sunduqchi and Guenther (1990). The sample size n' used in power calculations is equal to

$$n' = n/W,$$

where W is the Wilcoxon adjustment factor based on the assumed data distribution.

The adjustments are as follows:

| Distribution | W |
|--------------------|-----------|
| Uniform | 1 |
| Double Exponential | 2/3 |
| Logistic | $9/\pi^2$ |
| Normal | $\pi/3$ |

The power is calculated as follows for a directional alternative (one-tailed test) in which $\mu_1 > \mu_0$.

1. Find t_α such that $1 - T_{df}(t_\alpha) = \alpha$, where $T_{df}(t_\alpha)$ is the area under a central- t curve to the left of x and $df = n' - 1$.
2. Calculate: $X_1 = \mu_0 + t_\alpha \frac{\sigma}{\sqrt{n'}}$.
3. Calculate the noncentrality parameter: $\lambda = \frac{\mu_1 - \mu_0}{\frac{\sigma}{\sqrt{n'}}} = \frac{\delta_1}{\frac{\sigma}{\sqrt{n'}}}$.
4. Calculate: $t_1 = \frac{X_1 - \mu_1}{\frac{\sigma}{\sqrt{n'}}} + \lambda$.
5. Power = $1 - T'_{df,\lambda}(t_1)$, where $T'_{df,\lambda}(x)$ is the area to the left of x under a noncentral- t curve with degrees of freedom df and noncentrality parameter λ .

Example 1 – Power after a Study

This example will cover the situation in which you are calculating the power of a Wilcoxon Signed-Rank test on data that have already been collected and analyzed. For example, you might be playing the role of a reviewer, looking at the power of a Wilcoxon Signed-Rank test from a study you are reviewing. In this case, you would not vary the means, standard deviation, or sample size since they are given by the experiment. Instead, you investigate the power of the significance tests. You might look at the impact of different alpha values on the power.

Suppose an experiment involving 100 individuals yields the following summary statistics:

| | |
|-------------------------------|-------|
| Hypothesized mean (μ_0) | 100.0 |
| Sample mean (μ_1) | 110.0 |
| Sample standard deviation | 40.0 |
| Sample size | 100 |

Given the above data, analyze the power of a Wilcoxon Signed-Rank test which tests the hypothesis that the population median is 100 versus the alternative hypothesis that the population median is 110. Consider the power at significance levels 0.01, 0.05, 0.10 and sample sizes 20 to 120 by 20 when the assumed data distribution is Logistic.

Note that we have set μ_1 equal to the sample mean. In this case, we are studying the power of the Wilcoxon Signed-Rank test for a mean difference the size of that found in the experimental data.

Setup

If the procedure window is not already open, use the PASS Home window to open it. The parameters for this example are listed below and are stored in the **Example 1** settings file. To load these settings to the procedure window, click **Open Example Settings File** in the Help Center or File menu.

Design Tab

| | |
|---------------------------------------|--|
| Solve For | Power |
| Alternative Hypothesis | Two-Sided (H1: $\mu \neq \mu_0$) |
| Data Distribution | Logistic |
| Population Size | Infinite |
| Alpha..... | 0.01 0.05 0.10 |
| N (Sample Size)..... | 20 to 120 by 20 |
| μ_0 (Null or Baseline Mean) | 100 |
| μ_1 (Actual Mean) | 110 |
| σ (Standard Deviation)..... | 40 |

Wilcoxon Signed-Rank Tests

Output

Click the Calculate button to perform the calculations and generate the following output.

Numeric Reports

Numeric Results

Solve For: **Power**
 Hypotheses: $H_0: \mu = \mu_0$ vs. $H_1: \mu \neq \mu_0$
 Data Distribution: Logistic

| Power | Sample Size N | Mean | | | Standard Deviation σ | Effect Size | Alpha |
|---------|---------------|--------------|----------------|----------------------------|-----------------------------|-------------|-------|
| | | Null μ_0 | Actual μ_1 | Difference $\mu_1 - \mu_0$ | | | |
| 0.06416 | 20 | 100 | 110 | 10 | 40 | 0.25 | 0.01 |
| 0.15855 | 40 | 100 | 110 | 10 | 40 | 0.25 | 0.01 |
| 0.27019 | 60 | 100 | 110 | 10 | 40 | 0.25 | 0.01 |
| 0.38634 | 80 | 100 | 110 | 10 | 40 | 0.25 | 0.01 |
| 0.49768 | 100 | 100 | 110 | 10 | 40 | 0.25 | 0.01 |
| 0.59833 | 120 | 100 | 110 | 10 | 40 | 0.25 | 0.01 |
| 0.19362 | 20 | 100 | 110 | 10 | 40 | 0.25 | 0.05 |
| 0.36033 | 40 | 100 | 110 | 10 | 40 | 0.25 | 0.05 |
| 0.51014 | 60 | 100 | 110 | 10 | 40 | 0.25 | 0.05 |
| 0.63526 | 80 | 100 | 110 | 10 | 40 | 0.25 | 0.05 |
| 0.73462 | 100 | 100 | 110 | 10 | 40 | 0.25 | 0.05 |
| 0.81063 | 120 | 100 | 110 | 10 | 40 | 0.25 | 0.05 |
| 0.29827 | 20 | 100 | 110 | 10 | 40 | 0.25 | 0.10 |
| 0.48781 | 40 | 100 | 110 | 10 | 40 | 0.25 | 0.10 |
| 0.63672 | 60 | 100 | 110 | 10 | 40 | 0.25 | 0.10 |
| 0.74816 | 80 | 100 | 110 | 10 | 40 | 0.25 | 0.10 |
| 0.82865 | 100 | 100 | 110 | 10 | 40 | 0.25 | 0.10 |
| 0.88524 | 120 | 100 | 110 | 10 | 40 | 0.25 | 0.10 |

| | |
|-----------------|---|
| Power | The probability of rejecting a false null hypothesis when the alternative hypothesis is true. |
| N | The size of the sample drawn from the population. |
| μ_0 | The value of the population mean under the null hypothesis. |
| μ_1 | The actual value of the population mean at which power and sample size are calculated. |
| $\mu_1 - \mu_0$ | The difference between the actual and null means. |
| σ | The standard deviation of the population. It measures the variability in the population. |
| Effect Size | The relative magnitude of the effect. Effect Size = $ \mu_1 - \mu_0 /\sigma$. |
| Alpha | The probability of rejecting a true null hypothesis. |

Summary Statements

A single-group design will be used to test whether the location (distribution center) is different from 100 ($H_0: \mu = 100$ versus $H_1: \mu \neq 100$). The comparison will be made using a two-sided, one-sample Wilcoxon Signed-Rank test, with a Type I error rate (α) of 0.01. The underlying data distribution is assumed to be Logistic. The underlying distribution standard deviation is assumed to be 40. To detect a mean of 110 (corresponding to a difference of 10) with a sample size of 20, the power is 0.06416.

Wilcoxon Signed-Rank Tests

Dropout-Inflated Sample Size

| Dropout Rate | Sample Size N | Dropout- Inflated Enrollment Sample Size N' | Expected Number of Dropouts D |
|--------------|------------------|---|--|
| 20% | 20 | 25 | 5 |
| 20% | 40 | 50 | 10 |
| 20% | 60 | 75 | 15 |
| 20% | 80 | 100 | 20 |
| 20% | 100 | 125 | 25 |
| 20% | 120 | 150 | 30 |

| | |
|--------------|--|
| Dropout Rate | The percentage of subjects (or items) that are expected to be lost at random during the course of the study and for whom no response data will be collected (i.e., will be treated as "missing"). Abbreviated as DR. |
| N | The evaluable sample size at which power is computed (as entered by the user). If N subjects are evaluated out of the N' subjects that are enrolled in the study, the design will achieve the stated power. |
| N' | The total number of subjects that should be enrolled in the study in order to obtain N evaluable subjects, based on the assumed dropout rate. N' is calculated by inflating N using the formula $N' = N / (1 - DR)$, with N' always rounded up. (See Julious, S.A. (2010) pages 52-53, or Chow, S.C., Shao, J., Wang, H., and Lohknygina, Y. (2018) pages 32-33.) |
| D | The expected number of dropouts. $D = N' - N$. |

Dropout Summary Statements

Anticipating a 20% dropout rate, 25 subjects should be enrolled to obtain a final sample size of 20 subjects.

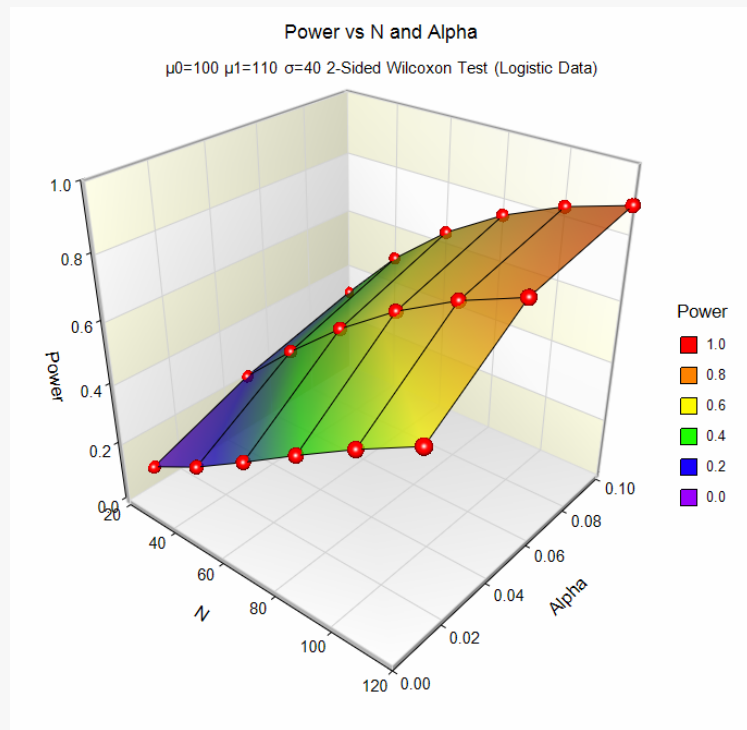
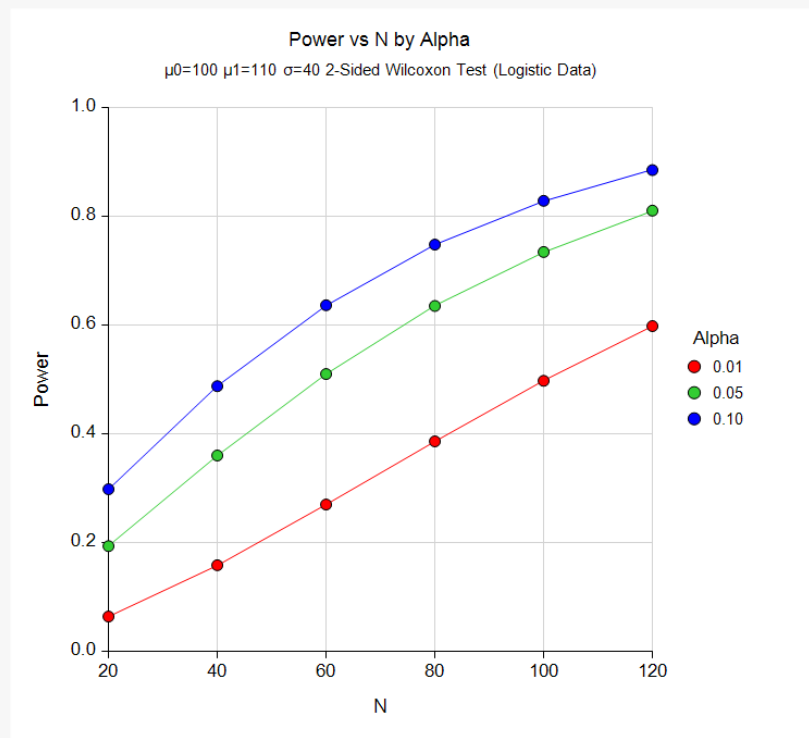
References

- Al-Sundugchi, Mahdi S. 1990. Determining the Appropriate Sample Size for Inferences Based on the Wilcoxon Statistics. Ph.D. dissertation under the direction of William C. Guenther, Dept. of Statistics, University of Wyoming, Laramie, Wyoming.
- Chow, S.C., Shao, J., Wang, H., and Lohknygina, Y. 2018. Sample Size Calculations in Clinical Research, Third Edition. Taylor & Francis/CRC. Boca Raton, Florida.
- Machin, D., Campbell, M., Fayers, P., and Pinol, A. 1997. Sample Size Tables for Clinical Studies, 2nd Edition. Blackwell Science. Malden, MA.
- Zar, Jerrold H. 1984. Biostatistical Analysis (Second Edition). Prentice-Hall. Englewood Cliffs, New Jersey.

This report shows the values of each of the parameters, one scenario per row. The values of power and beta were calculated from the other parameters.

Plots Section

Plots



These plots show the relationship between sample size and power for various values of alpha.

Example 2 – Finding the Sample Size

This example will consider the situation in which you are planning a study that will use the Wilcoxon Signed-Rank test and want to determine an appropriate sample size. This example is more subjective than the first because you now have to obtain estimates of all the parameters. In the first example, these estimates were provided by the data.

In studying deaths from SIDS (Sudden Infant Death Syndrome), one hypothesis put forward is that infants dying of SIDS weigh less than normal at birth. Suppose the average birth weight of infants is 3300 grams with a standard deviation of 663 grams. Use an alpha of 0.05 and power of both 0.80 and 0.90. How large a sample of SIDS infants will be needed to detect a drop in average weight of 25%? Of 10%? Of 5%? Note that applying these percentages to the average weight of 3300 yields 2475, 2970, and 3135.

Although a one-sided hypothesis is being considered, sample size estimates will assume a two-sided alternative to keep the research design in line with other studies. The assumed data distribution will be normal.

Setup

If the procedure window is not already open, use the PASS Home window to open it. The parameters for this example are listed below and are stored in the **Example 2** settings file. To load these settings to the procedure window, click **Open Example Settings File** in the Help Center or File menu.

Design Tab

| | |
|---------------------------------------|--|
| Solve For | Sample Size |
| Alternative Hypothesis | Two-Sided (H1: $\mu \neq \mu_0$) |
| Data Distribution | Normal |
| Population Size | Infinite |
| Power..... | 0.80 0.90 |
| Alpha..... | 0.05 |
| μ_0 (Null or Baseline Mean) | 3300 |
| μ_1 (Actual Mean) | 2475 2970 3135 |
| σ (Standard Deviation)..... | 663 |

Wilcoxon Signed-Rank Tests

Output

Click the Calculate button to perform the calculations and generate the following output.

Numeric Results

Solve For: [Sample Size](#)
 Hypotheses: $H_0: \mu = \mu_0$ vs. $H_1: \mu \neq \mu_0$
 Data Distribution: Normal

| Power | Sample Size N | Mean | | | Standard Deviation σ | Effect Size | Alpha |
|---------|---------------------|-----------------|-------------------|-------------------------------|-----------------------------------|----------------|-------|
| | | Null μ_0 | Actual μ_1 | Difference $\mu_1 - \mu_0$ | | | |
| 0.85339 | 9 | 3300 | 2475 | -825 | 663 | 1.244 | 0.05 |
| 0.90307 | 10 | 3300 | 2475 | -825 | 663 | 1.244 | 0.05 |
| 0.80426 | 36 | 3300 | 2970 | -330 | 663 | 0.498 | 0.05 |
| 0.90409 | 48 | 3300 | 2970 | -330 | 663 | 0.498 | 0.05 |
| 0.80105 | 136 | 3300 | 3135 | -165 | 663 | 0.249 | 0.05 |
| 0.90070 | 181 | 3300 | 3135 | -165 | 663 | 0.249 | 0.05 |

This report shows the values of each of the parameters, one scenario per row. Since there were three values of μ_1 and two values of power, there are a total of six rows in the report.

We were solving for the sample size, N . Notice that the increase in sample size seems to be most directly related to the difference between the two means. The difference in beta values does not seem to be as influential, especially at the smaller sample sizes.

Note that even though we set the power values at 0.8 and 0.9, these are not the power values that were achieved. This happens because N can only take on integer values. The program selects the first value of N that gives at least the values of alpha and power that were desired.

Example 3 – Finding the Minimum Detectable Difference

This example will consider the situation in which you want to determine how small of a difference between the two means can be detected by the Wilcoxon Signed-Rank test with specified values of the other parameters.

Continuing with the previous example, suppose about 50 SIDS deaths occur in a particular area per year. Using 50 as the sample size, 0.05 as alpha, and 0.80 as power, how large of a difference between the means is detectable?

Setup

If the procedure window is not already open, use the PASS Home window to open it. The parameters for this example are listed below and are stored in the **Example 3** settings file. To load these settings to the procedure window, click **Open Example Settings File** in the Help Center or File menu.

Design Tab

Solve For μ_1 (Search < μ_0)
 Alternative Hypothesis Two-Sided ($H_1: \mu \neq \mu_0$)
 Data Distribution Normal
 Population Size Infinite
 Power 0.80
 Alpha 0.05
 N (Sample Size) 50
 μ_0 (Null or Baseline Mean) 3300
 σ (Standard Deviation) 663

Output

Click the Calculate button to perform the calculations and generate the following output.

Numeric Results

Solve For: μ_1 (Search < μ_0)
 Hypotheses: $H_0: \mu = \mu_0$ vs. $H_1: \mu \neq \mu_0$
 Data Distribution: Normal

| Power | Sample Size N | Mean | | | Standard Deviation σ | Effect Size | Alpha |
|-------|---------------|--------------|----------------|----------------------------|-----------------------------|-------------|-------|
| | | Null μ_0 | Actual μ_1 | Difference $\mu_1 - \mu_0$ | | | |
| 0.8 | 50 | 3300 | 3023.2 | -276.8 | 663 | 0.417 | 0.05 |

With a sample of 50, a difference of $3023.2 - 3300 = -276.8$ would be detectable.

Example 4 – Validation using Chow, Shao, Wang, and Lokhnygina (2018)

Chow, Shao, Wang, and Lokhnygina (2018) presents an example on pages 45 and 46 of a two-sided one-sample *t*-test sample size calculation in which $\mu_0 = 1.5$, $\mu_1 = 2.0$, $\sigma = 1.0$, $\alpha = 0.05$, and $\text{power} = 0.80$. They obtain a sample size of 34.

The Wilcoxon Signed-Rank test power calculations are the same as the one-sample *t*-test except for an adjustment factor for the assumed data distribution. If we assume a uniform data distribution, we should get this same result. If we set the data distribution to Normal, we should get a result of $N = 34 \times \pi/3 = 35.605 = 36$.

Setup

If the procedure window is not already open, use the PASS Home window to open it. The parameters for this example are listed below and are stored in the **Example 4a** settings file. To load these settings to the procedure window, click **Open Example Settings File** in the Help Center or File menu.

Design Tab

Solve For **Sample Size**
 Alternative Hypothesis **Two-Sided (H1: $\mu \neq \mu_0$)**
 Data Distribution **Uniform**
 Population Size **Infinite**
 Power..... **0.80**
 Alpha..... **0.05**
 μ_0 (Null or Baseline Mean) **1.5**
 μ_1 (Actual Mean) **2**
 σ (Standard Deviation)..... **1**

Output

Click the Calculate button to perform the calculations and generate the following output.

Numeric Results

Solve For: **Sample Size**
 Hypotheses: $H_0: \mu = \mu_0$ vs. $H_1: \mu \neq \mu_0$
 Data Distribution: Uniform

| Power | Sample Size N | Mean | | | Standard Deviation σ | Effect Size | Alpha |
|---------|------------------|-----------------|-------------------|-------------------------------|--------------------------------|-------------|-------|
| | | Null μ_0 | Actual μ_1 | Difference $\mu_1 - \mu_0$ | | | |
| 0.80778 | 34 | 1.5 | 2 | 0.5 | 1 | 0.5 | 0.05 |

The sample size of 34 matches Chow, Shao, Wang, and Lokhnygina (2018) exactly.

Wilcoxon Signed-Rank Tests

If we now assume a Normal data distribution (**Example 4b** settings file), the results match our expected outcome.

Numeric Results

Solve For: [Sample Size](#)
 Hypotheses: $H_0: \mu = \mu_0$ vs. $H_1: \mu \neq \mu_0$
 Data Distribution: Normal

| Power | Sample Size N | Mean | | | Standard Deviation σ | Effect Size | Alpha |
|---------|---------------------|-----------------|-------------------|-------------------------------|-----------------------------------|----------------|-------|
| | | Null μ_0 | Actual μ_1 | Difference $\mu_1 - \mu_0$ | | | |
| 0.80778 | 36 | 1.5 | 2 | 0.5 | 1 | 0.5 | 0.05 |

The sample size of 36 matches the expected result.

Example 5 – Validation using Zar (1984)

Zar (1984) pages 111-112 presents an example in which $\mu_0 = 0.0$, $\mu_1 = 1.0$, $\sigma = 1.25$, $\alpha = 0.05$, and $N = 12$. Zar obtains an approximate power of 0.72 for the one sample t -test.

The Wilcoxon Signed-Rank test power calculations are the same as the one-sample t -test except for an adjustment factor for the assumed data distribution. If we assume a uniform data distribution, we should get the same power with $N = 12$. If we assume a Double Exponential data distribution with $N = 12 \times 2/3 = 8$, then the adjusted sample size would be $N' = 8/(2/3) = 12$ and we should get the same power as Zar (1984).

Setup

If the procedure window is not already open, use the PASS Home window to open it. The parameters for this example are listed below and are stored in the **Example 5a** settings file. To load these settings to the procedure window, click **Open Example Settings File** in the Help Center or File menu.

Design Tab

Solve For **Power**
 Alternative Hypothesis **Two-Sided (H1: $\mu \neq \mu_0$)**
 Data Distribution **Uniform**
 Population Size **Infinite**
 Alpha **0.05**
 N (Sample Size) **12**
 μ_0 (Null or Baseline Mean) **0**
 μ_1 (Actual Mean) **1**
 σ (Standard Deviation) **1.25**

Output

Click the Calculate button to perform the calculations and generate the following output.

Numeric Results

Solve For: **Power**
 Hypotheses: $H_0: \mu = \mu_0$ vs. $H_1: \mu \neq \mu_0$
 Data Distribution: Uniform

| Power | Sample Size N | Mean | | | Standard Deviation σ | Effect Size | Alpha |
|---------|---------------|--------------|----------------|----------------------------|-----------------------------|-------------|-------|
| | | Null μ_0 | Actual μ_1 | Difference $\mu_1 - \mu_0$ | | | |
| 0.71366 | 12 | 0 | 1 | 1 | 1.25 | 0.8 | 0.05 |

The difference between the power computed by **PASS** of 0.71366 and the 0.72 computed by Zar is due to Zar's use of an approximation to the noncentral t distribution. Notice that the results for the Wilcoxon signed-rank test with the uniform distribution match the one-sample t -test results.

Wilcoxon Signed-Rank Tests

If we now assume a double exponential data distribution and set $N = 8$ (**Example 5b** settings file), the results match our expected outcome.

Numeric Results

Solve For: **Power**
 Hypotheses: $H_0: \mu = \mu_0$ vs. $H_1: \mu \neq \mu_0$
 Data Distribution: Double Exponential

| Power | Sample Size N | Mean | | | Standard Deviation σ | Effect Size | Alpha |
|---------|---------------------|-----------------|-------------------|-------------------------------|-----------------------------------|----------------|-------|
| | | Null μ_0 | Actual μ_1 | Difference $\mu_1 - \mu_0$ | | | |
| 0.71366 | 8 | 0 | 1 | 1 | 1.25 | 0.8 | 0.05 |

The power of 0.71366 matches the expected result.

Example 6 – Validation using Machin (1997)

Machin, Campbell, Fayers, and Pinol (1997) page 37 presents an example in which $\mu_0 = 0.0$, $\mu_1 = 0.2$, $\sigma = 1.0$, $\alpha = 0.05$, and $\beta = 0.20$. They obtain a sample size of 199.

As in Example 4, if we set the data distribution to Uniform, we should get this same result. If we set the data distribution to double exponential, we should get a result of $N = 199 \times 2/3 = 132.6667 = 133$.

Setup

If the procedure window is not already open, use the PASS Home window to open it. The parameters for this example are listed below and are stored in the **Example 6a** settings file. To load these settings to the procedure window, click **Open Example Settings File** in the Help Center or File menu.

Design Tab

Solve For **Sample Size**
 Alternative Hypothesis **Two-Sided (H1: $\mu \neq \mu_0$)**
 Data Distribution **Uniform**
 Population Size **Infinite**
 Power **0.80**
 Alpha **0.05**
 μ_0 (Null or Baseline Mean) **0**
 μ_1 (Actual Mean) **0.2**
 σ (Standard Deviation) **1**

Output

Click the Calculate button to perform the calculations and generate the following output.

Numeric Results

Solve For: [Sample Size](#)
 Hypotheses: $H_0: \mu = \mu_0$ vs. $H_1: \mu \neq \mu_0$
 Data Distribution: Uniform

| Power | Sample Size N | Mean | | | Standard Deviation σ | Effect Size | Alpha |
|---------|---------------------|-----------------|-------------------|-------------------------------|-----------------------------------|----------------|-------|
| | | Null μ_0 | Actual μ_1 | Difference $\mu_1 - \mu_0$ | | | |
| 0.80169 | 199 | 0 | 0.2 | 0.2 | 1 | 0.2 | 0.05 |

The sample size of 199 matches Machin's result.

Wilcoxon Signed-Rank Tests

If we now assume a double exponential data distribution (**Example 6b** settings file), the results match our expected outcome.

Numeric Results

Solve For: [Sample Size](#)
 Hypotheses: $H_0: \mu = \mu_0$ vs. $H_1: \mu \neq \mu_0$
 Data Distribution: Double Exponential

| Power | Sample Size N | Mean | | | Standard Deviation σ | Effect Size | Alpha |
|---------|---------------------|-----------------|-------------------|-------------------------------|-----------------------------------|----------------|-------|
| | | Null μ_0 | Actual μ_1 | Difference $\mu_1 - \mu_0$ | | | |
| 0.80169 | 133 | 0 | 0.2 | 0.2 | 1 | 0.2 | 0.05 |

The sample size of 133 matches the expected result.